The Disturbing Interaction between Countercyclical Capital Requirements and Systemic Risk*

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November 29, 2013

Abstract

We present a model in which flat (cycle-independent) capital requirements are undesirable because of shocks to bank capital. There is a rationale for countercyclical capital requirements that impose lower capital demands when aggregate bank capital is low. However, such capital requirements also have a cost as they increase systemic risk taking: by insulating banks against aggregate shocks (but not bank-specific ones), they create incentives to invest in correlated activities. As a result, the economy’s sensitivity to shocks increases and systemic crises can become more likely. Capital requirements that directly incentivize banks to become less correlated dominate countercyclical policies as they reduce both systemic risk-taking and procyclicality.

*We thank participants at the FIRS conference in Dubrovnik, the workshop on “Understanding Macro-prudential Regulation” at Norges Bank, the workshop on systemic risk at Nottingham University, the workshop on “Liquidity, Banking and Financial Markets” at the University of Bologna, the conference on financial stability in Luxembourg, the summer workshop of the Hungarian Science Academy, the EFMA conference in Reading and a seminar at Tilburg University as well as Mathias Drehmann, Harry Huizinga, Peter Kondor, Enrico Perotti, Frédéric Malherbe, Xavier Freixas, John Theal and Javier Suarez for comments.

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Keywords: systemic risk, regulation, procyclicality

JEL classification: G01, G21, G28
1 Introduction

A key focus of the debate on the design of future financial regulation is on whether the financial system responds efficiently to shocks. While prior to the crisis of 2007-2009 the general view was that the economy adjusts optimally in the advent of shocks, there is a growing consensus that this view is inappropriate when it comes to the financial system. In particular, there is concern that the financial system exacerbates shocks, leading to excessive lending in boom times and sharp contractions in credit when conditions deteriorate. A common explanation for this is that agents in the financial system tend to be subject to constraints that can increase the impact of shocks, such as borrowing constraints that fluctuate with asset prices, risk-sensitive capital requirements or remuneration schemes based on relative performance.

In response to the experience of the recent crisis, there is now a broad move towards policies that mitigate procyclicality, the tendency of the financial system to amplify shocks over the cycle. For instance, the new Basel Accord incorporates capital buffers that are built up in good times and can be run down when economic conditions deteriorate. In addition, the liquidity coverage ratio of Basel III – which aims at safeguarding banks against short-term outflows – contains a countercyclical element to the extent that such liquidity buffers are released in bad times. On the accounting side, there is a discussion about whether mark-to-market accounting – which has the potential to amplify the impact of asset price changes – should be suspended when prices are depressed. There is also a growing debate about whether monetary policy should “lean against the wind” with respect to the financial cycle, that is, raise interest rates when the economy experiences excessive credit expansion and asset price inflation, but lower interest rates in times of significant contraction in lending or general stress in the financial system.

In this paper we argue that procyclicality cannot be separated from a second dimension of systemic risk: the extent to which institutions in the financial system are correlated with
each other.¹ Such correlation can arise through various channels: herding in investment activities, the use of common funding sources, interconnectedness through interbank linkages, but also through convergence in risk management practices and trading strategies. In particular, we show that there is a two-way interaction between these two dimensions of systemic risk: policies that target procyclicality affect the correlation of risks in the financial system and correlation (and policies that mitigate it) influence procyclicality. It is thus not possible to address the two dimensions of systemic risk in isolation, which has profound implications for the design of macroprudential regulation.

We consider an economy in which banks face shocks to their capital. There is a role for capital requirements because capital reduces moral hazard at banks (akin to Holmström and Tirole (1997)). Flat capital requirements create a very simple form of procyclicality: when there is a negative shock to bank capital it becomes expensive to fulfill the requirements, reducing welfare by more than in the absence of capital requirements. We show that welfare-maximizing capital requirements – for given correlation of risks in the financial system – are countercyclical: when there is sufficient capital in the economy, it is optimal to require banks to hold capital to contain moral hazard, while when capital is scarce it becomes optimal to forego the benefits of capital. Effectively, countercyclical capital requirements increase welfare by mitigating the impact of aggregate shocks to bank capital.

This result no longer holds in general when the correlation of risks is endogenous. We allow banks to choose between a common and a bank-specific project. Since a bank’s capital is determined by prior returns on its activities, capital conditions become more correlated when banks invest in the same project. At the same time, correlation makes it also more likely that banks fail jointly. In this case there is a cost as there are no longer sufficient funds in the economy for undertaking productive activities. Banks do not internalize this

¹It is common in the literature to see procyclicality and common risk exposures as the two key – but separate – dimensions of systemic risk (e.g., Borio (2003)).
cost, and hence may choose more correlation than socially optimal.

Countercyclical capital requirements worsen the problem of excessive correlation. The reason is simple: they insulate banks against common shocks, but not against bank-specific ones. The expected cost from exposure to aggregate risk hence falls relative to bank-specific exposures, increasing banks’ incentives to invest in the common project. A bank that continues to focus on bank-specific activities would run the risk of receiving a negative shock when aggregate capital is plenty, in which case it would be subject to high capital requirements precisely when it is most costly.

Countercyclical capital requirements thus trade off benefits from reducing the impact of a shock for given exposures in the financial system with higher correlation of risks in the financial system. Their overall welfare implications are hence ambiguous. Perversely, countercyclical policies may even increase the economy’s sensitivity to aggregate conditions. The reason is that by inducing banks to become more correlated, they make the financial system more exposed to aggregate shocks, which may result in a greater likelihood of joint bank failures. We show that the appeal of capital requirements that depend on the state of the economy is further reduced when there are commitment problems in capital regulation. This is because a regulator would always face the temptation of lowering capital requirements ex-post when capital is scarce – even though this may not be optimal ex-ante. Carrying out countercyclical policies in a discretionary fashion – as envisaged by Basel III\(^2\) – can hence induce inefficiencies.

There is an alternative macroprudential policy in our model: a regulator could directly incentivize banks to become less correlated (for example, by charging higher capital requirements for correlated banks). We show that such a policy (if feasible) dominates countercyclical policies. This is because it addresses the two dimensions of systemic risk at the same time: it discourages correlation but also makes the system less procyclical as more heterogenous institutions will respond less strongly to aggregate shocks. In contrast

\(^2\)See Basel Committee on Banking Supervision (2010).
as discussed before – countercyclical policies improve systemic risk along one dimension at the cost of worsening it along another one.

The key message of our paper is that the two dimensions of systemic risk (common exposures and procyclicality) are inherently linked. The consequence is that policies addressing one risk dimension will also affect the other – and possibly in undesired ways. While our model is set in the specific context of capital requirements and banks, the basic message also applies to other forms of countercyclical policies, such as macroeconomic stabilization policies. For example, a policy of “leaning against the wind” insulates banks against aggregate fluctuations in interest rates\(^3\) and likewise increase incentives for taking on common risk.

Our paper connects two strands of literature. The first investigates whether banking regulation should respond to the economic cycle.\(^4\) Kashyap and Stein (2004) argue that capital requirements that do not depend on economic conditions are suboptimal and suggest that capital charges for a given unit of risk should vary with the scarcity of capital in the economy. Repullo and Suarez (2013) demonstrate that fixed risk-based capital requirements (such as in Basel II) result in procyclical lending. They also show that banks have an incentive to hold pre-cautionary buffers in anticipation of capital shortages – but that these buffers are not effective in containing procyclicality. As a result, introducing a countercyclical element into regulation can be desirable. Malherbe (2013) considers a macroeconomic model where a regulator trades-off growth and financial stability and finds that optimal capital requirements depend on business cycle characteristics. Martínez-Miera and Suarez (2012) consider a dynamic model where (fixed) capital requirements reduce banks’ incen-

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\(^3\)Recent literature also suggests that central banks may want to vary interest rates in an (effectively countercyclical) way in order to reduce the cost of financial crises (e.g., Diamond and Rajan (2011) and Freixas et al. (2011)).

\(^4\)See Galati and Moessner (2011) for a general overview of macroprudential policies.
tives to take on aggregate risk (relative to investment in a diversified riskless portfolio). The reason is that capital requirements increase the value of capital to surviving banks in a crisis. This in turn provides banks with incentives to invest in safer activities in order to increase the chance of surviving when other banks are failing (the “last bank standing” effect).

A second strand of the literature analyzes the incentives of banks to correlate with each other. In particular, it has been shown that inefficient correlation may arise from investment choices (e.g., Acharya and Yorulmazer (2007)), diversification (Wagner (2011) and Allen et al. (2012)), interbank insurance (Kahn and Santos (2010)) or through herding on the liability side (Segura and Suarez (2011), Stein (2012) and Farhi and Tirole (2012)). In Acharya and Yorulmazer (2007), regulators cannot commit not to bail out banks if they fail jointly. Anticipating this, banks have an incentive to invest in the same asset in order to increase the likelihood of joint failure. In contrast, the effect in our paper is not driven by commitment problems but arises because there are benefits from letting capital requirements vary with the state of the economy. Another difference to Acharya and Yorulmazer (and most other papers on herding) is that correlation in the banking system – by itself – can be desirable as capital requirements that vary with aggregate conditions then better reflect the individual conditions of banks (by contrast, if bank conditions are largely driven by idiosyncratic factors, varying capital requirements with the aggregate state provides limited benefits). Farhi and Tirole (2012) consider herding in funding choices. They show that when the regulator lacks commitment, bailout expectations provide banks with strategic incentives to increase their sensitivity to market conditions. While in Farhi and Tirole (as well as Acharya and Yorulmazer (2007)) bank choices are strategic complements, in our setting they are not.

Our paper also relates to the long-standing literature on macroeconomic stabilization policies – as for example analyzed in the context of a textbook IS-LM model. This literature has focused on the ability of stabilization policies in insulating the economy from (aggre-
gate) shocks – taking as exogenous the risk exposures of firms (or banks) in the economy. Since stabilization policies reduce the cost of aggregate shocks in a similar way to countercyclical capital requirements, our analysis suggests that they may also have (potentially unintended) effects by changing the incentives of firms and banks to expose themselves to the aggregate cycle.

The remainder of the paper is organized as follows. Section 2 contains the model. Section 3 discusses the results. Section 4 concludes.

2 Model

2.1 Preview of the model

We present a simple model in which there is a role for state-dependent capital requirements as well as endogenous systemic risk. The scope for variable capital requirements comes from shocks to bank capital. In particular, a low return on an (existing) project reduces a bank’s capital. In such a situation, it is costly to use capital as a tool for mitigating moral hazard at the bank. When many banks have low capital, it may then be optimal for the regulator to reduce capital requirements.

Systemic costs arise because when banks fail at the same time, there is a shortage of funds to undertake productive opportunities in the economy. In our model this is because of the existence of a technology that requires a fixed amount of funds. Systemic risk and capital requirements interact because banks can affect the correlation of their projects. In particular, anticipation of capital requirements determines whether banks want to invest

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5Our view of bank capital is based on Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Martínez-Miera and Suarez (2012) in that (inside) bank capital derives from accumulated bank profits.

6More broadly, systemic costs would arise whenever the economy’s production function (or the utility of agents) is convex.
in the same project or not. This is in turn affects the likelihood of systemic crises where banks are failing jointly.

### 2.2 Setup

The economy consists of two bankers, a consumer and a producer. There are three dates (0, 1, 2).

Bankers (denoted with A and B) each have an endowment of one at date 0 and no endowments at the other dates. Bankers derive higher utility from consumption at earlier dates:

\[ u^b(c^b_0, c^b_1, c^b_2) = \alpha^2 c^b_0 + \alpha c^b_1 + c^b_2, \text{ with } \alpha > 1. \]  

(1)

The consumer is endowed with two units of funds at date 0 and has no time preference in consumption:

\[ u^c(c^c_0, c^c_1, c^c_2) = c^c_0 + c^c_1 + c^c_2. \]  

(2)

The producer has no endowment and consumes only at date 2:

\[ u^p(c^p_2) = c^p_2. \]  

(3)

At date 0 banker A has access to two projects: an economy-wide project (the “common” project) and a project that is only available to him (the “alternative” project). The project choice is not observable. Banker B has only access to the common project.\(^7\) The returns on the common and the alternative project are independently and identically distributed. Each banker can undertake only one project; we can hence summarize the projects in the economy by \(C\) (correlated projects) and \(U\) (uncorrelated projects).

A project requires one unit of funds at date 0. At date 1, it returns an amount \(\tilde{x}\), which is uniformly distributed on \([x, \bar{x}]\) (and hence has a mean of \(\mu := \frac{x + \bar{x}}{2}\)). At this date the banker can also decide to exert effort. Effort increases the expected return on the project.

\(^7\)This is without loss of generality since there is no benefit to having two alternative assets in our economy.
at date 2 but comes at a private cost of $z > 0$. At date 2 a project fails with probability $p_F$, in which case its return is zero. With probability $p_H$ the project reaches a high state and returns $R_H$ ($R_H > 1$). With probability $p_L$ ($p_F + p_L + p_H = 1$) it reaches the low state and returns $R_L$ ($R_L < 1$). If effort had been chosen, the likelihood of the high state increases by $\Delta p$ ($> 0$) and the one of the low state decreases by $\Delta p$.

The producer has a technology available which at date 2 converts $m$ ($m > 0$) units of funds into $m + \kappa$ ($\kappa > 0$) units. The technology cannot be operated with more or less than $m$ units. There is no storage technology in the economy.

At date 0 the banker has to decide to what extent to (initially) finance the project with own funds, denoted $k_0$. The remaining financing needs $(1 - k_0)$ can be raised in the form of one-period deposits from the consumer. Deposits are fully insured\(^8\) and the deposit insurance fund is financed by lump sum taxation from the consumer at date 2. At date 1, the deposits mature and the banker decides which amount of it to renew (because of the interim return, he may only partly renew the debt). If he wants to maintain capital of $k$ in the bank, this implies that he pays off $k - k_0$ of debt and consumes the remainder $(x - (k - k_0))$.

There is a regulator who maximizes utilitarian welfare. The regulator sets capital requirements at $t = 1$ (there is no scope for separate capital requirements at $t = 0$). The purpose of capital requirements is to induce efficient effort in the economy. We assume that the return on the common (economy-wide) project is observable (but not the one on the bank-specific project). The regulator can hence condition capital requirements on the return of the common project.\(^9\)

We make the following additional assumptions.

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\(^8\)Deposit insurance simplifies the analysis by making the interest on deposits independent of the expected likelihood of project success. It is not the key source of inefficiency in the economy (which is the systemic externality on the producer).

\(^9\)This captures that a regulator may be able to set capital requirements based on the state of the economy, but not on the conditions at an individual bank.
Assumptions

1. $\Delta p(R_H - R_L) > z$,

2. $\Delta p(R_H - 1 + \bar{p}) < z$,

3. $R_L > m$.

Assumption 1) ensures that effort is efficient. Assumption 2) is a condition that will ensure that the interim return (by itself) never suffices to induce effort. Assumption 3 states that the low-state output of a single bank is sufficient to operate the producer’s technology.

Timing

The sequence of actions is as follows. At date 0, the regulator announces how date-1 capital requirements will be set depending on the interim return of the common project, $x_C$. These capital requirements can be summarized by a function $k(x_C)$ (the special case of flat capital requirements arises when $k$ does not depend on $x_C$). Following this, bank $A$ makes its project choice. After the project choice has been made, banks learn the date-1 interim return of their project $x^i$ and decide on the amount of equity financing $k^i_0$ and raise $d^i_0 = 1 - k^i_0$ of deposits. At the end of the period, the consumer and the bankers consume.

At date 1, the interim return $x^i$ realizes. Each banker decides how much capital he wants to maintain ($k^i$), observing the regulatory constraint $k^i \geq k(x_C)$. The banker hence renews an amount $d^i = d^i_0 - (k^i - k^i_0)$ of deposits. Following this, banks decide to monitor and consumption takes place by bankers and the consumer.

At date 2, the returns $R^i$ ($R^i \in \{0, R_L, R_H\}$) realize. Each banker repays the consumer – in case there are sufficient funds. Any shortfall is financed by the deposit insurance fund. Following this, the producer makes an offer to the consumer and/or the bankers for $m$ unit of funds. If he succeeds, the producer operates his technology and repays the funds. In the final stage of date 2, all agents consume.
Figure 1 summarizes the timing.

![Timeline](image)

**Figure 1: Timeline**

### 2.3 Benchmark: Project choice is observable

To establish a benchmark, we first analyze an economy in which the project choice is observable and can hence be determined by the regulator. The regulator’s actions at the beginning of date 0 hence consist of setting capital requirements \( k(x_C) \) and the project type for bank \( A \).\(^{10}\) We solve the model backwards.

At date 2 the producer needs \( m > 0 \) funds to operate his technology. If the projects of both banks have failed, there are no funds in the economy. The technology can then not be operated and the producer’s consumption is hence zero. However, if there is no joint failure, total funds are at least \( R_L \), which is larger than \( m \) by Assumption 3. The producer can then raise \( m \) units of funds by offering a return of one per unit of funds to the consumer. After operating his technology and repaying the funds, he is left with \( \kappa \), which he then consumes.

At the end of date 1, each banker has to make the effort choice. Since a banker’s pay-off is \( R_H - d^i \) in the high state and \( \max\{R_L - d^i, 0\} \) in the low state (as he possibly defaults),

\(^{10}\)The benchmark is not identical to the constrained-efficient outcome in the economy – a regulator could always resolve the moral hazard problem by allocating the date-0 endowment of the consumer to the bankers.
the condition that effort is undertaken is
\[
\triangle p(R_H - d^i - \max\{R_L - d^i, 0\}) \geq z. \tag{4}
\]

When \(d^i\) is such that there is no default in the low state \((d^i \leq R_L)\), the effort condition boils down to \(\triangle p(R_H - R_L) \geq z\), which is fulfilled by Assumption 1. When there is default in the low state \((d^i > R_L)\), we have from (4) that the expected benefit from effort is positive whenever capital exceeds a threshold \(\bar{k}\), with
\[
\bar{k} := \frac{z}{\triangle p} - (R_H - 1). \tag{5}
\]

In this case the banker will exert effort if and only if \(k \geq \bar{k}\).

At the beginning of date 1, a banker has to decide how much capital to maintain in the bank by renewing (a part of the) deposits. The interest rate on deposits is zero because of deposit insurance. The banker has a strict preference for deposit financing over equity financing because he is impatient \((\alpha > 1)\) and because deposits are mispriced due to deposit insurance. He will hence only keep the minimum capital required: \(k^i = k(x_C)\). He thus does not renew \(k - k^i_0\) (we will use from now on \(k\) as a shortcut of the rule \(k(x_C)\)) of the initial amount of deposits and consumes \(x^i - (k - k^i_0)\).

At the end of date 0, the banker has to decide how much own funds (capital) to use to finance the project. Deposit can again be raised at an interest of zero. Given that banker is impatient, he will only use capital to the extent that this is required to fulfill regulatory requirements at date 1. Hence, if \(x^i \geq k(x_c)\) (that is, if the date-1 return alone is sufficient to fulfill capital requirements), he will use debt finance only: \(k^i_0 = 0\). By contrast, if \(x^i < k(x_c)\), he will use an amount of capital that, together with the interim return \(x^i\), just allows him to fulfill the capital requirements at date 1: \(k^i_0 = x^i - k(x_C)\).

**The regulator’s problem**

The regulator maximizes welfare \(W\), consisting of the utilities of bank owners, the consumer and the producer.
We first derive a banker’s utility. The consumption of banker \( i \) is \( 1 - k^i_0 \) at date 0, \( x^i - (k - k^i_0) \) at date 1 and \( \max \{ R^i - d^i, 0 \} \) at date 2. The banker’s total expected utility is hence \( u^{b;i} = \alpha^2 (1 - k^i_0) + \alpha (k^i_0 - (k - x^i)) + \max \{ R^i - d^i, 0 \} - Mz \), where \( M \in \{ 0, 1 \} \) indicating whether effort is exerted. Recalling that \( k^i_0 = \max (x^i - k(x_C), 0) \) and \( k^i = k(x_C) \) (and hence also that \( d^i = 1 - k^i = 1 - k(x_C) \)) this can be rewritten as

\[
  u^{b;i} = \alpha^2 - (\alpha^2 - \alpha) \max \{ k - x^i, 0 \} - \alpha (k - x^i) + \max \{ R^i - (1 - k), 0 \} - Mz. \tag{6}
\]

The utility of the consumer (before contribution to the deposit insurance fund) is simply one as he does not have a time preference and the interest rate is zero. The losses to the deposit insurance fund is \( \max \{ d^A - R^A, 0 \} + \max \{ d^B - R^B, 0 \} \). Using \( d^A = d^B = 1 - k \), we can write consumer’s total utility as

\[
  u^c = 2 - \max \{ 1 - k - R^A, 0 \} - \max \{ 1 - k - R^B, 0 \}. \tag{7}
\]

Let us define the total utility of a bank as the utility of its banker minus the impact of the bank on the deposit insurance fund. Recalling that the latter is \( \max \{ d - R, 0 \} = \max \{ 1 - k - R, 0 \} \), total utility for a bank of type \( t \) is given by

\[
  u^T_t(k(x_C)) := u^b_t - \max \{ 1 - k - R_t, 0 \}, \tag{8}
\]

where \( t = C \ (U) \) indicates whether the bank operates a correlated or uncorrelated project.

Taking expectations at date 0 we obtained for the total expected utility:

\[
  U^T_t(k(x_C)) := E[\alpha^2 - (\alpha^2 - \alpha) \max \{ k - x_t, 0 \} - \alpha (k - x_t) + R_t + (k - 1) - Mz]. \tag{9}
\]

The producer consumes \( \kappa \) whenever at least one bank survives, otherwise he obtains zero. His utility is hence

\[
  \kappa^p_2 = \begin{cases} 
    \kappa & \text{if } R^A + R^B > 0, \\
    0 & \text{otherwise.}
  \end{cases} \tag{10}
\]
Recalling that the producer obtains $\kappa$ if at least one bank does not fail, we have for his expected utility in the correlated and the uncorrelated economy:

\[
U_p^C = (1 - p_F)\kappa \tag{11}
\]

\[
U_p^U = (1 - p_F^2)\kappa. \tag{12}
\]

We can write welfare in the economy as the sum of the total (expected) utilities ($U_T(k)$) at the two banks, the consumer’s endowment (2) and producer’s utility ($U_p$). We obtain in the case of a correlated and an uncorrelated economy:

\[
W_C(k(x_C)) = 2U_T^C(k) + 2 + (1 - p_F)\kappa \tag{13}
\]

\[
W_U(k(x_C)) = U_T^C(k) + U_T^U(k) + 2 + (1 - p_F^2)\kappa. \tag{14}
\]

The regulator’s problem can then be formalized as \(\max_{t \in \{C,U\}, k(x_C)} W_t(k)\).

We first solve for the welfare-maximizing policy function, \(k^*(x_C)\), for given project choice in the economy (C or U).

**Proposition 1** Optimal capital requirements take the form

\[
k^*(x_C) = \begin{cases} 
\tilde{k} & \text{if } x_C \geq \hat{x}^* \\
0 & \text{otherwise},
\end{cases} \tag{15}
\]

where \(\hat{x}^*\) is given by

\[
\hat{x}^* = \begin{cases} 
\hat{x}_C = (\frac{1}{\alpha} + 1)\tilde{k} - \frac{\Delta p(R_H - R_L) - z}{\alpha^2 - \alpha} & \text{if projects are correlated} \\
\hat{x}_U = 2 \left( (\frac{1}{\alpha} + 1)\tilde{k} - \frac{\Delta p(R_H - R_L) - z}{\alpha^2 - \alpha} \right) - \mu & \text{if projects are uncorrelated}.
\end{cases} \tag{16}
\]

**Proof.** Conditional on the effort choice, capital requirements \(k\) reduce welfare because of the banker’s impatience. When \(k \geq x_t\), this is because higher capital requirements require the banker to give up date-0 consumption for date-2 consumption (from equation (6) we have for the utility impact: \(\frac{\partial u_b}{\partial k} = -\alpha^2\)). When \(k < x_t\), this is because the banker has to give up date-1 consumption for date-2 consumption (we have then \(\frac{\partial u_b}{\partial k} = -\alpha\)). It follows that the only benefit of capital requirements is to induce effort. This implies, first, that it
is not optimal to set capital requirements such that the bank does not default in the low state (if there is no default, the banker strictly prefers to exert effort by Assumption 1 and capital could be reduced without any cost). We can hence presume default and the effort choice is governed by the critical capital level $\overline{k}$ defined by (5). Second, any level of $k$ in the ranges $k \in (0, \overline{k})$ and $k \in (\overline{k}, \infty)$ is also suboptimal, because in these intervals capital can equally be reduced without affecting the effort choice. Thus, the regulator has to consider only two levels of capital requirements: $k = 0$ and $k = \overline{k}$.

We next derive the net (social) benefit from effort at a bank for a given $x_C$. For this define (equivalently to $U_t^T(k(x_C)))$ with $\tilde{U}_t^T(k(x_C), x_C) = E[u_t^b - \max\{1 - k - R_t, 0\}|x_C]$ the utility from pay-offs at a bank conditional on $x_C$. The net benefits from effort are then given by $\tilde{U}_t^T(\overline{k}, x_C) - \tilde{U}_t^T(0, x_C)$. Denoting these benefits by $\Delta \tilde{U}_t^T(x_C)$, we obtain:

$$\Delta \tilde{U}_t^T(x_C) = \Delta p(R_H - R_L) - z - (\alpha^2 - \alpha)(\overline{k} - E[x_t|x_C]) - (\alpha - 1)\overline{k}. \quad (17)$$

The first two terms $(\Delta p(R_H - R_L) - z)$ are simply the benefit from effort in the absence of an incentive problem. The other two terms are the costs of inducing effort through capital requirements. They arise because capital requirements force the banker to shift an amount of consumption $\overline{k}$ from date 1 to date 2, the cost of which is $(\alpha - 1)\overline{k}$. In addition, if the interim return at date 1 is insufficient to fulfill capital requirements ($x_t < \overline{k}$), he also has to give up consumption at date 0. The cost arising from this are $(\alpha^2 - \alpha)(\overline{k} - E[x_t|x_C])$.

Noting that $E[x_C|x_C]) = x_C$ and $E[x_U|x_C]) = \mu$, we can see that the benefits from effort are strictly increasing in $x_C$ for a common project and independent of $x_C$ for an alternative project. Since at least one project in the economy is common, it follows that effort benefits in the economy are always increasing in $x_C$. Hence, there will be a threshold $\hat{x}$, such that for $x_C \geq \hat{x}$ it is optimal to set $k = \overline{k}$ and for $x_C < \hat{x}$ it is optimal to set $k = 0$. When both banks are operating the common project, the policy maker is indifferent to inducing effort when $2 \Delta \tilde{U}_C^T(\hat{x}) = 0$. Solving this yields $\hat{x}_C$. When one project is alternative, the policy maker is indifferent if $\Delta \tilde{U}_C^T(\hat{x}) + \Delta \tilde{U}_U^T = 0$. Solving yields $\hat{x}_U$. ■
Proposition 1 implies that optimal regulation is countercyclical in the following sense. When the economy is in a good state (that is, when the common project pays off well in the interim), it is optimal to set high capital requirements ($k = \bar{k}$). Conversely, in bad states, it is optimal to set low (zero) capital requirements ($k = 0$).

**Corollary 1** Optimal regulation is countercyclical, that is,

$$\text{Cov}(k^*(x_C), x_C) > 0.$$  \hspace{1cm} (18)

**Proof.** See appendix.  

The intuition for this result is the following. While the benefits from effort are independent of the state of the economy, the cost of inducing effort is higher in bad states. This is because capital at banks is then low (because of low interim returns), making it more costly to induce effort using capital requirements.\(^{11}\) For sufficiently low capital it becomes then optimal to forego the benefits from effort.

Another implication of Proposition 1 is that the critical state of the economy where capital requirements should be lowered depends on the correlation of projects. This has the following consequences for optimal countercyclicality:

**Corollary 2** The optimal degree of countercyclicality is lower in the uncorrelated economy unless we are in the special case where $\mu$ equals \(\left(\frac{1}{\alpha} + 1\right)\bar{k} - \frac{\Delta p(R_H - R_L)}{\alpha^2 - \alpha} \). In this special case, countercyclicality is the same as in the correlated economy.

**Proof.** See appendix.  

The reason is that while in the correlated economy countercyclical capital requirements lower capital costs at both banks, in the uncorrelated economy they only do so at one

\(^{11}\)Capital requirements are here more costly in bad states since the pool of capital is then lower. A similar effect would arise if raising capital in bad states is more costly due to a more pronounced adverse selection problem.
bank. The gains from countercyclicality are thus lower in the uncorrelated economy and hence it is optimal to choose a lower degree of it.

Proposition 1 states the optimal policy rule for given projects. Whether it is optimal to have correlated or uncorrelated projects in the economy can then be determined by comparing the welfare levels that obtain in either case, presuming that the regulator implements the respective policy rules of Proposition 1.

In order to obtain an intuition for the determinants of the optimal project choice, let us presume for a moment that the regulator imposes the same capital requirement rule – characterized by a threshold $\hat{x} \in (x, \bar{x})$ – irrespective of the correlation choice. In this case we obtain from comparing (13) and (14) that a correlated economy provides higher welfare than an uncorrelated economy if and only if

$$U^T_C(k_{\hat{x}}(x_C)) - U^T_U(k_{\hat{x}}(x_C)) > (p_F - p_F^2) \kappa,$$

where $k_{\hat{x}}(x_C)$ denotes the policy function of the form of equation (15) with threshold $\hat{x}$.

The right-hand side of (19) is the expected cost of choosing correlated projects. It arises because there is a higher likelihood of joint bank failure in the correlated economy ($p_F$ instead of $p_F^2$). Joint failures are costly because the producer can then no longer operate his technology and the surplus $\kappa$ is lost.

The term $U^T_C(k_{\hat{x}}(x_C)) - U^T_U(k_{\hat{x}}(x_C))$ on the left-hand side of (19) represents the gains from correlation. These gains arise because in a correlated economy both banks can profit from countercyclical capital requirements (while in the uncorrelated economy only one bank can benefit). Using (9) we have that

$$U^T_C(k_{\hat{x}}(x_C)) - U^T_U(k_{\hat{x}}(x_C)) = (\alpha^2 - \alpha) \mathbb{E}[\max\{k_{\hat{x}}(x_C) - x_U, 0\} - \max\{k_{\hat{x}}(x_C) - x_C, 0\}] - (\alpha^2 - \alpha) \mathbb{E}[\max\{k_{\hat{x}}(x_C) - x_U, 0\} - \max\{k_{\hat{x}}(x_C) - x_C, 0\}]$$

For $k = 0$ both terms in the squared brackets are zero, while for $k = \bar{k}$ they are positive.
(because of Assumption 2). We can hence simplify

\[
U_C^T(k_{\bar{x}}(x_C)) - U_U^T(k_{\bar{x}}(x_C)) = (\alpha^2 - \alpha) \int_{\bar{x}}^{\bar{x}} (x_C - \mu) \frac{1}{\bar{x} - \bar{x}} dx_C = (\alpha^2 - \alpha) \frac{\text{Cov}(k_{\bar{x}}(x_C), x_C)}{\bar{k}}.
\]

(21)

\[U_C^T(k_{\bar{x}}(x_C)) - U_U^T(k_{\bar{x}}(x_C))\] is hence strictly positive whenever the policy rule is countercyclical (\(\text{Cov}(k_{\bar{x}}(x_C), x_C) > 0\)). The reason is that under countercyclical capital requirements common projects have lower costs as such capital requirements tend to be low when capital from common projects is scarce.\(^{12}\)

When the regulator tailors capital requirements to the correlation choice, additional effects arise because optimal capital requirements depend on correlation in the economy. From equations (13) and (14) we then have that welfare in the correlated economy is higher if and only if

\[
2U_C^T(k_{\bar{x}}(x_C)) - U_C^T(k_{\bar{x}}U(x_C)) - U_U^T(k_{\bar{x}}U(x_C)) > (p_F - p_F^2)\kappa.
\]

(22)

From this one can derive Proposition 2:

**Proposition 2** Correlation is optimal if and only if

\[
(\alpha^2 - \alpha) \frac{\text{Cov}(k_{\bar{x}}U, x_C)}{\bar{k}} + 2 \int_{\bar{x}}^{\bar{x}} \Delta \tilde{U}_C^T(x_C) \frac{1}{\bar{x} - \bar{x}} dx_C > (p_F - p_F^2)\kappa.
\]

(23)

**Proof.** See appendix. ■

As to be expected, condition (23) states that in order for correlated projects to be optimal, the costs of correlation in terms of a higher likelihood of joint failure, \((p_F - p_F^2)\kappa\), have to be low. Interestingly, for sufficiently small \(\kappa\) (the cost of a systemic crisis), correlation is always optimal.

\(^{12}\)The insight that correlation can be beneficial can be applied to other contexts as well. For instance, a monetary union benefits from its members being similar since interest rates set by the central bank then more easily reflect the individual conditions of the members.
2.4 Optimal capital requirements when project choice is unobservable

We now assume that the regulator cannot observe the project type. The consequence is that the correlation choice has to be privately optimal for bank $A$. Specifically, at date 0 the regulator announces the policy rule $k(x_C)$ and bank $A$ chooses a project depending on this policy rule. We constrain the analysis of capital requirements to step functions as in (15).

The financing decisions at date 0 and 1 are unchanged. At date 1, a bank will use an amount of equity financing to just fulfill the capital requirements ($k = k(x_C)$), while at date-0 a bank will have equity funding only to cover shortfalls at date 1 ($k_{0,t} = \min\{k(x_C)) - x_t, 0\}$). The effort choices of banks at date 1 are the same as well: a bank monitors if and only if capital requirements are at least $\bar{k}$, as defined in equation (5). There is also no change to the behavior of the producer.

This leaves to analyze the project choice of bank $A$. When deciding in which project to invest, the bank takes as given the policy rule $k(z(x_C))$. Writing the expression for banker’s utility (equation (6)) for a correlated and an uncorrelated project, taking difference and taking expectations at $t = 0$, we obtain the expected gains from choosing the common (as opposed to the alternative project)

\[
U^b_C(k(x_C)) - U^b_U(k(x_C)) = (\alpha^2 - \alpha)E[-\max\{k(z(x_C)) - x_C, 0\} + \max\{k(x_C)) - x_U, 0\}].
\]

(24)

Note that equation (24) is identical to the total utility difference from pay-offs at the bank (see equation (20)) under a fixed policy rule. Using (21) we hence have that

\[
U^b_C(k(x_C)) - U^b_U(k(x_C)) = (\alpha^2 - \alpha)\frac{\text{Cov}(k(x_C), x_C)}{\bar{k}}.
\]

(25)

Assuming a (weak) preference for uncorrelated projects, we obtain for the correlation choice:
Proposition 3  Banks choose correlated projects if and only if the policy rule is countercyclical \((\text{Cov}(k_{\hat{x}}(x_C), x_C) > 0)\).

\textbf{Proof.} Follows directly from (25). \(\blacksquare\)

The project choice is, however, not necessarily socially efficient. This is because a banker ignores the impact on the producer – who suffers in the event of joint failure. Since the likelihood of joint failure is higher for correlated projects, choosing the common project is associated with a negative externality.

This will result in an inefficient project choice whenever the policy rule is countercyclical (and bank \(A\) hence chooses correlation) but no correlation is welfare-optimal:

\textbf{Corollary 3} For a given policy rule \(k_{\hat{x}}(x_C)\), banks may choose correlated projects even though no correlation leads to higher welfare. This occurs precisely when \(\text{Cov}(k_{\hat{x}}(x_C), x_C) > 0\) and condition (23) is not fulfilled.

It follows that there are situations where the welfare level of the benchmark case can no longer be obtained. In fact, this happens whenever in the benchmark uncorrelated projects are welfare-maximizing. Since welfare-maximizing regulation (in the benchmark case) requires countercyclical capital requirements, banks would find it privately optimal to choose correlated projects, necessarily resulting in lower welfare:

\textbf{Corollary 4} Whenever condition (23) is not fulfilled, attainable welfare is lower than in the benchmark case.

\textbf{The regulator’s problem}

When correlation is optimal in the benchmark case (that is, condition (23) is fulfilled), the regulator can still obtain the same level of welfare as before. For this he simply sets (countercyclical) capital requirements to \(\hat{x}_C\) and banks (efficiently) choose correlated projects. In the case where the benchmark stipulates no correlation, we know that we can
no longer reach the welfare level of the benchmark case as optimal capital requirements are countercyclical and would hence induce banks to choose correlated projects (Corollary 4). This still leaves open what the regulator should do in this case.

Suppose first that the regulator implements correlation in the economy. In this case the regulator is not constrained by banks’ private incentives (since banks have a bias towards correlation). The regulator can hence set a threshold identical to the one in the benchmark case: \( \hat{x} = \tilde{x}_C \). Consider next that the regulator wants to implement an uncorrelated economy. In this case, the regulator is constrained by the incentive compatibility constraint of bank \( A \). Proposition 3 tells us that he then has to choose a policy that is not countercyclical. Since procyclical policies cannot be optimal, he will hence choose flat (state-independent) capital requirements. This implying that effort is either never or always induced.

Proposition 4 derives next the condition for when it is optimal to implement a correlated economy.

**Proposition 4** Correlation is optimal when condition (23) is met or when

\[
2 \left( \int_{\hat{x}_C}^{x} \frac{1}{x} \Delta \tilde{U}^T_C(x_C) \, dx_C - \max\{\Delta \tilde{U}^T_U, 0\} \right) \geq \kappa (p_F - p_F^2).
\]

The optimal policy rule is then \( \hat{x}_C \). Otherwise, no correlation is optimal and the policy rule is flat and given by

\[
\hat{x}_U = \begin{cases} 
\bar{x} & \text{if } \Delta \tilde{U}^T_U > 0, \\
\bar{x} & \text{otherwise}.
\end{cases}
\]

**Proof.** See appendix. □

Note that Proposition 4 implies that whenever it is optimal to implement uncorrelated projects, the regulator has to reduce the countercyclicality of capital requirements (compared to the benchmark case).
2.5 The role of commitment

We have assumed that at the beginning of date 0, the regulator can commit to a policy rule. In this section we relax this assumption. We assume that the regulator decides on the policy rule at the same time as when projects are chosen. Specifically, the regulator and bank $A$ play Nash at date 0: the regulator maximizes welfare taken as given the project choice of bank $A$, while banker $A$ maximizes his utility taken as given the policy function.

Consider first a (candidate) equilibrium with correlated projects. In such an equilibrium, the best response of the regulator is $\hat{x}_C$ (since $\hat{x}_C$, by Proposition 1, is the optimal policy given that projects are correlated). Since $\hat{x}_C$ is countercyclical, it is also optimal for bank $A$ to choose the common project (Proposition 3). Correlation and a policy rule of $\hat{x}_C$ thus form an equilibrium.

Consider next a (candidate) equilibrium with uncorrelated projects. The regulator’s best response to an uncorrelated economy is $\hat{x}_U$. However, since this policy is countercyclical, a bank would want to choose the common project. An equilibrium with uncorrelated projects hence cannot exist.

We summarize:

**Proposition 5** When the regulator lacks commitment, the unique equilibrium is one with correlated projects and a policy rule of $\hat{x}_C$.

In the case where no correlation was optimal without commitment problems, welfare is now lower compared to the commitment case. Lack of commitment thus amplifies the cost of countercyclical policies arising from banks’ correlation incentives.

3 Discussion

In this section we first discuss robustness of several aspects of the model. Following this, we discuss some implications of the model, including for policy.
Funding choices and the interim return. We assumed that banks make funding choices at date 0, knowing the return at date 1. This is not essential for the results. If \( x \) becomes known only at date 1, a bank has to use an amount of equity financing at date 0 that is sufficient for fulfilling capital requirements in all states of the world at date 1. Countercyclical capital requirements will lower this amount by reducing capital demands in states of the world where the capital stock is low.\(^{13}\)

Strategic interactions among banks. There is no role for strategic interaction among banks in our model. To see this, consider that bank \( B \) also has a project choice. Since the policy rule \( k(x_C) \) is set before the project choices, the project choices of bank \( A \) and \( B \) are interdependent. Hence, their strategies do not affect each other. Introducing a strategic interaction could either strengthen or weaken the correlation externality. For example, if banks benefit from bail-outs in the event of joint failures (Acharya and Yorulmazer (2007)), this will further increase their correlation incentives. Alternatively, higher correlation among banks can result in interbank externalities by eliminating the possibility for other banks to buy up assets of troubled banks (Wagner (2011)). Such interbank externalities will tend to result in higher correlation than socially optimal. Strategic incentives may also reduce correlation incentives because a surviving bank may enjoy higher benefits when the other bank fails. This may for instance arise because of reduced competition (the “last-bank-standing effect”, see Perotti and Suarez (2002)).

Cycle-dependent gains from consumption. We have assumed that the banker’s marginal utility at each date is constant, and hence independent of the state of the economy. It is conceivable that in bad (aggregate) states, the marginal utility is higher (because consumption is then lower). This would strengthen the rationale for countercyclical policies as it gives rise to an additional reason for lowering capital requirements (which have the

\(^{13}\)For a correlated bank, the capital needed to be transferred to date 1 is \( \max_{x_C \in [x]} \{k(x_c) - x_C\} \). For larger \( \text{Cov}(k(x_c), x_C) \) (that is, larger countercyclical), this expression will tend to be smaller.
effect of reducing consumption of the banker) in downturns.

**Cycle-dependent monitoring benefits.** In our model the benefit from monitoring is independent of the state of the economy. One may envisage a setting where monitoring is more effective in bad states of the world as assets are then more risky. This effect, if strong enough, could in principle lead to the optimality of procyclical capital requirements. In this case there would no longer be a trade-off between effort provision and correlation incentives.

**Deposit insurance.** Assuming the presence of a deposit insurance system has simplified the analysis but is not crucial for the results. In the absence of deposit insurance, there is no need for regulators to impose capital requirements for the purpose of inducing efficient effort. Rather, depositors themselves can require bankers to hold certain levels of capital at date 1 (if contractionally feasible). However, such capital requirements will not be socially efficient because they not address the externality on the producer (there will still be a tendency for inefficiently high correlation in the economy). Hence a rationale for regulation of capital at banks remains. The determination of optimal capital requirement will then be subject to the same trade-off as in the model (efficient effort versus systemic risk-taking).

**Systemic externality.** Our assumption that the producer can extract the full surplus on production is an extreme one. For the externality to hold, however, it is only important that banks (individually) can not extract the full surplus. In principle, the technology could also be operated by one of the banks. The externality would then become an interbank externality. This is because one bank would ignore that if it decides to become correlated with the other bank it reduces the likelihood that the other banks has sufficient resources to carry out the project.\(^{14}\)

\(^{14}\)Calculations available on request from the authors.
Interbank markets. The capital endowments of banks can differ at date 1. Nevertheless, there are no gains from trade and hence no scope for interbank markets where banks can borrow and lend to each other. This is because addressing the moral hazard problem requires inside equity, funds obtained from the other bank cannot improve incentives.

Bank-specific capital requirements. The cost of countercyclical policies (in the form of higher correlation) could be avoided entirely if capital requirements can be made contingent on bank’s individual project returns ($x^A$ and $x^B$) instead of the return on the common project only (as we have assumed). In this case regulators can isolate each bank against shocks to its own capital, and there is hence no longer an incentive to increase exposure to common risk. However, such capital requirements do not seem attractive for several reasons. First, they have high informational requirements as the regulator then needs to observe individual bank conditions. Second, there are issues of inequality and competition as weaker banks would be subjected to less stringent regulation. Third, it creates obvious moral hazard problems to the extent that banks can influence the return on their projects.

Reducing procyclicality versus reducing cross-sectional risk. Our model suggests that if tools are available that can directly influence the correlation choices of banks,\(^{15}\) they are to be preferred over countercyclical measures. This is because reducing correlation has two benefits. First, it lowers the likelihood of a systemic crisis (joint bank failure) and the costs associated with it. Second, it lowers the sensitivity of bank capital to shocks (the volatility of aggregate bank capital is lower in the uncorrelated economy), reducing the need for countercyclical policies.

Countercyclical capital requirements, in contrast, have the cost of increasing correlation risk – as we have shown. Perversely, they can even increase the sensitivity of the economy to

\(^{15}\) Examples of such tools include capital requirements based on measures of banks’ systemic importance, such as the CoVar (Adrian and Brunnermeier, 2011) or the Systemic Expected Shortfall (Acharya et al., 2012).
aggregate conditions. To see this, consider that starting from flat capital requirements, the regulator (marginally) increases countercyclicality. The economy will then move from an uncorrelated to a correlated equilibrium (Proposition 3). This will increase the likelihood of joint failures but also increase the sensitivity of aggregate bank capital to shocks. The latter is because shocks now affect both banks equally – while the (marginal) increase in countercyclicality will only have a second-order effect.

**Managerial herding.** The mechanism that leads to higher correlation in our model (arising because countercyclical policies reduce expected capital costs at banks) is only one possible one of many. For instance, countercyclical policies may also be conducive to herding by bank managers. This is because such policies make it more likely that following alternative strategies results in underperformance relative to peers as the manager then cannot benefit from the smoothing of shocks enjoyed by other banks that expose themselves predominantly to aggregate shocks.

**Countercyclical policies in developing countries.** Our analysis suggests a positive relation between the extent to which regulators use macroprudential tools to offset economic fluctuations and the extent to which banks correlate with each other. While with the exception of Spain, capital requirements have not been consistently used for macroprudential purposes, Federico et al. (2012) show that many developing countries have made active use of reserve requirements over the business cycle. Defining countercyclicality as the correlation of reserve requirements with GDP, they find that the majority of these countries used reserve requirements in a countercyclical fashion.
Consistent with the predictions of our model, we can indeed observe a positive relationship between countercyclicality and bank correlation: the correlation coefficient is 0.38 (albeit insignificant due to the small number of observations).

4 Conclusion

We have developed a simple model in which there is a rationale for regulation in reducing the impact of shocks on the financial system. In addition, in this model aggregate risk is

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\[^{16}\text{Correlations are calculated based on the weekly stock returns of all listed banks in the year prior to September 2012. Six countries had to be dropped due to an insufficient number of listed banks.}\]
endogenous since banks can influence the extent to which they correlate with each other. We have shown that countercyclical macroprudential capital requirements – while reducing the impact of shocks on the economy ex-post – provide banks with incentives to become more correlated ex-ante. This is because such capital requirements lower a bank’s cost from exposure to aggregate risk – but not the cost arising from taking on idiosyncratic risks. The overall welfare implications of countercyclical policies are hence ambiguous.

Our results have important consequences for the design of macroprudential policies. First, policy makers typically view different macroprudential tools in isolation: there are separate policies for dealing with procyclicality (e.g., countercyclical capital buffers) and correlation risk (e.g., higher capital charges for Systemically Important Financial Institutions as under Basel III). Our analysis suggests that there are important interactions among these tools. In particular, policies that mitigate correlation are a substitute for countercyclical policies since lowering correlation also means less procyclicality (while the reverse is not true). This suggests that if regulators prefer to employ a single policy instrument (for political or for practical reasons), they should focus on reducing cross-sectional risk rather than on implementing countercyclical measures.

Second, Basel III envisages countercyclical capital buffers that are imposed when (national) regulators deem credit expansion in their country excessive. Such discretionary buffers create a new time-inconsistency problem since a regulator will always be tempted to lower capital requirements in bad times, while it will be difficult for regulators to withstand pressure and raise capital requirements in boom times. Our analysis suggests in that context that providing domestic regulators with the option to modify capital requirements during the cycle may be counterproductive for the objective of containing systemic risk as it may increase banks’ correlation incentives.

\[17\] BCBS (2010) and Drehmann et al. (2011) recommend the buffer be linked to the gap between the credit-to-GDP ratio of a country and its trend. Repullo and Saurina (2011) warn that overreliance on such measures can lead to increased procyclicality because of imperfections in the credit-to-GDP gap measure.
Finally, while our model considers capital requirements as policy tool, any alternative policy that smooths the impact of aggregate shocks will likewise suffer from the problem that it increases correlation incentives in the economy. Our argument hence applies to a wide range of policies, ranging from countercyclical liquidity and reserve requirements, suspension of mark-to-market pricing in times of stress to general macroeconomic stabilization policies (such as “leaning against the wind” by the central bank).
References


Appendix

Proof of Corollary 1. We have that  
\[ \text{Cov}(k^*(x_C), x_C) = \int_{\tilde{x}}^{\bar{x}} (k^*(x_C) - E[k^*(x_C)]) (x_C - \mu) \frac{1}{x-x} \, dx_C, \]
which can be simplified to  
\[ \text{Cov}(k^*(x_C), x_C) = k \int_{\tilde{x}}^{\bar{x}} (x_C - \mu) \frac{1}{x-x} \, dx_C = \frac{k}{4} \frac{(\bar{x}-\tilde{x})^2}{x-x} > 0 \]
for \( \tilde{x} \in (\underline{x}, \bar{x}) \). □

Proof of Corollary 2. From  
\[ \text{Cov}(k^*(x_C), x_C) = \frac{k}{4} \frac{(\bar{x}-\tilde{x})^2}{x-x} \]
(see proof of Corollary 1) we have that the covariance attains its minimum at  
\( \tilde{x} = \frac{\bar{x}+\underline{x}}{2} = \mu \) and is a monotonous function on the intervals \([\underline{x}, \mu]\) and \([\mu, \bar{x}]\). The corollary then follows from the fact that for  
\( \tilde{x}_C < \mu \) we have \( \hat{x}_U < \hat{x}_C \) and that for  
\( \tilde{x}_C > \mu \) we have \( \hat{x}_U > \hat{x}_C \). □

Proof of Proposition 2. If  
\( \hat{x}_C > \mu \) (and hence  
\( \hat{x}_C < \hat{x}_U \) since we have  
\( \hat{x}_U = 2\tilde{x}_C - \mu \)) by equation (16)) we obtain  
\[ U_C^T(k_{\tilde{x}_C}(x_C)) = U_C^T(k_{\tilde{x}_U}(x_C)) + \int_{\tilde{x}_C}^{\tilde{x}_U} \Delta U_C^T(x_C) \frac{1}{x-x} \, dx_C. \]  
Using in addition equation (21) (written for  
\( \hat{x} = \hat{x}_U \)) to substitute  
\( U_C^T(k_{\tilde{x}_U}(x_C)) - U_U^T(k_{\tilde{x}_U}(x_C)) \), we can rewrite equation (22) as  
\[ \frac{(\alpha^2 - \alpha)}{k} \text{Cov}(k_{\tilde{x}_U}, x_C) + 2 \int_{\tilde{x}_C}^{\tilde{x}_U} \Delta U_C^T(x_C) \frac{1}{x-x} \, dx_C > \frac{PF - p_F^2}{\kappa}. \]  
Similarly, if  
\( \hat{x}_C < \mu \) (and hence  
\( \hat{x}_C > \hat{x}_U \)), we can rewrite equation (22) as  
\[ \frac{(\alpha^2 - \alpha)}{k} \text{Cov}(k_{\tilde{x}_U}, x_C) - 2 \int_{\tilde{x}_U}^{\tilde{x}_C} \Delta U_C^T(x_C) \frac{1}{x-x} \, dx_C > \frac{PF - p_F^2}{\kappa}. \]  
Combining (29) and (30) gives (23). □

Proof of Proposition 4. The optimality of correlation when condition (23) is fulfilled (that is, correlation is optimal in the benchmark case) is obvious as then the incentive constraint of bank A is irrelevant. Consider next that condition (23) is not fulfilled.

If the regulator wants to implement correlation, he is still not constrained by the incentive constraint of bank A, and can hence choose the same policy as in the benchmark case:  
\( \hat{x} = \hat{x}_C \). If he wants to implement an uncorrelated outcome, he has to choose a policy that is either procyclical or flat. Procyclical policies are always dominated by flat policies as the
former require higher capital when capital is scarce but have no benefits. The regulator hence chooses a flat policy, of which there are two: either he always sets \( k = 0 \) (that is, a threshold of \( \tilde{x} = \tilde{x} \)) or \( k = \bar{k} \) (that is, a threshold of \( \tilde{x} = \bar{x} \)). Which of the two dominates depends on whether in expectation it is beneficial to always induce effort or not, that is, on the sign of 

\[
E[\Delta \tilde{U}_C^T(x_C)] + E[\Delta \tilde{U}_U^T(x_C)] = \Delta \tilde{U}_C^T(\mu) + \Delta \tilde{U}_U^T = 2 \Delta \tilde{U}_U^T. \]

If \( \Delta \tilde{U}_U^T > 0 \), then setting \( k = \bar{k} \) is optimal, otherwise \( k = 0 \) is optimal.

In order to determine whether correlation is optimal, we have to compare welfare for the threshold \( \tilde{x}_C \) (correlation) with welfare under the two flat capital requirements (no correlation). Thus, we have to compare \( W_C(k_{\tilde{x}_C}(x_C)) \) with the maximum of \( W_U(k_{\tilde{x}}(x_C)) \) and \( W_U(k_{\bar{x}}(x_C)) \). The three respective welfare levels are given by:

\[
W_C(k_{\tilde{x}_C}(x_C)) = 2(\alpha + \mu + p_H R_H + p_L R_L) + 2 \int_{\tilde{x}_C}^{\bar{x}} \Delta \tilde{U}_C^T(x_C) \frac{1}{\bar{x} - \tilde{x}} \text{d}x_C - p_F \kappa \tag{31}
\]

\[
W_U(k_{\tilde{x}}(x_C)) = 2(\alpha + \mu + p_H R_H + p_L R_L) - p_F^2 \kappa \tag{32}
\]

\[
W_U(k_{\bar{x}}(x_C)) = 2(\alpha + \mu + p_H R_H + p_L R_L) + 2 \Delta \tilde{U}_U^T - p_F^2 \kappa. \tag{33}
\]

Rearranging \( W_C(k_{\tilde{x}_C}(x_C)) > \max\{W_U(k_{\tilde{x}}(x_C)), W_U(k_{\bar{x}}(x_C))\} \) using (31)-(33) yields (26).