Optimal monetary and fiscal policies in a heterogeneous country-size monetary union

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Abstract

In the sequence of the 2008-09 financial and economic crisis, public debt has increased considerably in many countries of the European and Monetary Union. A high level of government indebtedness may enlarge stabilization costs and influence their uneven distribution among small and large union country members.

Extending a standard New Keynesian open-economy model to a monetary union where atomistic economies coexist with a large country, we explore how the level of government indebtedness shapes full-optimal discretionary policies and stabilization outcomes, under different policy regimes.

Numerical results show that, in general, higher public debt levels hamper business cycle stabilization for the union as a whole and are more likely to penalize the stabilization performance of small country members. In this case, mechanisms to enforce cooperation are clearly recommended, not only because, under a high debt environment, union-wide welfare costs become meaningful if non-cooperation prevails, but also because political support for cooperation may be hard to achieve. Indeed, while cooperation and monetary leadership is preferable to fiscal leadership for the union as whole and for the small countries, the big country clearly prefers fiscal leadership, where it can explore a larger strategic power vis-a-vis the common monetary policy authority. In turn, under low debt levels, cooperative stabilization outcomes are relatively similar to the non-cooperative ones.

Keywords: Monetary union; Optimal monetary and fiscal rules; Asymmetric-size countries; Debt levels.
JEL Class.: E52; E61; E62; E63.

1 Introduction

In the sequence of the latest financial and economic crisis, public debt has increased considerably in many European and Monetary Union (EMU) countries. A higher level of government indebtedness may enlarge the budgetary consequences of the shocks and further constraint fiscal policy on business cycle stabilization. In addition, as Leith and Wren-Lewis (2013) showed, it boosts the effectiveness of monetary policy on debt-stabilization while it reduces that of fiscal policy. In turn, the time-consistency requirement under lack of commitment determines that permanent effects of shocks are fully eliminated and that debt

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returns to its (efficient) steady-state level, under optimal discretionary policies. Therefore, the optimal stabilization policy-mix that emerges under discretion crucially depends on the level of public debt. Additionally, there could be efficiency stabilization gains, resulting from the increasing competence of monetary policy on debt-stabilization and of fiscal policy on short-run stabilization, as the debt-to-output ratios increase. If these gains outweigh the costs of stabilizing larger budgetary consequences, it is possible that welfare stabilization costs could evolve non-monotonically with debt under discretion, as found by Blake and Kirsanova (2011).

In turn, in a monetary union such as the EMU, a common monetary policy coexists with decentralized fiscal policies and business cycle stabilization could be seriously hampered by strategic interactions between non-coordinated policies. Such strategic policy interactions are shaped by the level of government indebtedness and the relative size of the countries that belong to the monetary union, as the policy spillovers of large and small countries are of different magnitude. A small, rather open, country is more likely to suffer to a larger extent the effects of country-specific shocks and, thus, to experience a worse stabilization performance than a large, rather closed, one. Furthermore, as the policy spillovers of a very small country are negligible, incentives to deviate from cooperation are smaller than that for a big country.

In this paper we intend (i) to assess how the level of public debt shapes full optimal discretionary policies in a heterogeneous country-size monetary union and affects macroeconomic stabilization performance in cooperative and in alternative non-cooperative regimes (simultaneous-move, monetary leadership and fiscal leadership); and (ii) to appraise the implications for very small (atomistic size) and large country-members.

To address these issues we use a multi-country DSGE model of a monetary union, with monopolistic competition and sticky prices and where fiscal policy is allowed to have demand and supply-side effects, as in Leith and Wren-Lewis (2013), under different debt scenarios. In turn, while maximizing, respectively, the union-wide welfare and their national counterparts, the monetary authority and the fiscal authorities are assumed to engage in discretionary policy games. Most of the existing literature on optimal policies in a monetary union relies either in a two-country model (e.g., Beetsma and Jensen, 2004, 2005 and Ferrero, 2009) or in a multi-country model where the union is made up of a continuum of small open economies (e.g., Gali and Monacelli, 2008 and Leith and Wren-Lewis, 2011). This allows for the analysis of policy interactions between economies where domestic shocks and policy decisions have either significant impact or negligible impact on other member states and on the union. We intend to fill a gap in the literature, with a multi-country model that allows the analysis of monetary and fiscal policy interactions in a more realistic monetary union environment, where fiscal authorities of a large country and very small countries coexist. Our model provides a more
general framework that also matches the cases of a monetary-union made up by two large economies and by a continuum of small economies.

Several works have addressed how policy interactions and stabilization outcomes in a monetary union are shaped by country-size asymmetry (e.g., Canzoneri et al., 2005, and Mykhaylova, 2011) and by the government debt (e.g., van Aarle et al., 2004, Kirsanova et al., 2007, Argentiero, 2009, Ferrero, 2009, Leith and Wren-Lewis, 2007, 2011, Blueschke and Neck, 2011, Pappa, 2012, and Vogel et al., 2013). For instance, Canzoneri et al. (2005) show that a common monetary policy, responding to union-wide inflation, produces asymmetric effects on countries within the union, depending on whether they are large or small, or whether they have high or low levels of government indebtedness. They found that the (non-optimal) monetary policy favors larger countries in the Euro Area and that high debt levels lead to welfare costs.

As regards strategic interactions between different policy authorities, an important branch of this literature has considered the case of non-cooperation but still a scant part uses dynamic models, which are more appropriate to analyze the role of public debt in policy interactions. In this spirit, van Aarle et al. (2002), Beetsma and Jensen (2005) and Forlati (2009), for instance, analyze non-cooperative monetary and fiscal policies under Nash, Kirsanova et al. (2005) and Orjasniemi (2014) examine the case of monetary leadership, while Machado and Ribeiro (2010, 2011), Blueschke and Neck (2011) and Adam and Billi (2008, 2014) consider both Nash and leadership solutions. The issue of the desirability of policy cooperation is naturally a related one. For instance, van Aarle et al. (2002) argue that EMU increases the need for macroeconomic policy cooperation.

Our results show that government debt levels crucially shape the need for adopting policy cooperation in a monetary union. While under low debt levels, cooperative outcomes are relatively similar to the non-cooperative ones, under a high debt environment, union-wide welfare costs become meaningful if non-cooperation prevails. In this case, mechanisms to enforce cooperation are clearly recommended, once the big country may strongly oppose to it. A highly indebted big country clearly prefers fiscal leadership, where it can explore a larger strategic power vis-a-vis the common monetary policy authority; this regime, however, imposes substantial welfare costs on the small countries and on the union as a whole.

The work is organized as follows. Section 2 describes the theoretical currency union model. We derive the structural equations of the mode, the social loss function and discuss the policy games considered in the analysis. In Section 3, we study the design, the performance and the

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2 See Beetsma and Giulidori (2010) for a recent survey on this.
implications of fully optimal policies across different public debt levels and policy regimes, appraising the consequences for a very small and a large country. Section 4 concludes.

2 A Currency Union Model

We model the currency union as a closed system, made up of two blocks of countries, populated by a continuum of agents \( \in [0, 1] \). One of these blocks is a big country, indexed by \( B \), with a relative size of \((1-n)\), \( n \in [0, 1] \). The other block, indexed by \( S \), has dimension \( n \) and is made up of a continuum of small countries, each of them of measure zero, indexed by \( s \in [0, n] \).

Having identical economic structures, each country has a separate fiscal authority, but they are subjected to a common monetary policy. The big country (\( B \)) is made up of a continuum of small geographic units, indexed by \( b \), on the interval \([n,1]\). In terms of population (households and firms), each one of these \( b \) geographic units is equivalent to a small country, but, differently from the latter, they are subjected to the same shocks and share the same fiscal authority.\(^3\)

Households and firms in each geographic unit (\( s \) or \( b \)) are indexed by \( h \) on the interval \([0, 1]\). Firms constitute a monopolistic competitive sector that produces a continuum of differentiated final goods, with price stickiness. With regard to factor markets, labor is the only input of production and it is immobile across countries. Firms are wage-taker in segmented labor markets. Each household supplies a differentiated labor input, specializing in the production of a specific final good (indexed by \( h \)). Wages are perfectly flexible and settled by workers of each type of labor. We consider a cashless economy as in Woodford (2003, Chapter 2).

2.1 Households

Each country is inhabited by an infinitely-lived representative household \( h \) seeking to maximize lifetime utility \( U_0(h) \). We assume full asset markets, such that, through risk sharing, all the households inhabiting a given country face the same budget constraint and make the same consumption plans. The representative household \( h \) inhabiting a small country in the \( S \) block (say, country \( i \in [0,n] \)), seeks to maximize

\[
U_0^i(h) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ u(C_t^i) + V(G_t^i) - v(L_t^i(h)) \right] \right\},
\]

where:

\[
u(C_t^i) = \frac{\sigma}{\sigma - 1} \left( C_t^i \right)^{\frac{\sigma - 1}{\sigma}},
\]

\[V(G_t^i) = \psi_0 \left( G_t^i \right)^{\frac{\psi - 1}{\psi}}, \psi_0 \geq 0,
\]

\(^3\) This is similar to Forlati (2015) model’s strategy, but, differently from us, she assumes that all regions are very small countries.
\[ v\left( L_t^i(h) \right) = \chi_0 \frac{1}{1+\chi} \left( L_t^i(h) \right)^{1+\chi}, \chi_0 > 0. \]

The utility function is additively separable and \( C_t^i, G_t^i \) and \( L_t^i(h) \) denote, respectively, real private consumption, real per capita public consumption and hours of work.

Parameter \( \beta \) stands for the intertemporal discount factor, \( \chi \) is the inverse of the Frisch elasticity of labor supply, \( \sigma \) and \( \psi \) are, respectively, the intertemporal elasticity of substitution of private and public consumption.

Similarly, for the representative household \( h \) living in geographic unit \( b \) of the big country \( B \):

\[ U_0^{B,b}(h) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ u(C_t^B) + V(G_t^B) - v(L_t^B(h)) \right] \right\}, \forall b \in B. \tag{2} \]

\( C_t^B, G_t^B \) and \( G_t^B \) are composite consumption indexes, described below.

### 2.1.1 Consumption

From the viewpoint of a representative household inhabiting small country \( i \in S \), \( C_t^i \) is an Dixit-Stiglitz (1977) aggregator of home and foreign goods, defined as

\[ C_t^i \equiv \left[ \lambda_5 \gamma(C_{i,t}^i)^{\frac{\gamma-1}{\gamma}} + (1 - \lambda_5) \gamma(C_{i,t}^i)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma}}, \text{where:} \]

\[ C_{i,t}^i \equiv \left( \int_0^1 \left[ c_{i,t}^i(h) \right]^{\frac{\gamma-1}{\gamma}} \frac{\epsilon}{1-\epsilon} dh \right)^{\frac{\epsilon}{\gamma-1}}, \]

\[ C_{i,t}^i \equiv \left[ (1 - n) \gamma(C_{B,t}^i)^{\frac{\gamma-1}{\gamma}} + (n) \gamma(C_{S,t}^i)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma}}, \text{with:} \]

\[ C_{B,t}^i \equiv \left( \int_{1-n}^{\frac{1}{n}} \frac{\epsilon}{\gamma-1} \left[ c_{B,t}^i \right]^{\frac{\gamma-1}{\gamma}} db \right)^{\frac{\epsilon}{\gamma-1}}; C_{B,t}^i \equiv \left( \int_0^1 \left[ c_{B,t}^i(h) \right]^{\frac{\gamma-1}{\gamma}} \frac{\epsilon}{1-\epsilon} dh \right)^{\frac{\epsilon}{\gamma-1}}, \]

\[ C_{S,t}^i \equiv \left( \int \frac{\epsilon}{\gamma-1} \left[ c_{S,t}^i \right]^{\frac{\gamma-1}{\gamma}} ds \right)^{\frac{\epsilon}{\gamma-1}}; C_{S,t}^i \equiv \left( \int_0^1 \left[ c_{S,t}^i(h) \right]^{\frac{\gamma-1}{\gamma}} \frac{\epsilon}{1-\epsilon} dh \right)^{\frac{\epsilon}{\gamma-1}}. \]

\( \gamma > 0 \) is the elasticity of substitution between domestic and foreign goods, \( C_{i,t}^i \) is a composite consumption index of domestic goods and \( C_{i,t}^i \) is a composite consumption index of imported goods (we use "-i" to denote "not i"). Following Benigno and De Paoli (2010) and Gali and Monacelli (2008), \( (1 - \lambda_5) = (1-0)\alpha = \alpha \) is the imported share, which is function of the relative size of the small economy (zero, in this case) and the degree of trade openness \( \alpha \in [0,1] \). \( C_t^i(h) \) is the quantity of domestic good \( h \) consumed by country \( i \)'s representative household, and \( \epsilon > 1 \) is the elasticity of substitution between goods produced within a given country. \( C_{B,t}^i \) and \( C_{S,t}^i \) are, respectively, composite indexes of imported goods from blocks \( B \) and \( S \). In turn, \( C_{B,t}^i \) and
$C_{s,t}^i$ are, respectively, composite indexes of imported goods from geographic units $b \in B$ and $s \in S$.

To a representative household inhabiting country $B$, independently of the geographic unit where she or he lives, $C_{t}^B$ is defined as

$$C_{t}^B \equiv \left[ \frac{1}{\lambda_B} \left( C_{b,t}^B \right)^{\frac{\gamma}{1 - \gamma}} + \left( 1 - \lambda_B \right) \left( C_{s,t}^B \right)^{\frac{\gamma}{1 - \gamma}} \right]^{\frac{1}{\gamma}},$$

where $\lambda_B = n \alpha$ is the share of imported goods from block $S$. $C_{b,t}^B$ is a composite consumption index of domestic goods while $C_{s,t}^B$ is a composite index of domestic goods produced in geographic unit $b \in B$, defined in the same way as $C_{b,t}^i$. $C_{s,t}^B$ is a composite consumption index of imported goods whereas $C_{s,t}^B$ is the specific composite index of imported goods from country $s \in S$, defined in the same way as $C_{s,t}^i$.

2.1.2 Prices

The consumer price index (CPI) for a small economy $i \in S$ is given by

$$P_{c,t}^i \equiv \left[ \lambda_S \left( P_t^i \right)^{1-\gamma} + \left( 1 - \lambda_S \right) \left( P_t^{-i} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}},$$

where

$$P_t^i \equiv \left( \int_0^1 [P_t^i(h)]^{1-\varepsilon} \, dh \right)^{\frac{1}{1-\varepsilon}},$$

$$P_t^{-i} \equiv \left( \int_0^1 [P_t^{-i}(h)]^{1-\varepsilon} \, dh \right)^{\frac{1}{1-\varepsilon}},$$

with

$$P_t^B \equiv \left( \int_n^1 [P_t^B(h)]^{1-\varepsilon} \, dh \right)^{\frac{1}{1-\varepsilon}}; P_t^S \equiv \left( \int_0^n [P_t^S(h)]^{1-\varepsilon} \, dh \right)^{\frac{1}{1-\varepsilon}},$$

where $P_t^i$ is the aggregate price index for goods produced in country $i$, and $P_t^{-i}$ is the aggregate price index for foreign (and union as whole) products. The law of one price holds but, given the home biased preferences, the purchasing power parity does not hold for aggregate consumer price indexes. $P_t^i(h)$ is the price of country $i$’s home-produced good $h$; $P_t^B$ and $P_t^S$ are, respectively, aggregate price indexes for the bundle of goods imported from blocks $B$ and $S$. In turn, $P_t^B$ and $P_t^S$ are, respectively, aggregate price indexes for the bundle of goods imported from geographic units $b \in B$ and $s \in S$.

Similarly, $P_{c,t}^B$ is given by
\[ P^B_{C,t} \equiv \left[ \lambda_B (P^B_t)^{1-\gamma} + (1-\lambda_B) \frac{1}{\gamma} \int_0^\infty [P^x_s]^{1-\gamma} ds \right]^{1/\gamma}, \forall b \in B. \]  

Notice that the aggregate price index for goods produced within a geographic unit of country \( B \) is the same across all geographic units \((P^B_t = P^B_{C,t}, \forall b \in B)\). Combined with identical preferences and the law of one price, the aggregate consumer price index is also the same: \( P^B_{C,t} = P^B_{C,t}, \forall b \in B \).

### 2.1.3 Budget Constraints

Period budget constraints faced by representative households living in geographic units \( i \in S \) and \( b \in B \) are defined, respectively, as:

\[ P^B_{C,t} C^i_t + E_t \{ Q_{t,t+1} D^i_{t+1} \} \leq D^i_t + W^i_t(h) L^i_t(h) + \Gamma^i_t - T^i_t, \quad (7) \]

\[ P^B_{C,t} C^B + E_t \{ Q_{t,t+1} D^B_{t+1} \} \leq D^B_t + W^B_t(h) L^B_t(h) + \Gamma^B_t - T^B_t, \quad \forall b \in B. \]

\( W^i_t(h) \) represents the nominal wage in period \( t \), \( D^i_{t+1} \) is the nominal payoff in period \( t+1 \) of a portfolio of state-contingent securities held at the end of period \( t \), \( \Gamma^i_t \) stands for after-tax nominal profits from ownership of the firms, and \( Q_{t,t+1} \) denotes the stochastic discount factor for one-period ahead nominal payoffs, common across countries. It is assumed that households have access to a complete set of state-contingent securities that span all possible states of nature and are traded across the union. \( T^i_t \) denotes per capita lump sum taxes in country \( i \).

### 2.1.4 Labor Supply, Wage Setting and Optimal Consumption

The representative household \( h \) inhabiting a small country \( i \in S \), maximizes lifetime utility (1) with respect to \( C^i_t, D^i_{t+1} \) and \( L^i_t(h) \), subject to (7). In turn, the representative household \( h \), inhabiting geographic unit \( b \in B \), maximizes lifetime utility (2) with respect to \( C^B_t, D^B_{t+1} \) and \( L^B_t(h) \), subject to (8). The first-order conditions are, respectively,

\[ \frac{W^i_t(h)}{p^i_{C,t}} = \left( 1 + \mu^i_{w,t} \right) \frac{u_{c_t}(c^i_t)}{u_{c_t}(c^i_t)} \]

\[ \frac{W^B_t(h)}{p^B_{C,t}} = \left( 1 + \mu^B_{w,t} \right) \frac{u_{c_t}(c^B_t)}{u_{c_t}(c^B_t)}, \quad \forall b \in B, \]

\( Q_{t,t+1} = \beta \left[ \frac{u_{c_{t+1}}(c^j_{t+1})}{u_{c_t}(c^j_t)} \right] \left( \frac{p^j_{C,t}}{p^j_{C,t+1}} \right), j = B, s(including \ i) \in S. \)

which are assumed to hold for all periods and states of nature (at \( t \) and \( t+1 \), in the case of equation (9b)). Under the assumption of complete financial markets, equation (9b) will hold for the representative household in any country \( j \). Labor markets are characterized by an exogenous country-specific wage markup. \( \mu^i_{w,t} > 0 \) is the net wage markup, capturing monopolistic
distortions in input supply. Following Clarida, Galí and Gertler (2002), we allow for exogenous variation in the wage markup in order to allow for cost-push shocks.

Let $R_t^i$ denote the gross nominal yield on a riskless one-period discount bond. Then by taking the expectations of each side of equation (9b), we obtain the following conventional stochastic Euler equation:

$$1 = \beta R_t^i E_t \left( \frac{c_{t+1}^i}{c_t^i} \right) = \frac{\delta^i}{\sigma} \frac{p_{t+1}^i}{p_{t+1}^i} \left( \frac{c_{t+1}^i}{c_t^i} \right),$$

(10)

∀j∈B, s∈S, where $(R_t^s)^{-1} = E_t \{Q_{t,t+1} \}$ is the price of the riskless one-period discount bond. For future reference, a superscript "star" will be used to denote union-wide variables.

### 2.1.5 International Risk Sharing

Combining the Euler equations for each country under the assumption of complete financial markets, we obtain the following international risk sharing condition: $(C_t^i)^{\frac{1}{\delta}} = \delta_s (C_t^i)^{\frac{1}{\delta}} = \delta_B (C_t^i)^{\frac{1}{\delta}}$, ∀i,s∈S, all t, and where $\delta_s$ and $\delta_B$ are constants, depending on initial conditions. Without loss of generality, we assume symmetric initial conditions with $\delta_s = \delta_B = \delta = 1$. After log-linearizing the international risk sharing condition and integrating over all households, we get

$$c_t^i = c_t^s + \sigma(1 - \alpha) tt_t^i,$$

(11a)

$$c_t^B = c_t^s + \sigma(1 - \alpha) tt_t^B,$$

(11b)

where lowercase letters denote (natural) logs of the corresponding variables. $tt_t^i$ and $tt_t^B$ are the effective terms of trade for country $i \in S$ and for country $B$, respectively. Following Galí (2008, p. 155), the effective terms of trade can be approximated up to a first-order log-linear approximation around a symmetric (zero-inflation) steady state by:

$$tt_t^i = p_t^{-i} - p_t^i = p_t^s - p_t^i = (1 - n) tt_{\delta, t} + \int_0^n tt_{s, t} ds$$

and

$$tt_t^B = p_t^s - p_t^B = \frac{1}{n} \int_0^n tt_{s, t} ds. \quad \text{(4)}$$

Making use of equations (5) and (6), $p_{t+1}^i = (1 - \lambda_s) tt_t^i + p_t^i$ and $p_{t+1}^B = (1 - \lambda_B) tt_t^B + p_t^B$.

We can also establish a relation between domestic production inflation and the CPI inflation as:

$$\pi_{t+1}^i = \pi_t^i + (1 - \lambda_s) \Delta tt_t^i \text{ and } \pi_{t+1}^B = \pi_t^B + (1 - \lambda_B) \Delta tt_t^B.$$

We define bilateral terms of trade between countries $x_1$ and $x_2$ as the price of country $x_2$’s goods in terms of country $x_1$’s goods, i.e., $TT_{x_2,x_1} = \frac{p_t^2}{p_t^1}$. 

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2.1.6 Aggregate Demand

In each country, the demand for home-produced good \( h \) is the sum of three components: the demands of domestic and foreign households (private consumption) and government (public consumption). For simplicity, it is assumed that government expenditures are fully allocated to domestically produced goods:

\[
G^i_t \equiv \left( \int_0^1 \left[ G^i_{t,1}(h) \right] \frac{\epsilon-1}{\epsilon} \, dh \right)^{\frac{\epsilon}{\epsilon-1}},
\]

\[
G^B_t \equiv \left( \frac{1}{1-n} \int_1^n \left[ G^B_{t,1}(h) \right] \frac{\epsilon-1}{\epsilon} \, db \right)^{\frac{\epsilon}{\epsilon-1}},
\]

where \( G^i_t \) and \( G^B_t \) are composite indexes representing real per capita public consumption.\(^5\)

Total demand for the generic good \( h \) produced, respectively, in geographic units \( i \in S \) and \( b \in B \) are given by

\[
Y^d_{i,t}(h) = C^i_{t,1}(h) + \int_0^1 [C^i_{t,1}(h)] \, ds + (1-n) [C^B_{t,1}(h)] + G^i_t(h),
\]

\[
Y^d_{b,t}(h) = (1-n) [C^B_{b,1}(h)] + \int_0^1 [C^B_{b,1}(h)] \, ds + G^B_t(h),
\]

\( \forall h \in [0,1], \forall b \in B \), where we take into account the identical behavior of households within each geographic unit - symmetric equilibrium, given identical preferences and initial conditions.

Aggregate demand is normalized by population size, i.e., it is expressed in per capita terms. To obtain aggregate demand for countries \( i \in S \) and \( B \), we need to aggregate over all varieties \( h \) in equations (14) and (15), using Dixit-Stiglitz aggregators.

Aggregate demand in country \( i \in S \) is given by

\[
Y^d_{i,t} \equiv \left( \int_0^1 \left[ Y^d_{i,t}(h) \right] \frac{\epsilon-1}{\epsilon} \, dh \right)^{\frac{\epsilon}{\epsilon-1}}.
\]

For country \( B \) as a whole, aggregate demand is \( Y^d_{B,t} = Y^d_{b,t}, \forall b \in B \); total aggregate demand is

\[
(1-n) Y^d_{B,t}.
\]

The demand for the generic good \( h \), produced in country \( i \in S \), is a function of relative prices and aggregate demand: \( Y^d_{i,t}(h) = \left( \frac{p^i(h)}{p_t^B} \right)^{-\epsilon} Y^d_{i,t}, \forall h \in [0,1] \). The aggregate demand in geographic unit \( b \in B \)

is

\[
Y^d_{b,t} \equiv \left( \int_0^1 \left[ Y^d_{b,t}(h) \right] \frac{\epsilon-1}{\epsilon} \, dh \right)^{\frac{\epsilon}{\epsilon-1}}.
\]

For country \( B \) as a whole, aggregate demand is \( Y^d_{B,t} = Y^d_{b,t}, \forall b \in B \); total aggregate demand is

\[
(1-n) Y^d_{B,t}.
\]

The demand for the generic good \( h \), produced in \( b \in B \), is defined as

\[
Y^d_{b,t}(h) = \left( \frac{p^B(h)}{p_t^B} \right)^{-\epsilon} Y^d_{B,t}, \forall h \in [0,1], \forall b \in B.
\]

\(^5\) \( G^i_{t,1}(h) \) is the quantity of domestic good \( h \) purchased by the government of small country \( i \) and \( G^B_{t,1} \) represents a composite index of country \( B \)'s government expenditures on goods produced in geographic unit \( h \), defined in the same way as \( G^B_{t,1} \).
2.2 Firms

Each firm located in country \( i \in S \) produces a differentiated good \( h \) with a linear technology represented by the production function \( Y_i^f(h) = A_i^fL_i^f(h), \forall h \in [0,1] \), where \( Y_i^f(h) \) denotes country \( i \)'s production of good \( h \) (firm \( h \)'s production), \( A_i^f \) represents the level of technology of country \( i \), and \( L_i^f(h) \) is the firm-specific labor input.

Independently of the geographic unit where they are located, all firms in country \( B \) have access to the same linear technology represented by \( Y_B^b(h) = A_B^bL_B^b(h), \forall h \in [0,1], \forall b \in B \), where \( A_B^b \) represents the level of country \( B \)'s technology.

Let \( A_i^f, j=B, s \in S \), including small country \( i \), represent country \( j \)'s technology. It is assumed that \( a_i^f = \log A_i^f \) follows a stationary AR(1) process: \( a_i^f = \rho_a a_{i-1}^f + \varepsilon_i^f, j = B, s \in S \), where \( \rho_a \in [0,1] \) and \( \varepsilon_i^f \) is white noise.

It is assumed that firms set prices in a staggered fashion \( a \) la Calvo (1983): \( (1-\theta) \) is the probability of a firm re pricing its price in any given period, while \( \theta \) is the probability of a firm keeping its price unchanged. Let country \( j \)'s index of price rigidity, \( \theta^j \), be defined as \( \theta_s^j \) for \( j = s, \forall s \in S \), and \( \theta_B^j \) for \( j = B \).

Formally, in each country \( j, j=B, s \in S \), a wage-taker firm able to set a new price \( P_{t+1}^j \) in period \( t \) faces the following optimality condition:

\[
\sum_{k=0}^{\infty} (\theta^j)^k E_t \left\{ Q_{t,t+k}^{oj} Y_{t+1+k}^{oj}\left(1 - \tau_{t+k}^j \right)\left[ \frac{p_{t+1+k}^j}{P_{t-1}^j} - (1 + \mu_p)MC_{t+k}^{oj} \frac{P_{t+1+k}^j}{P_{t-1}^j} \right] \right\} = 0,
\]

where \( Y_{t+k|t}^{oj} = \left( \frac{p_{t+k}^j}{P_{t+k}^j} \right)^{-e} Y_{j,t+k} \), for \( j = B, s \in S, k = 0, 1, 2, \ldots \), \( \mu_p \equiv \frac{1}{1-\varepsilon_1} > 0 \) is the optimal net price markup, \( MC_{t+k}^{oj} \equiv \frac{(1-\varepsilon_1)w_{t+k}^{oj}}{A_{t+k} \left(1-\tau_{t+k}^j \right)} \) is the real marginal cost of firms reoptimizing price in period \( t \), \( Q_{t,t+k} = \beta^k \left( \frac{c_{t+k}^j}{c_t^j} \right)^{-\frac{1}{\gamma}} \frac{p_{t+k}^j}{p_{t+k}^j} \) is the stochastic discount factor for nominal payoffs, \( \tau_{t+k}^j \) is a revenue tax and \( \zeta_{t+k}^j \) is an employment subsidy. This employment subsidy is financed by lump-sum taxes and is designed to remove monopolistic and tax rate distortions, ensuring the efficiency of the steady-state output level.\(^6\)

\( Y_{t+k|t}^{oj} \) represents the output in period \( t+k \) for a firm that last reset its price in period \( t \), \( Y_{j,t+k}^{oj} \) is the amount of labor employed, and \( Y_{j,t+k}^{oj} \) represents country \( j \)'s aggregate demand in period \( t+k \).
Taking into account the optimal condition for labor supply (9a), the production function and demand constraints, \( MC_t^{2j} \equiv MC_t^j \left( \frac{p_t^{2j}}{p_t^j} \right)^{-\epsilon X} \), where \( MC_t^j \equiv \frac{(1-\epsilon_j^d)\lambda_j^d(1+\mu_j^d)(\epsilon_j^d)^2\chi_0}{(\lambda_j^d)^2(1-\epsilon_j^d)p_t^j} (Y_t^j)^X \) is the average/aggregate real marginal cost in country \( j \), considering an index analogous to the one used in the definition of aggregate domestic prices, and that \( Y_{j,t}^d \equiv Y_t^j \) in equilibrium.\(^7\)

### 2.3 Private Sector Equilibrium

#### 2.3.1 Goods Market Clearing Conditions

Aggregating over all varieties \( h \) in equations (14) and (15) and taking into account that in equilibrium \( Y_{j,t}^d \equiv Y_t^j \), \( j = B, s \in S \), a first-order log-linear approximation around the symmetric steady state yields a similar condition for each country \( j \) (using the definitions of \( tt_t^i \) and \( tt_t^B \)):

\[
\begin{align*}
\dot{\hat{y}}_t^j &= (1 - \varphi)\hat{c}_t^j + (1 - \varphi)\Phi(p_t^j - p_t^j) + \varphi\hat{g}_t^j \iff \quad (17a) \\
\dot{\hat{y}}_t^j &= (1 - \varphi)\hat{c}_t^j + (1 - \varphi)[\sigma(1 - \alpha) + \Phi]|p_t^j - p_t^j| + \varphi\hat{g}_t^j, \quad (17b)
\end{align*}
\]

where: \( \Phi \equiv \alpha[y - (1 - \alpha)(-y + \sigma)] \).

Country \( j \)'s output is positively related to private and public consumption and inversely related to domestic prices relative to union's average prices. The symbol "\(^ \wedge \)" is used to denote the log deviation of a variable from its steady state value, e.g., \( \hat{y}_t^j = y_t^j - y^j \), and \( \varphi \equiv \frac{\bar{g}^j}{\gamma^j} \) denotes the steady-state government spending share. Aggregating (17b) over all countries, we obtain the union-wide goods market clearing condition: \( \hat{y}_t^j = (1 - \varphi)\hat{c}_t^j + \varphi\hat{g}_t^j \).\(^8\)

#### 2.3.2 Aggregate Price Dynamics

Given the assumption about firm's price-setting decisions and the evolution of the aggregate domestic price index, a New Keynesian relationship between inflation and real marginal cost aggregate over all goods can be obtained:

\[
\pi_t^j = \beta E_t \{\pi_{t+1}^j\} + \phi_j^m \hat{mc}_t^j, \quad j = B, s \in S, \quad \text{where: } \phi_j^m \equiv \frac{(1-\theta_j^B)(1-\theta_j^B)}{\theta_j(1+\epsilon X)} \quad (18)
\]

\(^6\) Following Leith & Wren-Lewis (2013), \( \zeta_j^d \), eliminates linear terms in the welfare function without losing the possibility of using the revenue tax as fiscal instrument.

\(^7\) Like aggregate demand, aggregate output of country \( j \), \( Y_t^j \), \( j = B, s \in S \), is normalized by population size.

\(^8\) Notice that: \( \hat{y}_t^j = \int_0^\infty \hat{y}_t^j ds + (1 - n)\hat{y}_t^j; \hat{c}_t^j = \int_0^\infty \hat{c}_t^j ds + (1 - n)\hat{c}_t^j; \hat{g}_t^j = \int_0^\infty \hat{g}_t^j ds + (1 - n)\hat{g}_t^j. \)
In the zero-inflation steady state, the real marginal cost faced by country \( j \)'s firms is constant, and accordingly to the optimality condition (16), \( MC^j = \frac{1}{1+\mu_J} \). After some manipulations:

\[
\hat{m}^j_{w,t} = \hat{\rho}_w^j + \left( \frac{1}{(1-\varphi)(1-\alpha)+\Phi} \right) \hat{y}^j_t - \log(1-\tau^j_t) - \frac{\varphi}{(1-\varphi)(1-\alpha)+\Phi} \hat{a}^j_t + \left( \frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)(1-\alpha)+\Phi} \right) \hat{y}^*_t - \varphi \left( \frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)(1-\alpha)+\Phi} \right) \hat{a}^*_t - (1+\chi) a^j_t. \tag{19}
\]

where \( \log(1-\tau^j_t) = \log(1-\tau^j_t) - \log(1-\tau^j_t) \), and \( \hat{a}^j_t = a^j_t \), \( j=B,s \in S \). Following Clarida, Galí and Gertler (2002), the "cost-push shock" obeys to a stationary AR(1) process:

\[ \phi^j \hat{a}^j_{w,t} = \rho^j (\phi^j \hat{a}^j_{w,t-1}) + \varepsilon^j_t. \]

### 2.4 Policy Instruments and Budget Constraints

The monetary policy instrument is the nominal interest rate \( r^*_t (= \log R^*_t) \), which is set for the whole union by the common central bank. As for fiscal policy, we assume that national governments choose the mix between government spending, revenue taxation and one-period nominal risk-free debt. Government spending and the revenue tax rate are the fiscal policy stabilization instruments, encompassing demand and supply-side effects.

Since it is assumed that lump-sum taxation is used exclusively to finance the steady state employment subsidy, the national government of country \( j=B,s \in S \), faces the following flow government budget constraints: \( D^j_{g,t} = R^*_t - D^j_{g,t-1} - P^j_t (\tau^j_t Y^j_t - G^j_t), \forall t, j=B,s \in S \), where \( D^j_{g,t} \) represents the end of period per capita issues in nominal terms of country \( j \)'s risk-free debt.

Defining \( d^j_{g,t} = \frac{R^*_t D^j_{g,t}}{P^j_t} \), which denotes the real value of country \( j \)'s debt (expressed in consumer prices) at maturity in per capita terms, and taking a first-order log-linear approximation to the symmetric steady state, we get

\[
\log \left( d^j_{g,t} \right) = \hat{r}^*_t + \frac{1}{\beta} \left\{ \log \left( d^j_{g,t-1} \right) - \pi^j_t + \frac{\varphi}{\alpha} \left( \varphi \hat{g}^j_t - \tau^j_t \hat{y}^j_t - \tau^j_t \log(\tau^j_t) \right) \right\} + a(p^*_t - 1 - \left( \frac{1}{1+\tau^j} \right) a(p^*_t - p^*_t^j).
\]

\[ j=B,s \in S, \forall t, \text{ where } R^* = \frac{1}{\beta}, d^j_{g}, \tau^j, Y^j \text{ and } G^j = \varphi Y^j \text{ are the steady-state values for the corresponding variables. In what follows, we consider variable } \hat{b}^j = \log \left( d^j_{g,t} \right) \times \left( \frac{d^j_{g}}{Y^j} \right) \text{ when referring to country } j \text{'s debt, denoting the absolute change in debt in percentage of the steady-state output, as in Kirsanova and Wren-Lewis (2012).} \]
In order to solve for the optimal policies, monetary and fiscal authorities have to take into account both the private sector behavior, obtained from optimization of (1), (2) and (16), as well as the budget constraints described above. These conditions can be written in gap form, presented in Appendix A.  

\[\text{2.5 Policy Objectives}\]

Since the social planner ignores nominal inertia and distortionary taxation in describing optimal allocations, the solution to the social planner problem can be used as a benchmark for optimal policy. Moreover, the solution to the social planner’s problem allows us to obtain the complete solution for the efficient equilibrium (see Appendix A).

Under a full cooperation context, benevolent authorities seek to maximize welfare for the currency union as a whole - the discounted sum of the utility flows of the households, given the set of equations describing the dynamic structure of the economies. Following Woodford (2003), we compute the second-order approximation of the welfare objective around a deterministic steady state, obtaining the following union-level loss with variables in gaps:

\[L^* = E_0\left(\sum_{t=0}^{\infty} \beta^t L_t^*\right), \text{with } L_t^* = \int_0^n L_t^B ds + (1 - n) L_t^B, \text{where:}\]

\[L_t^S = \left(\frac{1}{2}\right) \left\{ \frac{\varepsilon}{\phi_S (\pi_t^S)^2} + (1 - \varphi) \left[ \frac{\alpha}{\sigma} + (1 - \varphi) \chi \right] (\varepsilon_t^S)^2 + \varphi \left( \frac{1}{\psi} + \varphi \chi \right) (\bar{g}_t^S)^2 + (1 - \varphi) [\gamma \alpha (\alpha - 2) + 2 \Phi + (1 - \varphi) \Phi^2 \chi] (\tilde{e}_t^S)^2 + 2 \varphi (1 - \varphi) \chi \varepsilon_t^S \bar{g}_t^S + (1 - \varphi) [2 \alpha + 2 (1 - \varphi) \Phi \chi] \tilde{e}_t^S \hat{e}_t^S + 2 \varphi (1 - \varphi) \Phi \chi \bar{g}_t^S \tilde{e}_t^S \right\}, \forall s \in S,\]

\[L_t^B = \left(\frac{1}{2}\right) \left\{ \frac{\varepsilon}{\phi_B (n_t^B)^2} + (1 - \varphi) \left[ \frac{\alpha}{\sigma} + (1 - \varphi) \chi \right] (c_t^B)^2 + \varphi \left( \frac{1}{\psi} + \varphi \chi \right) (\bar{g}_t^B)^2 + (1 - \varphi) [\gamma \alpha (\alpha - 2) + 2 \Phi + (1 - \varphi) \Phi^2 \chi] (n_t^B)^2 + 2 \varphi (1 - \varphi) \chi c_t^B \bar{g}_t^B + (1 - \varphi) [2 \alpha + 2 (1 - \varphi) \Phi \chi] \varepsilon_t^B (n_t^B) + 2 \varphi (1 - \varphi) \Phi \chi \bar{g}_t^B (n_t^B) \right\} ,\]

\[\phi_S \equiv \frac{(1 - \theta_S) (1 - \epsilon_S)}{\theta_S (1 + \epsilon_S)}; \phi_B \equiv \frac{(1 - \theta_B) (1 - \epsilon_B)}{\theta_B (1 + \epsilon_B)}; \Phi \equiv \alpha [\gamma - (1 - \alpha) (-\gamma + \sigma)].\]

\(^9\) For a generic variable \(X_t\), its gap is defined as \(\bar{x}_t = \bar{x}_t - \bar{x}_t\), where \(\bar{x}_t\) and \(\bar{x}_t\) denote, respectively, their effective and efficient values in log-deviations from the efficient steady-state.
As expected, welfare losses associated to inflation are larger for a higher degree of nominal rigidly \((\theta^1)\) and increase with the elasticity of substitution between goods produced in the same country and with the inverse of the labor supply elasticity \((\chi)\). Private and public consumption gaps imply welfare losses in line with the household’s risk aversions \((1/\sigma \text{ and } 1/\psi)\) respectively and with the inverse of the labor supply elasticity. There are costs associated to misallocation of goods at the monetary union level, captured by the terms-of-trade gaps. These costs increase with the trade elasticity \((\gamma)\) and with \(\chi\); while decreases with \(\sigma\), with the degree of home bias \((1-\alpha)\), and, in the particular case of the large economy, decreases with its dimension. There are also cross-terms between public and private consumption and between these and the terms-of-trade that represent additional welfare costs (see Beetsma and Jensen, 2005, for arguments).

In what follows, we assume that all policymakers share the same per-period social loss function \(L_t^s\) under cooperation. Alternatively, under non-cooperation, we assume that national fiscal authorities are exclusively concerned with their own citizens and, hence, their objective functions only include their national counterparts. We approximate the national welfare counterparts through welfare losses obtained from splitting the union-wide loss function. Following Leith and Wren-Lewis (2011), we set the linear terms contained in the country-specific loss functions to zero. These linear terms capture the desire of national governments to manipulate their terms-of-trade to obtain additional national gains, but this manipulation is ineffective if all countries proceed in the same manner.

Accordingly, for benevolent cooperative policymakers: \(L_t^s = \alpha L_t^{\text{big country}} = L_t^{\text{central bank}} = L_t^{\text{small country}}\), \(\forall s \in S\). As for benevolent non-cooperative policymakers: \(L_t^s = \alpha L_t^{\text{big country}} = L_t^{\text{central bank}} = L_t^{\text{country}}\) and \(L_t^{\text{small country}} = L_t^B\), \(\forall s \in S\).

### 2.6 Policy Games

As a benchmark, we assume that policymakers are benevolent and cooperate under discretion. The conflict of policy objectives allows for strategic interactions between policymakers and different equilibriums, depending on the timing structure of the policy games.

We consider both simultaneous-move (Nash) and leadership (monetary and fiscal leadership) equilibria. In these different setups, the timing of the events is as follows (cf. figure 1): 1) private sector forms expectations; 2) the shocks are realized; 3) depends on the policy scenario: under Nash, the central bank sets the interest rate and fiscal authorities define fiscal policy instruments simultaneously; under monetary leadership, the central bank sets the interest rate before fiscal authorities choose the right amount of fiscal instruments (fiscal authorities play a Nash between them), while the reverse occurs under fiscal leadership; 4) finally, the private sector reacts. To solve for these dynamic policy games we follow the methodology developed in
the seminal work of Söderlind (1999) and in the work of Kirsanova and co-authors (Blake and Kirsanova, 2011, and Kirsanova et al., 2005)

Figure 1: Policy regimes and intra-period timing of moves

2.7 Baseline Calibration

Relative to the union’s structure, we assume that country B and block S have identical dimension (n=0.5). The model is calibrated at a quarterly frequency: the intertemporal discount factor is set $\beta=0.99$, which implies a 4% annual basis steady-state interest rate. We choose $\sigma=0.4$, which implies a 40% share of domestic consumption allocated to imported goods for the small countries, but for the large economy this value is only of 20%. We assume $\psi = \sigma = 0.4$, which implies a coefficient of risk aversion for private and public consumption equal to 2.5, as in Beetsma and Jensen (2005). The inverse of the labor supply elasticity, $\chi$, is equal to 3, following Kirsanova and Wren-Lewis (2012). The elasticity of substitution between goods produced in the same country, $\varepsilon$, is equal to 11, implying a price mark-up of 10%. Following Ferrero (2009), we set the elasticity of substitution between domestic and foreign goods, $\gamma$, to 4.5. The steady-state share of public consumption in output, $\varphi$, is 0.25, a value commonly used in the literature. We consider that $\theta_S = \theta_B = 0.75$, in order to get an average length of price contracts equal to one year. As to the steady-state debt-to-output ratio, we adress a low- and a high-debt scenario (15% and 60%, respectively). Finally, we assume that technology and cost-push shocks are independent. Technology shocks follow an AR(1) process with common persistence of 0.85 (for instance, Ferrero, 2009, considers $\rho_\alpha=0.815$), while cost-push shocks are assumed to be non-persistent ($\rho_\mu=0$). Following Chadha and Nolan (2007), we assume the same standard deviation for technology and cost-push shocks, which is common in the literature, and set it to 1%.
3 Optimal Stabilization Policies

This section analyses optimal discretionary policy to non-symmetric technology shocks which, by their persistence and related trade-offs, reveal to be welfare-dominating. As a benchmark, we consider the full cooperation setup. Then, we analyse non-cooperative scenarios, where conflicting policy objectives generate strategic interactions between policymakers and different equilibria for different time-structure of the policy. Using union-wide and national losses as metrics, we assess welfare stabilization costs across different debt levels and policy regimes. Finally, we conduct a robustness analysis to selected model parameters.

3.1 Cooperation

Time-consistent requirement under lack of commitment determines that, under optimal discretionary policies, permanent effects of shocks are fully eliminated and all variables, including debt, return to their efficient steady-state levels (see Leith and Wren-Lewis, 2013). The policy-mix that allows debt-stabilization and the speed of this adjustment depend on steady-state government debt.

The effectiveness of monetary policy in promoting debt-stabilization increases with debt-to-output ratios, because of the higher first-order effects of interest and inflation rates on debt. Conversely, as steady-state public debt increases, fiscal policy instruments – particularly, the tax rate – become relatively less effective to promote debt-stabilization, while they become more apt in offsetting inflationary consequences (cf. Leith and Wren-Lewis, 2013).

Following a shock that boosts debt and inflation, the conventional monetary and fiscal policy assignments apply if debt levels are low enough (debt-to-output ratio < 28%): the interest rate gap increases on impact to control for inflation (“active” monetary policy) and government spending gaps diminish while the tax rate gaps increase to stabilize debt. However, for large enough government debts (debt-to-output ratio ≥ 28%), monetary policy moves towards debt-stabilization and the interest rate gap decreases on impact (“passive” monetary policy). In turn, fiscal policy becomes relatively less effective in promoting debt-adjustment and eventually moves towards an inflation-stabilization assignment. Hereafter, “low-debt” (debt-to-output ratio

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10 Under discretion and for, e.g., a steady-state debt-to-output ratio of 60%, stabilization costs (union’s per capita welfare losses) of asymmetric cost-push shocks represent only 0.11% of those of asymmetric technology shocks under discretion; if we assume the same persistency for cost-push shocks as that of technology shocks (i.e., ρμ = 0.85), this value rises 7.68%. This large difference in terms of stabilization costs is mainly due to the fact that cost-push shocks cause smaller policy trade-offs than technology shocks when the tax rate is an available policy instrument for stabilization purposes.

11 Higher steady-state debt levels lead to higher steady-state tax rates and, therefore, the impact of the tax rate on inflation further increases.

12 We follow Leeper’s (1991) categorization: a policy is “passive” when it promotes debt-stabilization; when policy instruments promote short-run stabilization instead, policy is said to be “active”.
< 28%) refers to an environment where conventional policy assignments apply, while in the “high-debt” scenario (debt-to-output ratio ≥ 28%), monetary policy promotes debt-stabilization.

In our setup, a non-symmetric technology shock generates policy trade-offs because of its budgetary consequences and the existence of nominal rigidities. In what follows, we first examine its impact on the efficient equilibrium, to assess the different budgetary consequences it produces domestically and abroad. Because a shock hitting a big country has different consequences than one hitting a small country, we provide a separate analysis for each case.

### 3.1.1 Technology shock at the Big Country

A negative technology shock at the big country B, by increasing the work effort to produce unit output, leads to a decrease on impact of the efficient levels of domestic output and, to a lesser extent, of the utility-enhanced government spending. The terms-of-trade also fall, since the domestically-produced goods (B-goods) become more expensive relative to foreign goods (S-goods). As domestic and foreign goods are substitutes in the utility function, the reduction in the terms-of-trade increases output in the small countries. Therefore, targeting efficient outcomes produces opposite budgetary consequences domestically and abroad: a primary government budget deficit at B, where output and tax revenues decrease, and a surplus at the small countries, where the reverse occurs. Moreover, as the efficient interest rate increases on impact to ensure a lower efficient level of union-wide private consumption, debt service costs rise. This further enlarges the B’s deficit while it mitigates S’s surplus. As a result, with positive steady-state debt levels, overall budgetary consequences are higher for country B than for the small countries and, thus, union-wide debt increases on impact. Moreover, higher steady-state debt levels require higher steady-state tax rates which amplify the changes in the tax base. Hence, due to larger interest payments and to a larger decrease in tax revenues, the budgetary consequences of the shock increase with B’s steady-state debt. For a small country, this only occurs for high enough debt levels, i.e., when debt-service costs surpass primary budget surplus.

Nominal rigidities preclude that the terms-of-trade fall as much as their efficient level. This leads to a positive terms-of-trade gap that inefficiently shifts demand from S-goods to B-goods, causing a positive output gap and reinforcing inflation in country B while the opposite occurs in the small countries.

As the shock causes a primary budget deficit in B and a surplus in the small countries, the reaction of the fiscal policy in the first period requires a positive tax rate gap and a negative government spending gap at country B, while the reverse occurs at the small countries (see

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13 Domestic and foreign goods are substitutes (complements) in the utility function when trade elasticity is larger (smaller) than intertemporal elasticity of substitution.
Figure 2, which plots impulse responses of selected variables across different debt-to-output ratios). The higher distortionary tax rate gap at country B fuels firms’ marginal costs and further increases inflation. The reverse occurs for a small country.

The initial debt level is critical for monetary and fiscal policy responses:

- In a **low-debt** monetary union (and except for the zero-debt case where symmetric primary and total budget effects require symmetric fiscal policies), the reaction of the fiscal policy in the Big country dominates. To lessen the subsequent union-wide inflation, the interest rate gap increases in the first period which further enlarges B’s budgetary deficit while it mitigates S’s surplus. Moreover, as steady-state debt level increases, the
extent to which this shock affects national debts further amplifies this asymmetry and, thus, country B’s fiscal policy becomes progressively more debt-adjusting while the reverse occurs in the small countries. Therefore, welfare stabilization costs increase in B while they decrease in the small countries. The former dominates the latter and welfare deteriorates for the union as a whole, as the steady-state of debt increases. This is shown in Figure 3 that depicts, for different debt levels, the welfare stabilization losses for the big and a representative small country (L_B, L_s) and for the union (L_U) following a technology shock at B.

![Figure 3: Union-wide and country-specific per capita welfare losses across debt levels, under cooperative discretionary policy responses to a technology shock at the Big country](image)

- In a **high-debt** monetary union, monetary policy moves towards debt-stabilization and the interest rate gap decreases, on impact, to reduce the union-wide debt level. The higher the steady-state of the debt, the greater the union-wide fiscal consequences of the shock and the larger becomes the effectiveness of monetary policy for debt-stabilization. Therefore, as government indebtedness increases, monetary policy becomes progressively more debt-adjusting, mitigating the budgetary consequences of the shock at country B while aggravating those at a small country. As a consequence, for a sufficiently high debt level, fiscal policy becomes progressively less debt-adjusting and welfare improves in
country B; the opposite occurs in a small country. The net effect is, for the union as a whole, a monotonically increase in welfare stabilization costs as debt rises (cf. Figure 3).

3.1.2 Technology shock at a Small Country

A shock at a very small country (zero-dimension) has no external effects, bringing only domestic implications. Thus there is no monetary policy adjustment nor a fiscal policy reaction from any other country. A negative technology shock requires the same qualitative fiscal policy from the small home country: the government spending gap decreases and the tax rate gap increases, on impact, to make debt returning to its initial (efficient) level (cf. Figure 4).

![Figure 4: Responses to a 1% negative technology shock at a small country under optimal cooperative discretionary policy (debt-to-output ratios: 0%, 15% and 60%)](image)

Fiscal policy becomes more debt-adjusting as the initial debt level increases, because the budgetary consequences of the shock enlarge and fiscal policy instruments become less effective on controlling debt. Therefore, welfare stabilization costs monotonically increase with the level of the steady-state debt (cf. Figure 5).

In sum, at the union level, welfare stabilization costs increase with the level of government indebtedness, as the aggregate budgetary consequences of the technology shocks also increase. Therefore, the potential efficiency stabilization gains, resulting from the higher effectiveness of monetary policy on debt-stabilization and of fiscal policy on short-run stabilization in higher-debt environments, are not sufficiently large to overcome the costs of stabilizing the larger budgetary consequences of shocks.

14 This reduction on debt allows for monetary policy to raise the interest rate gap in the second period which, by lowering inflation expectations, contributes to contain current union-wide inflation.
As for the perspective of the big and the small countries in a monetary union, there are meaningful differences on the level and evolution with debt of their welfare stabilization costs. In line with the findings of Canzoneri *et al.* (2005) or Machado and Ribeiro (2010), a small country always faces higher welfare stabilization costs than a big one, in face of non-correlated shocks.\textsuperscript{15} Furthermore, for realistic one-period debt levels, welfare improves at small countries while it worsens at the big country, when the level of government indebtedness increases, uniformly, in a monetary union.\textsuperscript{16} The reverse occurs, but only for sufficiently high debt levels.\textsuperscript{17}

![Chart: Small country welfare loss across debt, under cooperative discretionary policy response to a domestic technology shock](image)

Figure 5: Small country welfare loss across debt, under cooperative discretionary policy response to a domestic technology shock

### 3.2 Non-Cooperation

Here, the benevolent monetary authority still seeks to maximize the union-wide welfare but, differently from cooperation, national fiscal authorities are exclusively concerned with their national counterparts. Since shocks occurring only at a small country produce no external effects and no reaction from the central bank, there are no significant differences between alternative policy regimes. Hence, hereafter we focus on technology shocks hitting solely the big country.

\textsuperscript{15} In a setting close to Canzoneri *et al.* (2005), Mykhaylova (2011) found, in a monetary union calibrated to the EMU, that welfare costs are virtually the same for small and large union-member countries, due to highly correlated technological processes and trade openness.

\textsuperscript{16} Notice that in our model, all government debt has a one-period maturity, lending monetary policy high leverage over debt service. A given one-period debt level of our setup should correspond to a higher debt level in a more realistic structure of debt.

\textsuperscript{17} These results are in accordance with those of Machado and Ribeiro (2011) for the case of an asymmetric two-country monetary union.
3.2.1 Simultaneous-move (Nash)

In a non-cooperative setup, policy outcomes can diverge from those under cooperation because fiscal authorities do not internalize the cross-border effects of their policies. Relative to cooperation, nationally-oriented fiscal policies react more (less) intensively to a shock when they cause negative (positive) externalities. Naturally, the magnitude of these deviations is expected to be different for small and large countries, since policy externalities are of different size. Since externalities caused by a small country with zero-dimension are negligible, it is not expectable its fiscal policy reaction to be meaningfully different from that under cooperation. Conversely, since externalities caused by the fiscal policy of a big country are large, it is expectable non-internalization to make fiscal policy reacting differently to shocks.

Consider the case of a negative technology shock at country B that, by causing a primary budget deficit domestically, requires a positive tax rate gap and a negative government spending gap to promote debt stabilization. The first-period tax rate response further increases inflation at country B, which, by mitigating the positive terms-of-trade gap, helps to close the small countries’ negative output gaps. This positive externality coexists with a negative one, since it also enlarges the positive budgetary consequences of the shock at the small countries. The reverse occurs for the government spending response: the first-period negative government spending gap further enlarges the negative output gaps in small countries while it simultaneously attenuates the budgetary consequences of the shock at the small countries. In what follows, we denote the former as “standard” externality, and the later as “debt-related” externality.

The initial debt level is critical for the externality that dominates, and determines how country’s B fiscal policy response under non-cooperation diverges from cooperation.

- In low-debt monetary union, where budgetary consequences are less significant, the “standard” externality prevails. Therefore, relative to cooperation, a negative technology shock at country B requires a smaller tax rate gap and a larger government spending gap, in the first period, as the two policy instruments cause opposite externalities. Compared with cooperation, this allows for a lower inflation at B and, thus, for a higher terms-of-trade gap that reinforces the negative output gap at the small countries. Since output increases by less in the small countries, primary budget surplus also increase by less and, therefore, fiscal policy becomes relatively less debt-adjusting in these countries. In turn, since lower inflation in country B transmits to lower inflation at the union level, monetary

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18 The country-specific loss is the national counterpart of the union-wide loss and, thus, there is no difference between policy objectives beyond that of not including the remainder counterparts, see e. g. Leith and Wren-Lewis (2011).
policy becomes less “active” than under cooperation. From inspection of Figure 6, it is apparent that, except for the private and public consumption, all the other welfare-relevant variables in B display shorter volatility under Nash compared to cooperation. The \textit{per capita} welfare losses of this shock, reported in Table 1, show the welfare-superiority of Nash for all countries, under low-debt. This finding confirms that policy cooperation can be counterproductive in the presence of pre-existing distortions.\footnote{The argument follows from the key contribution of Rogoff (1985), according to which cooperation among a subset of players (all policymakers) could lead to adverse reactions of the outsiders (\textit{e.g.}, private sectors) that all players would be better off by not cooperating. Non-cooperation may alleviate time-consistency problems. See Beetsma \textit{et al.} (2001) for a review on the desirability of policy coordination.}

\textbf{Figure 6:} Responses to a 1\% negative technology shock at the \textbf{Big} country – cooperation (Coop) versus non-cooperation (Nash): low debt-to-output ratio = 15\%
Figure 7: Responses to a 1% negative technology shock at the Big country – cooperation (Coop) versus non-cooperation (Nash): high debt-to-output ratio=60%

- In a **high-debt** monetary union, where the budgetary consequences are considerable, the “debt-related” externality dominates. Therefore, relative to cooperation, a negative technology shock at country B requires, domestically, a larger (smaller) tax rate gap (government spending gap) in the first period, as the dominant “debt-related” externality is negative (positive). Compared with cooperation, this allows for a higher inflation at B, thus a lower terms-of-trade gap, increasing the demand for small countries’ goods and enlarging their primary budget surpluses. As a result, fiscal policy in a small country is required to be more debt-adjusting under Nash. At the aggregate level, fiscal policy turns to be less debt-adjusting than in cooperation and, thus, the central bank closes the interest
rate gap by more, in the first period, to ensure the stabilization of aggregate debt. This allows the central bank to raise the interest rate by more in the second period, without an adverse effect on debt and helping to lower country B’s inflation. Figure 7 illustrates these differences in a high-debt scenario (debt-to-output ratio of 60%). It shows that the small countries’ welfare-relevant variables display higher volatility under the simultaneous-move regime than under cooperation. Therefore, for the small countries cooperation is welfare-superior to Nash, in a high-debt monetary union, as it is shown in Table 1. This result also holds for the union as a whole, despite the lower welfare costs achieved by the big country B under the non-cooperative regime.

3.2.2 Leadership

Consider now the possibility of a policy authority to be a first mover. In what follows, the analysis under (fiscal or monetary) leadership to a negative technology shock at country B will take as reference the policy reaction under the simultaneous-move regime (Nash). Thus we will now focus on the differences arising from exploiting a first-moving advantage.

3.2.2.1 Fiscal Leadership (FL)

Only the fiscal authority of the big country may face incentives to explore a first-moving advantage towards the monetary authority, given the zero-dimension of a small country.

- Compared to the simultaneous-move regime, in a low-debt monetary union, the leading big country’s fiscal authority chooses to be relatively more debt-adjusting (especially through the costless fiscal instrument – the tax rate)\(^{20}\), relying on the monetary policy to control for excessive inflation at the union-wide level. Thus, at country B and relative to Nash, the tax rate gap increases by more, generating higher domestic (and union-wide) inflation that is further enlarged by a smaller decrease in the public consumption gap. This results in a lower terms-of-trade gap that boosts demand towards the small countries’ goods, improving primary budget surplus. Conversely, a higher union-wide inflation induces a higher interest rate gap, which mitigates small countries’ budgetary surplus. For sufficiently low debt levels, the latter effect is less effective than the former and, thus, fiscal policy in a small country is required to be more debt-adjusting under FL relative to Nash. The fiscal policy-mix that emerges at the aggregate level – higher positive tax rate gap and lower negative government spending gap – aggravates union-wide inflation and requires monetary authority to set a higher interest rate gap in the first-

\(^{20}\) As Leith and Wren-Lewis (2013) have shown, the effectiveness of the tax rate in promoting debt-stabilization is higher for lower levels of public debt.
period (cf. Figure 8 and Figure 6, which plot impulse responses under FL and under Nash, respectively). Welfare stabilization costs reveal to be higher under FL than under Nash for all countries, under the selected low-debt scenario (see Table 1).

Figure 8: Responses to a 1% negative technology shock at the Big country: fiscal leadership (FL) and monetary leadership (ML) relative to Nash (low debt-to-output ratio = 15%)

- In a high-debt monetary union, in contrast to the low-debt case, the leading big country’s fiscal authority, by relying on a more debt-adjusting monetary policy, turns to be relatively less debt-adjusting compared to the simultaneous-move regime. Therefore, at country B, and relative to Nash, both fiscal policy instruments move by less, globally contributing to reduce domestic inflation and to enlarge the positive terms-of-trade gap. Budgetary surplus at a small country is expected to be larger, because the interest rate gap
is expected to decrease by more, dominating the budgetary consequences of a higher positive terms-of-trade gap. Hence, fiscal policy in small countries is required to be more debt-adjusting under FL relative to Nash. It is clear, from inspection of Figure 9 and Figure 7, that country B’s inflation displays lower volatility than under Nash, while the reverse occurs in a small country. Under the high-debt scenario reported in Table 1, FL delivers the best stabilization outcome for the big country B and the worst for a small country. Union-wide, FL is slightly superior to the simultaneous-move regime.

**Figure 9:** Responses to a 1% negative technology shock at the Big country: fiscal leadership (FL) and monetary leadership (ML) relative to Nash (high debt-to-output ratio = 60%)
3.2.2.2 Monetary Leadership (ML)

Under monetary leadership, the common central bank is aware of the big country’s fiscal authority incentives to deviate from cooperation. As deviation from cooperation may either hamper or improve the welfare of the union, monetary policy is set accordingly, maximizing union’s welfare.

- In a **low-debt** monetary union, the leading monetary authority anticipates that the incentives faced by the fiscal authority of country B under non-cooperation will result in a union-wide welfare-superior equilibrium. Hence, the interest rate gap rises even less than in Nash and, thus by lowering debt-service costs, it further enlarges the budget surplus of the small countries while it mitigates the government deficit in B. As a consequence, relative to the simultaneous-move regime, the fiscal policy in small countries (country B) has to be more (less) debt-adjusting under the monetary leadership regime (*cf.* Figure 8, blue lines). Figures 6 and 8 show that, in general, country-specific variables of country B display lower volatility under ML than under Nash, while the reverse occurs for a small country. So, relative to Nash, ML reveals to be welfare-improving for the big country B and welfare-deteriorating for small countries (*cf.* Table 1). For the union as a whole, ML is the welfare-superior regime in a low-debt scenario.

**Table 1:** *Per capita* welfare losses under cooperation and non-cooperation – asymmetric technology shock at the Big country

<table>
<thead>
<tr>
<th>Debt-to-Output Ratio</th>
<th>15%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Big Country Loss (L_B)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cooperation</td>
<td>1.7421</td>
<td>2.1939</td>
</tr>
<tr>
<td>Nash</td>
<td>1.7032</td>
<td>1.8826</td>
</tr>
<tr>
<td>Fiscal Leadership</td>
<td>1.7068</td>
<td>1.7086</td>
</tr>
<tr>
<td>Monetary Leadership</td>
<td>1.6287</td>
<td>2.2202</td>
</tr>
<tr>
<td><strong>Small Country Loss (L_S)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cooperation</td>
<td>1.5525</td>
<td>1.2836</td>
</tr>
<tr>
<td>Nash</td>
<td>1.5290</td>
<td>2.2403</td>
</tr>
<tr>
<td>Fiscal Leadership</td>
<td>1.6276</td>
<td>2.3987</td>
</tr>
<tr>
<td>Monetary Leadership</td>
<td>1.5850</td>
<td>1.6460</td>
</tr>
<tr>
<td><strong>Union-wide Loss (L_U)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cooperation</td>
<td>1.6473</td>
<td>1.7388</td>
</tr>
<tr>
<td>Nash</td>
<td>1.6161</td>
<td>2.0615</td>
</tr>
<tr>
<td>Fiscal Leadership</td>
<td>1.6672</td>
<td>2.0537</td>
</tr>
<tr>
<td>Monetary Leadership</td>
<td>1.6069</td>
<td>1.9331</td>
</tr>
</tbody>
</table>

Note: grey (blue) cells show the best (worst) result for each “country-debt” combination.
In a high-debt monetary union, the monetary authority, anticipating that the incentives faced by the fiscal authority of country B under non-cooperation are welfare-decreasing for the union, chooses to reduce by more the interest rate gap in the first period, in order to induce the big country’s fiscal authority to be relatively less debt-adjusting. This monetary policy reaction mitigates country B’s budget deficit while it enlarges small countries’ surplus, inducing fiscal authority of country B (small country) to implement a relatively less (more) debt-adjusting policy-mix under ML (cf. Figure 9, blue lines). The resulting equilibrium ends up by delivering, relative to Nash, higher volatility in country B while the reverse occurs in a small country.\textsuperscript{21} In the high-debt case, small countries and the union as a whole experience welfare stabilization gains from having a leading monetary authority under non-cooperation (cf. Table 1).

3.3 Welfare Stabilization Costs across Different Debt Levels and Policy Regimes

Here, we extend the debt range and consider all technology shocks, to appraise how welfare stabilization costs for the union and for the different-size country members evolve across debt levels. This analysis is meaningful within the context of the Euro Area, where formal limits to debt are set homogeneously on a supranational basis. We conjecture, from previous results, that the institutional environment of both monetary and fiscal policy in the EMU is not welfare neutral for defining limits on public debt.

3.3.1 Union-wide Welfare Losses

Figure 10 plots union-wide welfare losses for a range of debt-to-output levels for which the system converges to a unique solution under all policy regimes. From its inspection we can conclude that, in general, higher government indebtedness hampers business cycle stabilization. Moreover, there seems to be clear stabilization gains from promoting policy cooperation in a high-debt monetary union, while this could be counterproductive in a low-debt monetary union.

Non-cooperation may alleviate time-consistency problems of optimal discretionary policies and allow for a better stabilization performance than cooperation, when monetary policy is attached to an inflation-stabilization assignment in low-debt environments. Overall, fiscal policy is less debt-adjusting under non-cooperation and this alleviates union inflation pressures, requiring a less “active” monetary policy.

Conversely, in high-debt environments, where monetary policy is attached to a debt-stabilization assignment, a less debt-adjusting fiscal policy under non-cooperation requires a

\textsuperscript{21} For this result it is crucial the relatively higher (lower) government spending gap at country B (s).
more “passive” monetary policy with negative consequences for union’s welfare. Therefore, cooperation is welfare superior relative to non-cooperation in a high-debt monetary union. In turn, monetary leadership delivers a better stabilization performance than fiscal leadership, independently of the level of government indebtedness.

![Figure 10: Union-wide welfare loss (L_U) across different policy regimes and debt levels (all technology shocks)](image)

### 3.3.2 Big Country Welfare Losses

Domestic shocks are the only that cause stabilization costs for the big country B, as a shock at a country with zero-dimension produces no external effects. The dominance of domestic shocks is crucial for the observed non-monotonic relationship between welfare stabilization costs and debt across policy regimes, despite the increasing budgetary consequences of the shocks with debt (cf. Figures 11 and 3). This follows from the fact that, when debt becomes high enough, the increased effectiveness of monetary policy in promoting debt-stabilization may allow for domestic fiscal policy to be progressively less debt-adjusting. When the gains of alleviating domestic fiscal policy from time-consistency problems are sufficiently large, welfare stabilization costs decrease with the level of government indebtedness at country B.
As for the union as a whole, for sufficiently low-debt levels, non-cooperation dominates cooperation and monetary leadership is the welfare-superior regime for the big country. Fiscal leadership is preferable to Nash, only for very low debt levels (cf. Figure 11). Non-cooperation still dominates cooperation only for sufficiently high-debt levels. If debt is high enough, the positive “debt-related” externality dominates for government spending and allows for a lower country B’s government spending gap under non-cooperation. In contrast, for intermediate-debt levels, the negative “standard” externality dominates and the negative government spending gap is larger under non-cooperation than cooperation, at country B. In a high-debt monetary union, FL dominates for the big country (cf. Figure 11).

3.3.3 Small Country Welfare Losses

For a small country, only external shocks at a big country matter for the differences between cooperation and non-cooperation and for the non-monotonic relationship observed between welfare losses and debt (cf. Figure 12 and Figure 3). Domestic shocks, despite their higher stabilization costs, do not produce relevant externalities.

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22 This is probably a consequence of how effectiveness towards debt-stabilization of the two fiscal policy instruments – government spending and tax rate – changes with debt (see Leith and Wren-Lewis, 2013).
Figure 12: Small country welfare loss ($L_s$) across different policy regimes and debt levels (all technology shocks)

For low-debt levels, as seen before, the budgetary consequences at small countries of shocks at the big country diminish with the pre-shock debt level. Therefore, welfare stabilization costs decrease in small countries, as the steady-state debt level increases in a low-debt scenario.

However, in a high-debt scenario, the “passive” monetary policy reaction to the shock enlarges a small country’s primary budget surplus and, for a high enough debt level, fiscal policy becomes progressively more debt-adjusting. As a consequence, welfare stabilization costs increase with government indebtedness for a small country (cf. Figure 12).

In general, a small country faces lower stabilization costs under cooperation than under non-cooperation. Only for a small range of low debt levels, Nash is welfare-superior. Under non-cooperation, small countries benefit from monetary leadership in a high-debt monetary union. Fiscal leadership is the welfare-inferior regime for a wide range of debts (cf. Figure 12).

3.4 Sensitivity Analysis

In this section, we check the robustness of our results to changes in selected model parameters, in addition to debt-to-output ratio. Given its welfare-dominance, we keep the focus on non-symmetric technology shocks.
3.4.1 Nominal Rigidity

A reduction on the degree of nominal rigidity impinges directly on welfare, because it reduces the costs attached to inflation volatility, but it also affects welfare indirectly by changing the relative volatility of welfare-related variables. Furthermore, the degree of nominal rigidity has non-negligible budgetary consequences and, as shown by Leith and Wren-Lewis (2011, 2013), affects the absolute and relative contribution of the different policy instruments to debt stabilization. Therefore, welfare consequences of higher price flexibility are not clear-cut.

In fact, focusing on optimal cooperative discretionary outcomes across different debt levels and a wide range for nominal rigidity, we find a non-monotonic relationship between welfare stabilization costs and nominal rigidity (see Figure B1, Appendix B). However, simulations of cooperative and non-cooperative outcomes for an alternative calibration of the degree of nominal rigidity ($\theta = 2/3$), does not change meaningfully the welfare rankings between policy regimes relative to the baseline calibration (cf. Figure B2, Appendix B).

3.4.2 Elasticity of Labor Supply

In baseline calibration, the inverse of the labor elasticity is set to $\chi = 3$. We now assess welfare implications for two alternative values for $\chi$: 1.5 and 5 (as in Erceg et al., 2007).

It is expected that a lower elasticity of labor supply (higher $\chi$) results in higher stabilization costs since fluctuations in work effort, arising from misallocations caused by inflation, become more costly. Moreover, this calls for a stronger response of monetary policy to dampen fluctuations in the work effort observed at the aggregate level. A lower value of the elasticity of labor supply reduces the cost of private consumption fluctuation relative to inflation fluctuation and, thus, it reduces the incentives for monetary policy to stabilize debt. Consequently, the threshold debt levels for which monetary policy becomes “passive” increase with smaller values for the elasticity of labor supply (higher $\chi$). Thus cooperation becomes welfare superior to non-cooperation for higher debt levels as the elasticity of labor supply decreases, but key qualitative results are preserved (cf. Figure B3, Appendix B).

3.4.3 Trade Elasticity

If the elasticity of substitution between domestic and foreign goods ($\gamma$) is bigger (smaller) than the intertemporal elasticity of substitution ($\sigma$), goods are substitutes (complements). We now examine the implications of the goods being complements, by setting $\gamma = 0.2$ ($< \sigma = 0.4$).

Under complementarity, one should expect a negative technology shock at country B to cause opposite budgetary consequences, both domestically and abroad, from those observed
under substitutability. Differences on policy reactions do not change the qualitative results obtained under baseline calibration of this parameter (cf. Figure B4, Appendix B).

3.4.4 Alternative Size for the Big Country

Under the baseline calibration, the large economy represents 50% of the monetary union. We will now consider two alternative dimensions for country B: \( n_B = 0.35 \) and \( n_B = 0.65 \).

As the size of the large economy increases, the larger are the externalities produced to small countries and also the larger is the impact on aggregate variables to which monetary policy reacts. Our experiments show that this renders the distribution of welfare costs across countries even more asymmetric: the welfare stabilization costs of a small (big) country are higher and increasing (lower and decreasing) with the size of country B, independently of the debt level (cf. Figure B5 and B6, Appendix B). Our results also suggest that it may be harder to get the approval of a very big country for a cooperative arrangement. Conversely, and for a wide range of debt levels, cooperation is beneficial for small countries and the union as a whole, the larger the big country is. In general, the welfare rankings of the different policy regimes reveal to be robust to the relative size of the big country.

4 Concluding Remarks

We fill a gap in the literature by proposing a model that allows the analysis of fiscal and monetary policy interactions between countries either with negligible or with meaningful impact on the outcomes of a monetary union as well as on the counterparts’. Besides providing a more adequate scenario in regards to mimicking heterogeneous-size EMU member countries, our analysis provides further insights on how debt scaling, in the sequence of the 2008-2009 crisis, may shape policy interactions between national governments and the ECB. The size of the steady-state level of debt qualifies both (i) the budgetary consequences of the shocks and (ii) the effectiveness of the different policy instruments towards debt-adjustment. In a high-debt environment, the potential stabilization gains for the union, resulting from the higher effectiveness of monetary policy to stabilize debt and of fiscal policy to promote short-run stabilization, are not sufficiently large to overcome the costs of stabilizing the larger budgetary consequences of technology shocks. According to our results, in general, higher government indebtedness hampers union-wide business cycle stabilization.

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23 In the Euro area, for instance, Germany’s GDP represents approximately 28.5% of the union’s GDP; in turn, the three major economies (France, Germany and Italy) represent approximately 66.2% of the Euro area’s GDP (Eurostat, 2013).

24 The threshold debt level for which cooperation delivers the worst solution for the big country is lower the larger the big country is (see Figure B6, Appendix B).
Shocks at larger economies are the most relevant to assess how country-specific welfare stabilization react in alternative scenarios for government debt. They are crucial for the uneven distribution of stabilization costs between small and big countries across debt levels: welfare costs decrease with debt for a small country and increase for the big country, in a low-debt monetary union; the reverse occurs in a high-debt monetary union. Thus, a higher level of debt, as the one experienced in the EMU is more likely to penalize the stabilization performance of the small country-members than that of the large.

Strategic policy interactions, arising from different policy objectives set by nationally-oriented fiscal authorities, also disclose different welfare consequences for the large and the small countries. In general, the non-internalization, under non-cooperative policy regimes, of the externalities produced by the big country’s fiscal policy imposes higher stabilization costs to the small countries. The large country can benefit from non-cooperation if debt levels are either high or low enough, but not for intermediate ranges. For the union as a whole there are clear stabilization gains from promoting policy cooperation under a high-debt scenario, though this could be counterproductive in a low-debt monetary union.

The leadership structure has also critical stabilization consequences. Fiscal leadership imposes higher stabilization costs than monetary leadership, for the union as whole and for the small countries. In turn, in high-debt environments, the big country clearly prefers fiscal leadership, where it can explore a larger strategic power vis-a-vis a debt-accommodative monetary authority, suggesting that it may be hard to get political support for a cooperative arrangement that enhances union-wide welfare stabilization gains.
References


Appendix A

A.1 Model Equations

In the following equations, a union-wide variable, $x_t^*$, is defined as $x_t^* = nx_{t-1}^* + (1-n)x_t^B$, where $x_t^S$ is in turn defined as $x_t^S = \frac{1}{n} \int_0^n x_{t-1}^S ds$.

A.1.1 Equations of the Forward Looking Variables

- **Euler Equation**

  Log-linearising equation (9b), aggregating over all countries, and rewriting in gaps, yields

  $$\gamma_t^* = E_t \{ \gamma_{t+1}^* \} - \sigma(\gamma_t^* - E_t \{ \gamma_{t+1}^* \}), \quad \text{(A.1)}$$

  where $\gamma_{t+1}^* = \gamma_{t+1}^*$, and assuming that the efficient union-wide inflation level is zero.

- **Phillip's Curves**

  $$\pi_t^* = \beta E_t \{ \pi_{t+1}^* \} + \phi^j \left( \frac{1}{(1-\phi)(\sigma(1-\alpha)+\Phi)} + \chi \right) \gamma_t^j$$

  $$- \phi^j \left( \frac{1}{(1-\phi)\sigma(1-\alpha)+\Phi} \right) \gamma_t^j + \phi^j \left( \frac{1}{\sigma(1-\phi)} - \frac{1}{(1-\phi)\sigma(1-\alpha)+\Phi} \right) \gamma_t^j$$

  $$- \phi^j \left( \frac{1}{(1-\phi)\sigma(1-\alpha)+\Phi} \right) \gamma_t^j + \phi^j \left( \frac{1}{(1-\phi)} \right) \gamma_t^j,$$

  where: $\phi^j = \left( \begin{array}{c} \phi_s \equiv \frac{(1-\theta_s\beta)(1-\theta_s)}{\theta_s(1+\epsilon \chi)}, \text{for } j = s, \forall s \in S, \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon \chi)}, \text{for } j = B, \end{array} \right)$

  $\Phi \equiv \alpha \gamma - (1-\alpha)(-\gamma + \sigma)$.

A.1.2 Equations of the Predetermined Variables

- **Market Clearing**

  $$\gamma_t^S = (1-\phi)\gamma_t^S + (1-\phi)[\sigma(1-\alpha) + \Phi]n \tilde{\gamma}_t^S + \phi \tilde{\gamma}_t^S, \forall s \in S, \quad \text{(A.3)}$$

  $$\gamma_t^B = (1-\phi)\gamma_t^B + (1-\phi)[\sigma(1-\alpha) + \Phi]n \tilde{\gamma}_t^B + \phi \tilde{\gamma}_t^B, \quad \text{(A.4)}$$

  where: $\Phi \equiv \alpha \gamma - (1-\alpha)(-\gamma + \sigma)$.

- **International risk sharing condition**

  $$\tilde{c}_t^S = \tilde{c}_t^S + \sigma(1-\alpha)[\tilde{\gamma}_t^S - n \tilde{\gamma}_t^S], \forall s \in S, \quad \text{(A.5)}$$

  $$\tilde{c}_t^S = \tilde{c}_t + \sigma(1-\alpha)[\tilde{\gamma}_t^S - n \tilde{\gamma}_t^S], \forall s \in S, \quad \text{(A.6)}$$

- **Law of motion for the terms-of-trade gaps**

  Taking into consideration the relation between domestic prices and consumer prices, that

  $$tt_t^i = p_t^i - p_t^i, \forall s \in S \text{ and } ntt_t^B = p_t^B - p_t^B, \text{ we obtain}$$

  $$\left( \tilde{\gamma}_t^S + \tilde{\gamma}_t^S \right) - \left( \tilde{\gamma}_t^S + \tilde{\gamma}_t^S \right) = \pi_t^* - \pi_t^*, \forall s \in S, \quad \text{(A.7)}$$
\[
\left( \tilde{t}_t^B + \tilde{t}_t^B \right) - \frac{\pi_t - \pi_t^B}{n} = \text{(A.8)}
\]

- **Flow Budget Constraints**

\[
\log\left( d_{i,t}^e \right) = \tilde{r}_t^e + \frac{1}{\beta} \left\{ \log\left( d_{i,t-1}^e \right) - \pi_t^e + \frac{\gamma^e}{\alpha^e} \left[ \psi \tilde{y}_{i,t}^e - \tau^e \tilde{y}_{i,t}^e - \tilde{t}_t^e \right] + \alpha \tilde{t}_{t-1}^e - \left( \frac{1}{1+\rho} \right) \alpha \tilde{t}_{t}^e \right\} + \tilde{r}_{t-1}^e + \frac{1}{\beta} \left\{ \psi \tilde{y}_{i,t}^e - \tau^e \tilde{y}_{i,t}^e - \tilde{t}_t^e \right\} + \alpha \tilde{t}_{t-1}^e - \left( \frac{1}{1+\rho} \right) \alpha \tilde{t}_{t}^e, \forall s \in S. \quad \text{(A.9)}
\]

\[
\log\left( d_{i,t}^B \right) = \tilde{r}_t^B + \frac{1}{\beta} \left\{ \log\left( d_{i,t-1}^B \right) - \pi_t^B + \gamma^B \left[ \psi \tilde{y}_{i,t}^B - \tau^B \tilde{y}_{i,t}^B - \tilde{t}_t^B \right] + \alpha \tilde{t}_{t-1}^B - \left( \frac{1}{1+\rho} \right) \alpha \tilde{t}_{t}^B \right\} + \tilde{r}_{t-1}^B + \frac{1}{\beta} \left\{ \psi \tilde{y}_{i,t}^B - \tau^B \tilde{y}_{i,t}^B - \tilde{t}_t^B \right\} + \alpha \tilde{t}_{t-1}^B - \left( \frac{1}{1+\rho} \right) \alpha \tilde{t}_{t}^B. \quad \text{(A.10)}
\]

- **Exogenous Processes**

\[
a_t^j = \rho_a a_{t-1}^j + \epsilon_{t}^j, j = B, s \in S, \quad \text{(A.11)}
\]

\[
\phi_j \hat{\mu}_{w,t} = \rho_a (\phi_j \hat{\mu}_{w,t-1}) + \epsilon_{t}^j, j = B, s \in S, \quad \text{(A.12)}
\]

### A.2 The Social Planner’s Problem and the Efficient Equilibrium

The social planner is concerned with real allocations, ignoring nominal inertia and distortionary taxation. Thus, she or he simply decides how to allocate private and public consumption and production of goods in each economy within the union, seeking to maximize welfare for the currency union as a whole \(W^*\) - the discounted sum of the utility flows of the households belonging to the union, subject to the existent technology, the resources constraints and all the constraints that arise from operating in a monetary union, e.g., the international risk sharing condition. Notice that in the absence of nominal rigidities there is no price dispersion and, hence, the social planner will chose to produce equal quantities of the different goods in each country. Consequently, all households work the same number of hours in each country. Furthermore, the aggregation over all agents cancels out the budget constraints and, thus, the solution to the social planner’s problem is not constrained by them. The complete solution for the efficient equilibrium is given by the following expressions, in deviations from the steady state (the main steps in their derivation is available upon request):

- **Private consumption**

\[
\tilde{c}_t^e = \frac{(1-\alpha)(1+\chi)\sigma}{1+\chi(\psi(1-\phi)(1+\sigma)(1-2\alpha+\alpha^2))} a_t^e (A.13)
\]

\[
\tilde{c}_t^e = \frac{(1+\chi)(\psi(1-\phi)(1-2\alpha+\alpha^2))}{1+\chi(\psi(1-\phi)(1-2\alpha+\alpha^2))} a_t^e, \epsilon_t^j = B, s \in S.
\]
\section*{Public consumption}

\begin{equation}
\tilde{g}_t^j = \frac{(1+\chi)\psi}{1+\chi[\phi\psi+(1-\phi)\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)]} a_t^j - \frac{(1+\chi)\phi(1-\phi)\chi(\sigma-\gamma)(2\alpha-\alpha^2)}{[1+\chi[\phi\psi+(1-\phi)\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)]]} a_t^j, j = B, s \in S, \tag{A.14}
\end{equation}

\begin{equation}
\tilde{g}_t^* = \frac{(1+\chi)\psi}{1+\chi[\phi\psi+(1-\phi)\sigma]} a_t^*. \tag{A.15}
\end{equation}

\section*{Output}

\begin{equation}
\tilde{f}_t^j = \frac{(1+\chi)[\phi\psi+(1-\phi)\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)]}{1+\chi[\phi\psi+(1-\phi)\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)]} a_t^j + \frac{(1+\chi)(1-\phi)(\sigma-\gamma)(2\alpha-\alpha^2)}{[1+\chi[\phi\psi+(1-\phi)\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)]]} a_t^j, j = B, s \in S, \tag{A.16}
\end{equation}

\begin{equation}
\tilde{f}_t^* = \frac{(1+\chi)[\phi\psi+(1-\phi)\sigma]}{1+\chi[\phi\psi+(1-\phi)\sigma]} a_t^* = (1 - \phi)\tilde{c}_t^* + \phi \tilde{g}_t^*. \tag{A.17}
\end{equation}

\section*{Terms-of-trade}

The efficient terms-of-trade levels follow from a combination of the international risk sharing condition with the efficient levels of private consumption:

\begin{equation}
(p_t^* - \bar{p}_t^j) = \frac{(1+\chi)}{1+\chi[\phi\psi+(1-\phi)\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)]} (a_t^j - a_t^*), \quad j = B, s \in S \tag{A.18},
\end{equation}

\begin{equation}
\tilde{t}_t^s = (\tilde{p}_t^* - \bar{p}_t^s), s \in S \tag{A.19}
\end{equation}

\begin{equation}
\tilde{t}_t^B = \frac{1}{n} (\tilde{p}_t^* - \bar{p}_t^B). \tag{A.20}
\end{equation}

\section*{Interest rate}

The efficient interest rate follows from the Euler equation, assuming that the efficient union-wide inflation is zero:

\begin{equation}
\tilde{r}_t^* = \frac{(1+\chi)}{1+\chi[\phi\psi+(1-\phi)\sigma]} \tilde{E}_t (a_{t+1}^* - a_t^*), \tag{A.19}
\end{equation}

where it is implicit that the steady state nominal (and real) interest rate is \( r^* = \rho = -\log(\beta) \).

\section*{Revenue tax rate}

Recall that it is assumed that each firm receives an employment subsidy of \( c_{w}^j \) that removes steady state distortions and is fully financed by lump sum taxes. This employment subsidy eliminates linear terms in the social welfare function without losing the possibility of using the revenue tax \( \tau_t^j \) as fiscal instrument. Additionally, we assume that the efficient revenue tax rate eliminates marginal cost deviations from its steady state level arising from pure cost-push shocks. Thus,

\begin{equation}
\tilde{\tau}_t^j = -(1 - \tau^j) \tilde{c}_w^j, j = B, s \in S, \tag{A.20}
\end{equation}

where \( \tau^j \) represents the revenue tax steady-state value.
A.3 Steady State Equilibrium and the Employment Subsidy

To compute the employment subsidy, notice that in the absence of price rigidities profit maximization behavior (16) implies that real marginal cost

\[ MC^j_t = \frac{1}{(1+\mu_p)} j = B, S \in S, \] 

that is, firms would choose a constant markup over the nominal marginal cost, with \( \mu_p \equiv \frac{1}{\epsilon - 1}. \) From the definition of the real marginal cost, \( MC^j = \)

\[ \frac{(1-\epsilon_j^l)(1+\mu_p)}{(1-\tau^l)} (C^j)^{\frac{1}{\epsilon}} x_0 (y^j)^{\frac{1}{\epsilon}}, j = B, S \in S, \]

and from the optimality conditions of the social planner problem we get that in the steady state \( (C^j)^{\frac{1}{\epsilon}} = x_0 (y^j)^{\frac{1}{\epsilon}}, j = B, S \in S. \) Thus, \( MC^j = \)

\[ \frac{(1-\epsilon_j^l)(1+\mu_p)}{(1-\tau^l)}, j = B, S \in S. \]

To ensure that in the steady state \( MC^j = \frac{1}{(1+\mu_p)} \) the employment subsidy in country \( j \) is assumed to take the value

\[ \epsilon^j_w = 1 - \frac{(1-\tau^l)}{(1+\mu_p)(1+\mu_w^l)} = 1 - \frac{(\epsilon-1)(1-\tau^l)}{\epsilon(1+\mu_w^l)}, j = B, S \in S. \quad (A.21) \]
Appendix B

Figure B1: Union-wide welfare loss ($L_U$) under cooperative discretionary policy, across different debt levels and different degrees of nominal rigidity (all technology shocks)
Figure B2: Union-wide welfare loss (L_U) across different policy regimes and debt levels, considering two degrees of nominal rigidity (all technology shocks)
Figure B3: Union-wide welfare loss \( (L_U) \) across different policy regimes and debt levels, for alternative elasticities of labor supply (all technology shocks)
Figure B4: Union-wide welfare loss (L_U) across different policy regimes and debt levels, for alternative elasticities of substitution between domestic and foreign goods (all technology shocks)
Figure B5: Welfare losses for the Big country (L_B) across different policy regimes and debt levels, for alternative dimensions of country B (all technology shocks)
Figure B6: Welfare losses for a small country ($L_s$) across different policy regimes and debt levels, for alternative dimensions of country $B$ (all technology shocks)