The Dynamics of Hours Worked and Technology∗

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Abstract

We study the relationship between hours worked and technology during the postwar period in the US. We show that the responses of hours to technological improvements have increased over time, and that the patterns captured by the SVAR are consistent with those obtained from an RBC model with a less than unitary elasticity of substitution between capital and labor. Data supports the hypothesis that the observed changes in the response of hours to a technology shock are attributable to changes in the magnitude of the degree of capital-labor substitution, \( \sigma \). We argue that the observed time-variation in \( \sigma \) can arise from changes in the structural composition of sectors (or factors) in a heterogeneous inputs production function or from biases in technological change.

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1 Introduction

One of the most controversial issues in business cycle theory regards the impact of technology shocks on hours worked. From a theoretical standpoint, the sign and size of the responses of hours to a positive technology shock was used as evidence to distinguish between theories, i.e. the RBC and the New Keynesian paradigms, see Galí (1999). This particular impulse response, it is argued, tells us a lot about whether we live in a world where output is largely driven by supply shocks or a Keynesian world in which output is largely demand-determined over the short-run due to price (and possibly wage) rigidities. If it is the former, then a positive technology shock induces people to work more. If it is the latter, then demand remains unchanged in the short-run and firms would lay off workers.

On pure measurement grounds, the estimated impact of technology shocks on hours using post-war US data is far from being generally agreed upon and it has been lively debated. Using long run restrictions on a VAR the sign of the response of hours is sensitive to the treatment of hours worked, i.e. if they enter in level or in first difference (see Christiano, Eichenbaum and Vigfusson (2003)). To overcome this ambiguity, researchers switched attention to different identification schemes such as sign, as opposed to zero, restrictions (see Uhlig (2004) and Dedola and Neri (2007)), richer information sets (see Fève and Guay (2010)), or alternative ways of measuring technology shocks (see Basu, Fernald and Kimball (2006) and Alexopoulos (2011)). However, sticking to the original information set and identification scheme of Galí (1999), Canova, Lopez-Salido and Michelacci (2010) and Fernald (2007), amongst others, showed that the results depend dramatically on the treatment of the long run movements of hours. If these are controlled for, with filters or by removing trend breaks, hours decline.

This result is important and sound. However, it relies on the crucial assumption that the impact of technology shocks has not changed over the post-war period. This assumption is difficult to entertain given the number of structural changes undergone by the US economy from the postwar era until recent years. Consistent with this belief, Galí and Gambetti (2009) show that the response of hours has substantially changed over time, i.e. it was typically negative at the beginning of the post-war sample and it turned positive or zero towards the end. They employ a SVAR with a densely parameterized structure where both the coefficients and volatilities are allowed to vary over time following a linear autoregressive process. While they propose insightful explanations for these changes, the SVAR framework they adopt allows only for limited structural interpretations.

Our objective is to gather further evidence on the time varying relationship between hours and technology and to offer a structural explanation for such changes. First, we document the time

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1 See Galí and Rabanal (2005) for a comprehensive overview of the literature.
2 See also Stiroh (2009) for an analysis of changing unconditional correlations between productivity growth and hours growth.
variation in the response of hours to technology shocks by estimating a SVAR on overlapping win-
dows of fixed length. Regardless of the amplitude of the windows and the filter used on hours, we
find that the response of hours worked has increased over the sample. The response is negative at
the beginning of the sample and turns positive or zero towards the end in line with the findings
of Gali and Gambetti (2009). Moreover, we complement the VAR analysis with a reduced form
framework where we study the correlation between hours and an empirical measure of TFP growth
over the same overlapping windows. The estimated regression coefficients have a qualitative pattern
comparable to the conditional correlation obtained from the SVAR. We then interpret such pat-
tern using a parsimonious structural model. The model is a standard RBC where the production
technology combines labor and capital with a constant – but not necessarily unitary – elasticity
of substitution. Within this framework, following Cantore, León-Ledesma, McAdam and Willman
(2012), the response of hours to a technology shock depends crucially on the value of the elasticity
of capital-labor substitution. When we bridge the model to the same overlapping subsamples of the
SVAR, we find that the response of hours worked obtained with the RBC model mimics fairly well
those from the SVAR. With a reverse engineering exercise a la Chari, Kehoe and McGrattan (2008),
we then compare the responses of hours in a SVAR model obtained using data generated by the
estimated theoretical model to those obtained using actual data. The results support the hypothesis
that both sets of responses are similar. The driving factor behind the increase in the conditional
correlation of hours is an increase in the estimate of the elasticity of substitution over time form a
value close to 0.2 in early samples to 0.9 towards the end of the period considered.

Related literature has also aimed at explaining the changing nature of some key data moments in
the US economy regarding output, hours and productivity. An explanation that has received much
attention is the well known change in monetary policy at the beginning of the 80’s. However, this
explanation is not free from criticism. For instance, Canova and Gambetti (2009) find little support
for the role of monetary policy changes in driving output and inflation dynamics and point towards
the potential importance of changes in private sector behavior. Along these lines, Nucci and Riggi
(2009) attribute changes in the response of hours to an increase in performance-related pay schemes
during the 1980s. Their model, however, can account for a reduction in the negative response of
hours to a technology shock but not for a sign switch. Similar considerations apply to models where
these changes are attributed to reduced labor market frictions (see Galí and Van Rens (2010)). An
alternative explanation offered by Lindé (2009) and Rotemberg (2003) relies on changes in the
diffusion of technology shocks. We can account for these changes in our setting, but find that they
did not play a key role in explaining the change in the response of hours. It has to be noted, as will
be argued below, that we focus on the response of hours to a productivity shock because: i) unlike

\footnote{See amongst other Clarida, Galí and Gertler (2000), Galí, López-Salido and Vallés (2003) and Cogley and Sargent (2005).}
other moments, its change is robust to different data definitions and filters and ii) because there is
a direct, one-to-one, mapping to theory shocks, unlike in the case of non-technology shocks.

In our model, the observed variation of the elasticity of capital-labor substitution means that it
is not a structural parameter in a strict sense. We thus offer alternative explanations to interpret
these changes. We conjecture that changes in the sectoral structure of the US economy or, related,
changes in the composition of the labor force (and biased technical change) might explain the
protracted increase in the degree of capital-labor substitutability through the sample. Since the
sectoral and labor force structure has deep implications for the elasticity of substitution between
labor and capital, it can affect the way technology shocks are transmitted into the labor market
and hence on aggregate hours worked. Of course, we do not rule out the potential relevance of
alternative explanations of changes in the conditional hours-productivity correlations. Frictions and
macroeconomic policies can play a relevant role that cannot be undermined. Our paper, however,
emphasizes the importance of deeper changes in the structure of the US economy as a key driver of
this crucial changing macroeconomic correlation.

The rest of the paper is organized as follows. Section 2 presents the empirical dynamics of hours
and technology and discusses why we focus on their correlation; Section 3 presents briefly the struc-
tural model, its time-varying estimation, an indirect inference exercise in the spirit of Chari et al.
(2008) and discusses alternative structural explanations; Section 4 offers possible explanations to
rationalize the observed changes in the elasticity of substitution and section 5 concludes.

2 The Empirical Dynamics of Hours and Technology

While there is a large literature documenting the changes in the second moments of various US
times series, here we focus on the response of hours worked to a technology shock. To identify the
technological process we adopt two strategies. The first approach relies on the production function,
which can be represented in its most general form as

\[ Y_t = A_t F(K_{t-1}, Z_t H_t), \]

where \( F(\ldots) \) is a constant returns to scale production function, \( K_{t-1} \) and \( H_t \) are effective capital and
labor input services employed at time \( t \) (even allowing for unobservable variations in utilization),
\( A_t \) is a (temporary) displacement to total factor productivity and \( Z_t \) represents the permanent
technological progress.\(^4\) We can rewrite the latter in terms of the log of output per hours worked

\[^4\text{With an homogeneous production function of degree one equation (1) is equivalent to}
Y_t = F(Z_t^h K_{t-1}, Z_t^h H_t)
where \( Z_t^h = A_t \) and \( Z_t^h = Z_t A_t \)\]
(i.e. labor productivity),

\[ p_t = \ln \frac{Y_t}{H_t} = \ln A_t + \ln Z_t + \ln F \left( \frac{K_{t-1}Z_t}{H_t}, 1 \right). \]

From economic theory (see Uzawa (1961) and King, Plosser and Rebelo (1988)), we know that the only source of growth consistent with a balanced growth path must be expressible in labor augmenting form. From econometric theory (see Quah (1992)), we know that any time series made up of permanent and transitory shocks is characterized by different spectra for the persistent and temporary component, i.e. at frequency zero (long run) the permanent shock has full power and the transitory null. At any other frequency the two process have non zero power. Thus, we can uniquely identify the labor augmenting technical process form the times series of labor productivity by assuming it to be the only shock having an impact in the long run. This specification was used first by Galí (1999). The second approach to measure the relationship between hours and technology is more empirical and relies on the regression of hours worked on an empirical measure of technology. We adopt the measure of utilization adjusted quarterly TFP growth proposed by Fernald (2012) and based on Basu et al. (2006) as proxy for technology.

Our data ranges from 1948:Q1 until 2009:Q1. The times series include output in the non-farm business sector (OUTNFB), which was obtained from the FRED database, and a measure of hours worked constructed by Francis and Ramey (2009). Labor productivity is computed as the ratio between the measure of output and hours, and we take logarithms of both series. For both approaches, we estimate parameters using Bayesian methods assuming flat priors. We estimate both setups on rolling windows of fixed length, starting from the sample [1948:Q2,1978:Q1], and repeating the estimation moving the starting date by one year and stopping at [1979:Q2,2009:Q1].

In an online appendix we document the data construction and show that our results are independent of the measure of hours worked and their transformations. We consider the specifications of hours used in Galí and Gambetti (2009), Chang, Doh and Schorfheide (2007) as well as Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko and Santaclária-Llopis (2009). All databases display similar patterns. Moreover, we carry out a robustness analysis by extracting different portions of fluctuations from each measure of hours worked. We consider, in turn, the growth rate of hours, where high frequency fluctuations are emphasized, HP filtered hours with a smoothing parameter of 1600, so that fluctuations larger then 32 quarters are dampened, and quadratic detrended hours whose spectrum contains a significant portions of medium term fluctuations. We show that all the results are insensitive to these transformations with the exception of the first difference case.

We considered also different windows sizes and the results do not change. It is important to stress that smaller windows introduce a large portion of high frequency components (noise) which is typically absorbed by the parameters estimates. This, in turn, makes the time variation of the impulse responses not very smooth and more prone to small sample bias.

Available from the authors web pages.

This outcome is not surprising. A first difference filter dampens both long run and business cycles frequencies.
conclude that the time variation in the relationship between hours worked and technology shocks is robust to both data construction and filtering techniques.

2.1 Structural VAR Analysis

A reduced form Vector of Autoregression (VAR) can be represented as

\[ x_t = A_0 + A_1 x_{t-1} + \ldots + A_p x_{t-p} + u_t \]

where \( u_t \) are i.i.d. zero mean normal shocks with covariance matrix \( \Sigma \). We assume that \( u_t = \Omega \epsilon_t \) where \( \epsilon_t = [\epsilon_s^t, \epsilon_d^t] \) is a normal i.i.d. shock with \( E(\epsilon_t \epsilon_t') = I \), and where \( \epsilon_s^t \) is the technology shock and \( \epsilon_d^t \) a non-technology shock. It follows from the assumptions that \( \Sigma = \Omega \Omega' \).

We consider \( x_t = [\Delta p_t, h_t] \) in estimation, where \( h_t = \ln H_t \). For exposition purposes it is more convenient to rewrite the system in a companion form

\[ z_t = \mu + B z_{t-1} + \epsilon_t \]

where \( z_t = [x_t', x_{t-1}', \ldots, x_{t-p+1}']' \), \( \epsilon_t = [u_t', 0, \ldots, 0]' \), \( \mu = [A_0', 0, \ldots, 0]' \), and \( B \) is the companion form matrix. The long run restriction implies that the impact matrix of cumulative effects of the shock on labor productivity has a Cholesky factor, i.e. the matrix \( F = \sum_{k=0}^{\infty} S_{2,2}(B^k) \Omega \) has a lower triangular structure where \( S_{2,2}(.) \) is a selection matrix that picks the first two rows and columns of matrix \( B^k \).

Figure 1 plots the response of hours worked to a technology shock in the estimated SVAR with three lags. The response of hours worked displays significant time variations. In fact, the impact response is negative in early samples, decreases until the mid-1970s, and then increases steadily thereafter. To aid visual analysis, figure 2 reports the impulse responses for selected sub-samples with 68% credible sets. As it clearly stands out, the response of hours worked to an identified technology shock has changed over time. In particular, while it was negative during the 1960s and 1970s on impact, it was negligible during the 1980s, and hours increase following a technology shock if we consider the samples that include entirely the 1990s.

2.2 A Complementary Empirical Analysis

We present an alternative but complementary framework to measure the relationship between hours and technology. We consider the following specification

\[ h_t = \alpha_0 + \alpha_1 t + \beta(L) h_t + \delta TFP_t + \epsilon_t \]

and tends to overemphasize high frequency fluctuations. The secular change in the conditional correlation of hours and technology has more likely occurred at medium run or business cycle frequencies of the spectrum rather than at noisy frequency. Hence, by taking first differences we downsize the impact of long run and business cycle frequencies, making it difficult to spot time variations. This argument is reinforced if we consider the response of HP filtered hours or quadratic detrended hours, where business cycle frequencies are left unchanged and where we do observe a time varying pattern.
Figure 1: Response of hours worked to a technology shock over overlapping windows, the median response is reported.

Figure 2: Response of hours worked to a technology shock for selected sub-samples with 68% credible sets. The red dashed line corresponds to the mean estimate while the blue line to the median.
where $\beta(L)$ is a polynomial in the lag operator and $TFP_t$ is a measure of utilization adjusted TFP growth proposed by Fernald (2012). We are interested on the estimates of $\delta$ over overlapping subsamples. Figure 3 reports the estimates of $\hat{\delta}$ with the 68% confidence sets (dotted lines) across overlapping windows. Regardless of the number of lags, the estimated response of hours tends to increase over time, moving from negative to zero values.

![Figure 3: Estimates of $\hat{\delta}$ with the 68% confidence sets (dotted lines) across overlapping windows. It is assumed three lags.](image)

While the two approaches deliver different quantitative results, especially in the final part of the sample, the qualitative picture is very similar and points to a time-varying relationship between hours and technology identifying a smooth increase in the reaction of hours to technological improvements over time. In all, these results confirm the existence of important changes in the short-run technology-hours correlations in the US over the post-war period.

2.3 Why do we focus on the Hours-Technology correlation?

Given the information set that we consider (hours, productivity and output), there is a wide array of conditional and unconditional correlations that might have experienced changes and breaks (see, for instance, Galí and van Rens, 2010 and Nucci and Riggi, 2009). It is then legitimate to question why we focus exclusively on the correlation between hours and technology. There are, in fact, two good reasons for this choice.

While there is little ambiguity in the measurement of output, the construction of a measure of hours worked is less trivial. As mentioned earlier, and as available in an online appendix, we analyzed four measures of hours worked based on Gali and Gambetti (2009), Chang et al. (2007), Ríos-Rull et al. (2009), and Francis and Ramey (2009). For each of these measures and different filtering techniques, we looked at their business cycle properties, changes in standard deviations,
changes in unconditional correlations, and changes in conditional correlations. We find that only a few of these moments display changes that are robust across datasets and spectrum frequencies. The response of hours worked to a technology shock is one of those. While there are quantitative differences, all databases and transformations (save for first differencing) display similar patterns. A second relevant robust result is the response of labor productivity to a so-called “non-technology” shock. It also displays important time variations across different samples. In particular, the impact response of productivity turns from positive to negative in the final part of the sample. The decline in the volatility of hours, output, and productivity when a large portion of medium and long run fluctuations are removed (see also Pancrazi (2012)), and the the increase in the volatility of hours relative to output at medium term and business cycles fluctuations also appear consistent across datasets. We do not detect any more robust time-varying patterns based on conditional and unconditional correlations. In particular, the vanishing procyclicality of labor productivity (measured as the correlation of labor productivity with hours or output) is difficult to spot in databases other then Galí and Gambetti (2009). This result, thus, appears to be sensitive to both, the measure of hours used, and the frequency of fluctuations we consider, i.e. whether high, business cycle, medium-run, or all fluctuations. These conclusions also appear to be robust to the rolling window size used.

The changing response of productivity to “non-technology” shocks is in itself an interesting and challenging finding. However, interpreting this evidence through the lens of a structural (DSGE) model is problematic. This is because there is no direct link between the non-technology shock of the SVAR and an equivalent shock in the DSGE model. The technology shock does have a unique mapping, i.e. it is the only shock that has full power spectra at the zero frequency both in the SVAR and in the DSGE model. While one might question if the latter is a ‘technology’ shock, it is unquestionable that it has the same stochastic properties in the two dynamic systems. However, the non-technology shock in the SVAR captures a transitory shock, which can be represented in the structural model in different ways such as a preference process, a government spending shock, or a temporary displacement of technology. Even if we assumed that there are only preference shocks, the way they are built in as either shocks to the discount rate or shocks to the MRS between consumption and leisure, would have consequences for their sign and size effects. A change in the response to the non-technology shock in the SVAR, thus, may simply be the consequence of changes in the proportion of the variance explained by these two different shocks, as the non-technology shock would be a composite. Absent any further restrictions, the non-technology shock of the SVAR might capture, in the best case, any of these structural shocks (or their combination) and, in the worse case, misspecification. Hence, and in line with the criticisms found in Chari et al. (2008), it would not be appropriate to draw conclusions from identifying, say, the RBC-based preference shock with the SVAR-based non-technology shock.
We consider a closed economy Real Business Cycles (RBC) model. The novelty of the model is that it features a Constant Elasticity of Substitution (CES) production function, which is characterized by two sources of fluctuations, a labor- and a capital-augmenting stochastic shift to the production frontier (Cantore et al. (2012)). The model is otherwise standard, it is a single good optimizing agent framework. On similar grounds, Francis and Ramey (2005) analyze deviations from Cobb-Douglas to explain the response of hours to technology shocks. However they examined the limiting Leontieff case of a zero substitution elasticity. While this offers a potential explanation for the negative response to hours, their setup is unable to generate positive and negative responses. The advantage of our setup is that, with an elasticity of capital-labor substitution that differs from unity (the Cobb-Douglas case), even in the canonical RBC model the response of hours to a labor-augmenting technology shock can be positive or negative. Cantore et al. (2012) show analytically that the sign of the response depends crucially on the relative magnitudes of the elasticity of substitution and the capital share. Hence, ex ante, the model is able to generate conditional correlations between hours and technology of either sign.

We then bridge the model to observed data on US productivity and hours worked. We estimate the parameters of the RBC-CES model on rolling windows of the same fixed length as the SVAR and we look closely at propagation mechanism of the structural shocks. Let the solution of the DSGE model be of the form,

\[
y_{t+1} = \Phi(\vartheta)y_t + \Psi(\vartheta)\eta_{t+1}
\]

\[
x_t = Sy_{t}^\dagger
\]

where the vector \(y_t^\dagger\) contains the endogenous variables of the model, \(S\) is a selection matrix that picks hours and productivity growth from the vector of endogenous variables, and \(\eta_t\) is the structural vector of innovations with zero mean and diagonal covariance matrix \(\Sigma_\eta\). \(\Phi\) and \(\Psi\) are matrices which are non-linear functions of the structural parameters of the model, \(\vartheta\). Since we have a unique mapping from the structural parameters of the model to the reduced form matrix, we can back out the ‘deep’ parameters responsible for the changes (if any) in the transmission of shocks. Then, we look closely at the time pattern of the estimated structural parameters and try to provide intuition for such changes. Finally, we perform a ‘reverse’ exercise in the same spirit of Chari et al. (2008). We ask whether the estimates of the SVAR on data simulated from our structural model would yield similar results to those obtained with the SVAR on actual data.
3.1 The RBC model and the Elasticity of Substitution

The representative household is characterized by a separable preferences in consumption and leisure.

\[ U_t = \ln C_t - V_t \frac{H_t^{1+\gamma}}{1+\gamma}, \] (2)

where \( C_t \) denotes consumption, \( H_t \) hours worked, \( \beta \) is the discount factor, \( \gamma \) is the inverse of the Frisch elasticity, \( V_t \) is a preference shock process that has an AR(1) representation, i.e. (in log deviations from the steady state) \( v_t = \rho_v v_{t-1} + \eta^v_t \eta^v_t \sim N(0, \sigma_v) \). The production is CES and presented in normalized form as in Cantore et al. (2012)\(^8\)

\[ Y_t = y \left[ \alpha \left( \frac{Z^k_t K_{t-1}}{k} \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left( \frac{Z^h_t H_t}{h} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{1}{\sigma - 1}}, \] (3)

where, as usual, output is produced by a combination of two factors, \( K_{t-1} \) the installed physical capital at time \( t \), and \( H_t \). \( y \) and \( k \) are the steady state values of output and capital re-scaled by the labor augmenting process, and \( h \) is the steady state value for hours. \( \alpha \) and \( \sigma \) are parameters controlling the capital share in production and the degree of substitutability between factors. As \( \sigma \to 0 \), factors are net complements, and the production function is Leontief. If \( \sigma \to \infty \) factors are net substitutes and the production function is linear. As \( \sigma \) approaches 1, we have a Cobb-Douglas production function. The CES production function encompasses two types technological change, i.e. the capital augmenting, \( Z^k_t \), and the labor augmenting technological process, \( Z^h_t \). We assume that capital-augmenting technology has an AR(1) representation, i.e. (in log deviations from the steady state) \( z^k_t = \rho_k z^k_{t-1} + \eta^k_t \eta^k_t \sim N(0, \sigma_k) \), where \( \rho_k < 1 \) to ensure the existence of a balanced growth path in accordance with Uzawa (1961). For the labor-augmenting shock we adopt an autoregressive process in growth rates, i.e. \( z^h_t - z^h_{t-1} = \rho_h (z^h_{t-1} - z^h_{t-2}) + \eta^h_t \), with \( z^h_t = \ln Z^H_t - \ln Z^H_0 \). With a technology process persistent enough in growth rates, Lindé (2009) showed that the response of hours can be negative even in a standard RBC model. Parameter \( \rho_h \) can, ex ante, also generate time variations in the response of hours in our setting. Hence, changes in the diffusion of the technology shock offer an alternative explanation for changes in the response of hours even in this simple setting. By estimating the two parameters jointly, we are able to quantify the importance of these two structural parameters for the observed changes. The model is then closed by assuming that capital depreciates at rate \( \delta \) and that the economy’s resource constraint is given by:

\[ Y_t = C_t + K_t - (1 - \delta) K_{t-1}. \] (4)

As mentioned, this model has the property that the capital intensity in production and the elasticity of factor substitution, \( \alpha \) and \( \sigma \), are the main drivers of the dynamics of output and hours worked.

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\( ^8\)Normalization is required to compare responses when we change the elasticity of substitution. Also, it allows us to interpret directly the share parameter \( \alpha \) as the capital income share at the point of normalization (the steady state in this case).
conditional on a labor augmenting technology shock.

By means of a sensible calibration exercise, we can study the impact of a labor augmenting technology shock on hours worked for different values of the capital-labor substitution elasticity. We let the capital-labor elasticity vary between 0.1 and 1, and we fix the capital share in production to 0.33. Figure 4 (left panel) reports the impulse response of hours worked to a labor-augmenting technology shock for different values of $\sigma$ and $\alpha = 0.33$. Approximately, when $\sigma > \alpha$ the response of hours to a labor augmenting technology shock is positive. However, hours worked decrease if $\sigma < \alpha$. The right panel of Figure 4 displays the instantaneous response of hours worked to a labor-augmenting technology shock for different values of $\sigma$ and $\alpha$. We let the value of $\alpha$ vary between 0.2 to 0.6. Thus, approximatively for values of $\sigma$ larger than 0.7 and close to the Cobb-Douglas specification, the response of hours is positive regardless of values of $\alpha$. The intuition behind the result is that the shock induces a substitution effect that reduces the demand for labor and a quantity effect that increases the demand for labor. Depending on the strength of these two, hours may increase or decrease.

### 3.2 The dynamic transmission of technology shocks

We estimate the structural parameters of model by combining prior information and the likelihood of the data in each of the rolling windows. The choice of priors is standard. We assume an inverse gamma for standard deviations, beta distributions for the autoregressive parameters centered in 0.5, a normal distribution for the inverse of the Frisch elasticity, $\gamma$, and for the capital intensity in production, $\alpha$. The prior for $\sigma$ follows a gamma distribution centered around 0.5 and with a loose
For conciseness, we do not report the posterior estimates either for the full sample or for the overlapping windows. The results, however, showed that that data are pretty informative regarding the parameters of interest. The key fact that our setup aims to explain is the time varying relationship between hours and technology shocks and, in particular, if the model is able to reproduce the patterns found using the SVAR model. Figure 5 plots the response of hours worked to a labor augmenting technology shock. The response of hours worked shows clear shape and sign variations along the sample. Taken literally, the very early samples are characterized by a negative response. Then, the reaction of hours steadily increases and settles around positive values in the last ten rolling windows. The resemblance with the SVAR evidence is striking. On impact, the the signs of the response of hours are correctly identified. Figure 6 plots the 90% credible sets around the instantaneous response of hours with the SVAR estimates and the RBC-CES. If the instantaneous response of hours were different in the two settings, we would observe windows with non overlapping bands. It is important to note that the bands around the RBC-CES are tighter than the SVAR, and this is partly due to the proper priors imposed on the structural parameters as opposed to the flat priors on the reduced from coefficients of the SVAR. The difference in the amount of uncertainty around the impact is also due to the structure of the DSGE model whose solution is typically embedded with tight restrictions on the autoregressive part of the dynamic model. Yet, we detect no significant difference in the contemporaneous response of hours between the estimates of the SVAR and the estimates of the RBC with CES production function.

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9Given the difficulty reported in the literature to identify parameter \( \sigma \) (see León-Ledesma, McAdam and Willman (2010)), we carried out number of experiments to check its identifiability with simulated data. We find that, with 100 data points, labor productivity and hours worked contain enough information to pin down the parameters of interest with a flat prior on \( \sigma \) and standard priors on the remaining parameters. Results can be consulted in the online appendix.

10The estimates over the full sample are available in the online appendix. Estimates for the overlapping subsamples are available upon request.
The question that follows is what are the driving parameters behind the change in the propagation mechanism. Since the impulse response is computed as the marginal impact of a structural innovation to a variable, we can rule out changes in the standard deviations of the structural shocks as responsible for such variations. Even if the model is very stylized, the CES production function allows us to disentangle the scenarios where hours increase (decrease) in response to a technology innovation because the degree of factor substitution is larger (smaller) than the capital share in production. While we find little variations in the estimates of the persistence of the growth process for technology, there are large variations in the posterior estimates of the elasticity of factor substitution, in absolute terms and relative to the capital share parameter in production, $\alpha$. Figure 6 (right panel) plots the posterior mean of the elasticity of substitution, the capital share and the autoregressive parameter in each of the sub-samples. Changes in the hours-technology conditional correlation on impact are associated with changes in the elasticity of capital-labor substitution.

A few things are worth mentioning. The variation in the estimates of $\sigma$ are significant but not very smooth. This is partly due to the non parametric approach we adopt and to the uniform weighting scheme we impose on each window. One way to smooth the estimates of $\sigma$ is to downsize the impact of sub-sample endpoints. As in the sample spectrum estimation (see Priestley (1982), Ch.5), we could design a bell shape distribution so that end-of-sample points would have milder impact on structural estimates. However, we preferred to be agnostic and to give priority to the observables without imposing any ad hoc weighting scheme. The other approach is to parameterize the changes in $\sigma$ by assuming that the capital-labor elasticity follows a slow moving exogenous process (i.e. an autoregressive process). Since first order approximations are insufficient to capture such process, higher order approximations are required. With higher order solutions the implied state space system is neither linear nor gaussian, and we would require particle filters to extract the likelihood. Despite important advances in this direction (see Fernandez-Villaverde and Rubio-Ramirez (2008)),
the estimation of time-varying structures is still computationally burdensome and difficult to handle. Given these constraints, and for comparison with our SVAR results, we study what a computationally less intensive yet intuitively appealing structural method could tell us about the time varying relation between hours and productivity.\footnote{For robustness purposes, we also estimated $\sigma$ using the normalized system approach advocated by León-Ledesma et al. (2010). Given the data requirements in that approach, we used annual data for two subsamples, namely 1952-1981 and 1982-2009. We also found a substantial increase in $\sigma$ from 0.47 to 0.75.}

### 3.3 Can changes in capital-labor elasticity reproduce the SVAR evidence?

The time-varying relationship between hours and technology identified by a SVAR with long-run restrictions is very similar to the one obtained from our RBC model with CES production function. However, Chari et al. (2008), amongst others, express concerns about the ability of SVARs with long-run restrictions to identify model shocks. This may then cast doubts about whether comparisons of model-based and SVAR-based impulse-responses constitute a reliable way to evaluate our model. To address this issue, we simulate 50 sets of data of 120 observations from the RBC-CES model using the mean estimates in each window. With each simulated dataset we estimate a SVAR with 3 lags and compute the impulse response. We then compare the data-based SVAR with the SVAR with model-simulated data. Now even if we believed that the SVAR was unable to identify technology shocks, both models would be compared on the basis of the same (potentially misspecified) metric. Figure 7 reports the median impulse responses of hours for the SVAR (on the right panel) with simulated data, and those obtained by a SVAR with actual data. A visual inspection reveals that the instantaneous response of hours obtained with a SVAR on simulated data is similar to the one obtained with SVAR using actual data. Overall, we conclude that changes in the elasticity of capital-labor substitution are able to generate the observed time varying path of a SVAR with long-

![Figure 7: Impulse responses of hours to a positive labor augmenting technology shock.](image-url)
run restrictions. We note at this point that our model has two shocks that do not affect productivity in the long run. Hence, a standard bi-variate SVAR would be capturing both shocks under the “non-technology” label. It is in this sense that we believe that the response to the non-technology shock is not an appropriate metric to focus on because there is no one to one mapping between the theory and the SVAR shocks.

3.4 Alternative structural interpretations

As mentioned in the introduction, there are a number of structural interpretations that could be advocated to explain the SVAR evidence. Since the sign switch of hours roughly coincides with onset of the ‘Great Moderation’ period, a popular view attributes time variations to the change of the Fed’s monetary policy at the beginning of the 1980s. In the 70s the monetary authority believed that there was an exploitable trade-off between inflation and output. Since output was low following the oil shocks of 70s, the temptation to inflate was strong. However, the option of keeping inflation temporarily high was unfeasible: in the medium run, inflation reached a higher level with output settling at its potential. Since the 1980s, the Fed learned that the output-inflation trade-off was not exploitable and concentrated on the objective of stabilizing inflation. A low inflation regime ensued and the better predictability of monetary policy made the macroeconomic environment less volatile, see Clarida et al. (2000). Galí et al. (2003) argue that the change in monetary policy stance has contributed to shape not only the second moments of inflation and the output gap but moments of other variables, such as the correlation between hours and technology shocks. However, their empirical evidence is restricted to two arbitrarily chosen subsamples and the structural interpretation is based on a calibrated New Keynesian model where the sign of the response of hours crucially depends on the values of the calibrated parameters, e.g. the intertemporal elasticity of substitution. While informative, their analysis lacks a formal statistical exercise testing whether data supports their conjecture. Moreover, with a structural VAR exercise Canova and Gambetti (2009) find little support for the role of monetary policy changes in driving output and inflation dynamics and point towards the potential importance of changes in the behavior of the private sector. In accordance, Nucci and Riggi (2009) attribute changes in the response of hours to an increase in performance-related pay schemes during the 1980s. Their results explain a reduction in the negative response of hours to a technology shock and also the change in the reaction of productivity to a non-technology shock. However, their model cannot account for a sign switch in the response of hours to technology shocks. Furthermore, as explained before, changes in the responses to non-technology shocks cannot be mapped to specific model shocks and may capture in the best of cases a combination of shocks.

Other possible ‘real’ structural channels that could explain the empirical findings are changes in

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12 This points towards the importance of a detailed analysis of the timing of these changes.
investment adjustment costs and the persistence of the technological diffusion (see Rotemberg (2003), Albonico, Kalyvitis and Pappa (2012) and Lindé (2009)). With large (small) enough investment adjustment costs it is possible to generate positive (negative) responses of hours to a technology shock on impact in a standard RBC model. However, regardless of the value of the adjustment costs, such model would be unable to generate fully negative dynamics of hours.\footnote{We carried out an a-priori sensitivity analysis of an RBC model with investment adjustment costs, where we show that the response of hours can be positive or negative on impact but it always reverts to positive values after few quarters. See the online appendix.} It is then inconsistent with the observed evidence in the VAR, for which the response of hours appears persistently negative (positive) in early (late) samples. Finally, the persistence of technology growth might shape the sign of the response of hours. In fact, in a standard RBC model, the response of hours is positive with $\rho_h = 0$ and negative with $\rho_h > 0$. This interpretation, however, does not seem to play an important role in our model since the estimate of $\rho_h$ varies very little across subsamples and, if anything, it goes in the opposite direction, see figure 6.

4 Rationalizing changes in the elasticity of substitution

Our analysis suggests that the driving factor behind the change in the response of hours is the increase in the elasticity of capital-labor substitution $\sigma$. This means that the parameter is not structural in a strict sense. Changes in deep parameters, such as e.g. the degree of risk aversion, are commonly used to explain the existence of instabilities in macroeconomic relationships. However, the observed change in $\sigma$ deserves further attention. We propose here an explanation based on a production function with heterogeneous inputs (be them factors or sectors) where either changes in the composition of inputs or their relative efficiency can lead to changes in the aggregate elasticity of capital-labor substitution. Note that our focus remains on the response of aggregate hours to technology shocks. In this sense, we focus on potential explanations for changes in the elasticity of substitution between capital and aggregate hours as in the RBC-CES model. We leave a more detailed empirical analysis of this heterogeneous inputs conjecture for future research.

Several authors such as Hicks (1932) and La Grandville (1989) have proposed that the elasticity of substitution can react endogenously to changes in the macroeconomy. This is formalized in Miyagiwa and Papageorgiou (2007), who present a multisector growth model where $\sigma$ is endogenously determined by economic development. Similarly, Álvarez-Cuadrado and Van Long (2011) analyze a multisector model of structural change where the aggregate elasticity of substitution is endogenous as capital intensity increases in the more flexible sectors (i.e. those with higher elasticity of substitution). Since the aggregate elasticity is a weighted average of sectoral elasticities, growth and structural change can lead to changes in aggregate $\sigma$.

We can think of a production with heterogeneous inputs in different ways. One is a production
function that uses capital and different kinds of labor (e.g. skilled and unskilled). Another is a final production function that assembles intermediate inputs from different economic sectors with, possibly, different factor intensities. Analytically, both cases can be treated in a similar manner. In a two-inputs CES production function, \( \sigma \) is constant. However, in the presence of heterogeneous inputs, the aggregate elasticity of substitution is not constant and will depend, among other things, on the share of these heterogeneous inputs. We focus here on the case of a CES with three inputs and use the common specification of a two-level nested CES function. The effects that changes in these shares ("structural change") have on \( \sigma \) will depend on the (constant) elasticities of substitution between the three inputs, and the type of nesting specified for the CES.\(^{14}\) Thus, we analyze the effect of changes in the shares of heterogeneous inputs in this setting.\(^{15}\) We then provide a numerical example considering heterogeneous labor.

Without loss of generality, and for simplicity, we ignore technological process terms and time subscripts and assume that all variables are measured at the normalization point. Assume that output \((Y)\) is obtained by combining a homogeneous input \((Z)\) and a heterogeneous input \((Q)\) composed of two classes \(J\) and \(M\). There are three possible CES nestings between inputs \(Z, J\) and \(M\). Here, we will consider only one particular nesting as it suffices for our analysis. It has to be noted, however, that the effect of structural changes on the aggregate elasticity of substitution between \(Z\) and \(Q\) depends on the nesting considered.\(^{16}\) The nesting we consider corresponds to:

\[
Y = \left[ \pi_X X^{\psi} + (1 - \pi_X) J^{\psi} \right]^{1/\psi}, \tag{5}
\]

\[
X = \left[ \pi_Z Z^{\theta} + (1 - \pi_Z) M^{\theta} \right]^{1/\theta}, \tag{6}
\]

where \(\psi\) and \(\theta\) are the inter- and intra-class substitution parameters, \(\pi_X\) is the income share parameter for aggregator \(X\) at the point of normalization, and \(\pi_Z\) is the share parameter of homogeneous input \(Z\) in \(X\) (also at the normalization point). The corresponding elasticities of substitution are \(\sigma_{Z,M} = \frac{1}{1-\theta}\) and \(\sigma_{Z,J} = \sigma_{M,J} = \frac{1}{1-\psi}\) with \(-\infty < \theta < 1\) and \(-\infty < \psi < 1\). It is worth noting that the Cobb-Douglas case occurs when \(\psi(\theta) = 0\), the Leontief case when \(\psi(\theta) = -\infty\), and the perfect substitutes case when \(\psi(\theta) = 1\). This particular nesting assumes that both \(Z\) and \(M\) are equally substitutable for \(J\) but not between them. Nesting (5)-(6) has been widely used in the capital-skill complementarity literature as discussed in Krusell, Ohanian, Ríos-Rull and Violante (2000). If we assume that capital is the homogeneous factor, skilled labor is \(M\), and unskilled labor is \(J\), capital-skill complementarity simply implies that \(\psi > \theta\).

In order to analyze the effect of changes in the shares of heterogeneous inputs in \(Q\), we define

\(^{14}\)See Papageorgiou and Saam (2008) for a theoretical examination of the properties of similar production technologies.

\(^{15}\)Note that, as will be apparent below, this is equivalent to analyzing the effects of changes in the relative efficiency of these inputs.

\(^{16}\)A full analysis of the different cases is available from the authors on request.
\[ n = \frac{J}{J + M} \text{ as the fraction of class } J \text{ in input } Q. \] Since \( Q = J + M \), we can write \( J = nQ \) and \( M = (1 - n)Q \). Now, we use the definition of the aggregate elasticity of substitution \( \sigma \) between \( Z \) and \( Q \):

\[ \sigma = \frac{\frac{pQ}{pZ}}{\frac{d(pQ/pZ)}{d(Z/Q)}}. \quad (7) \]

where \( p_i, i = Z, Q, \) is the rental price of factors \( Z \) and \( Q \). Note also that, at the normalization point, \( \frac{pQ}{pZ} = \frac{1 - \pi_X}{\pi_Z} \frac{Z}{Q} \). Using this and expression (7), Papageorgiou and Saam (2008) show that the aggregate elasticity of substitution between \( Q \) and \( Z \) is a harmonic mean of the elasticities of substitution in the nested CES functions that can be expressed as:

\[ \sigma = \frac{1}{(1 - \theta) + (\theta - \psi)g}, \quad (8) \]

\[ g = \frac{\pi_Z}{\frac{1 - \pi_X}{\pi_Z} + \pi_K}. \quad (9) \]

Since \( \theta \) and \( \psi \) are constants, we can analyze the effect of a change in \( (1 - n) \) on \( \sigma \) by obtaining the derivative of \( g \) with respect to \( (1 - n) \). We are then in a position to state the following lemma:

**Lemma 1** The aggregate elasticity of substitution between factors \( Z \) and \( Q \), \( \sigma \), is a positive function of the share of input \( M \), \( (1 - n) \), (and the efficiency of \( M \) relative to \( J \)) if \( |\theta| > |\psi| \).

**Proof.** See Appendix A. □

This condition would imply that if \( Z \) and \( M \) are complements (within the \( X \) aggregator), i.e. \( \theta < 0 \), and \( J \) is substitutes with \( Z \) and \( M \) (\( \psi > 0 \)), the degree of complementarity between \( Z \) and \( M \) has to be stronger than the degree of substitutability between \( J \) and the other two inputs. On the other hand, this would also be the case if all inputs are substitutes (\( \theta > 0 \) and \( \psi > 0 \)) but \( J \) is less substitutable for \( X \) than \( M \) and \( Z \) are between each other.

In order to give intuition to these results, we can think of these conditions as structural change that has favored sectors with a higher degree of flexibility to substitute factors of production. The 1970s and 1980s witnessed an accelerated process of technological change that was also associated with rapid structural change, with a fast decline in manufacturing and an increase in the share of business services. Another, related, important change is the increased importance of skilled workers in production as well as the prominence of skill-biased technical change as has been widely documented in papers such as Acemoglu (2002) and Acemoglu and Autor (2011).\(^{17}\) These trends appear as potential drivers of the protracted increase in \( \sigma \) that we observe throughout the sample.

In order to give a quantitative flavor of how these structural changes could have played into changes in \( \sigma \), we provide a simple numerical example using the case of heterogeneous labor. As

\(^{17}\) The structural change hypothesis has gained relevance in recent years as an explanation of changes in output volatility in the US, as reported in Carvalho and Gabaix (2010) and Moro (2011). Recently, Buera and Kaboski (2012) develop a model where demand shifts produce a simultaneous increase in the share of market services and the quantities and wages of high-skill workers.
mentioned above, the capital-skill complementarity literature discussed in Krusell et al. (2000) uses a similar CES nesting if we assume that capital is the homogeneous factor, skilled labor is $M$ and unskilled labor is $J$. Estimates of the skilled-unskilled workers substitution parameter $\psi$ usually range between 0.25 and 0.5.\footnote{For evidence on the elasticity of substitution between workers by skill level see, amongst many others, Katz and Murphy (1992), Autor, Katz and Krueger (1998), Ciccone and Peri (2005) and Autor, Katz and Kearney (2008). Most of these estimates range between 1.3 and 2.5, with consensus estimates around 1.5, corresponding to $\psi = 0.33$.} Regarding substitution between capital and skilled workers, estimates differ by study and are less abundant. Krusell et al. (2000) find an elasticity of 0.67 ($\theta \approx -0.5$).

However, given that aggregate $\sigma$ is estimated to be substantially below unity (see Chirinko (2008)) and our estimates for the full sample are below 0.2, this elasticity is likely to be even lower. Hence, the conditions for a positive effect of $1 - n$ on $\sigma$ are plausible.

We calibrate $\psi$ to a value of 0.33 (corresponding to an elasticity of 1.5). Baseline values for the shares are $\pi_X = 0.6$ and $\pi_Z = 0.5$, corresponding to a an aggregate capital income share of 0.27 and a skilled income share of 0.33. The initial share of skilled workers is 20% ($n = 0.2$). To be compatible with our low $\sigma$ estimate, we then set $\theta = -3$ corresponding to a plausible elasticity of 0.25. The value of the aggregate elasticity of substitution yields 0.32. We then analyze the impact of an increase of the share of skilled workers of 0.25 (25 percentage points) similar to that observed in the data (see Acemoglu and Autor (2011)). The corresponding new value for $\sigma$ is above 0.8. This large change is thus compatible with that observed in our estimates.\footnote{Recently, Balleer and van Rens (2011) analyze the effect of skill-biased technology shocks on the labor market using a SVAR identification scheme. Their findings show that the response of the wage premium to investment-specific shocks is incompatible with capital-skill complementarity. Their preferred model would display a strong capital-skill substitutability such that $\theta > \psi > 0$. This would also be compatible with the results from Lemma 1. Nevertheless, we note that this would imply an aggregate $\sigma$ much larger than 1, which clashes with a large body of evidence for the US where $\sigma \ll 1$. Also, this would imply a strongly pro-cyclical aggregate labor share. The correlation of the private sector labor share with output growth in the data, however, is about -0.4.}

Although this constitutes a simple quantitative example, the results are empirically consistent with the change in $\sigma$ observed in the data. We observe both a protracted increase in $\sigma$ accompanied by an increasing response of hours to technology shocks that is monotonic throughout the sample. Thus, the conjecture that a combination of sectoral reallocation and changes in the structure of the labor force may have led to changes in the conditional correlation between hours and productivity appears to offer a plausible rationalization of these dynamics.

5 Conclusions

We analyze the dynamics of the response of hours worked to technology shocks in the US economy over the last 60 years. We report evidence based on a SVAR model with long-run restrictions and a reduced form model using an empirical measure of TFP on rolling sub-samples. Consistent with previous results, we report that the conditional correlation between hours and the technology shock...
using both measures has increased over time becoming positive in the last parts of the sample. This pattern is also robust to data definitions and filtering methods. We then offer a structural interpretation of this change using a parsimonious RBC model with a Constant Elasticity of Substitution production function. Within this setting, the sign of the response of hours crucially depends on the magnitude of the elasticity of capital-labor substitution.

The model is estimated using Bayesian methods for overlapping samples of the same length as the SVAR. We find that there is a significant sign variation of the response of hours worked to a positive technology shock and that this time-varying impulse response tracks satisfactorily the changes observed in the data-based SVAR despite its parsimonious nature. Such variation is driven by a change in the magnitude of the elasticity of factor substitution: we observe an increase in the elasticity of capital-labor substitution towards the end of the sample that leads to a change in the sign and size of the response of hours. The elasticity, however, remains below one (the Cobb-Douglas case) along the different sub-samples. Finally, we show that the impact response of hours obtained from applying a SVAR on data simulated from the estimated RBC-CES model is close to that obtained with actual data.

We argue that that the observed increase in the aggregate elasticity of capital-labor substitution may be the result of structural changes in the supply side of the economy. These are associated with changes in the sectoral structure of the economy and related shifts in the skill composition of the labor force (and biased technical change). Although we do not claim that changes in frictions and macroeconomic policies do not play an important role contributing towards the dynamics of hours and technology, we emphasize the importance of deeper changes in the structure of the US economy as a key driver of this important time-varying macroeconomic correlation.
References


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A Proof of Lemma 1

We first need to use the following results:

\[
\frac{\partial \pi_X}{\partial X} = \frac{\psi_X}{X}(1 - \pi_X), \tag{A.1}
\]

\[
\frac{\partial X}{\partial (1 - n)} = \frac{1 - \pi_Z}{1 - n} X, \tag{A.2}
\]

\[
\frac{\partial \pi_Z}{\partial (1 - n)} = -\theta \frac{\pi_Z(1 - \pi_Z)}{1 - n}, \tag{A.3}
\]

which, since, for any variables \((x_1, x_2, x_3)\), \(\partial x_1/\partial x_2 = (\partial x_1/\partial x_3)(\partial x_3/\partial x_2)\), immediately implies

\[
\frac{\partial \pi_X}{\partial (1 - n)} = \psi \frac{\pi_X(1 - \pi_X)(1 - \pi_Z)}{1 - n}. \tag{A.4}
\]

With these results we can then calculate the partial derivative of \(g = \frac{\pi_Z}{1 - \pi_X + \pi_Z}\). After some tedious algebra, we can write this expression as:

\[
\frac{\partial g}{\partial (1 - n)} = \frac{\pi_Z(1 - \pi_Z)(\theta + \psi)}{(1 - \theta)(1 - \pi_X)(1 - \pi_Z + \pi_Z)^2}, \tag{A.5}
\]

Since \(\frac{\partial \sigma}{\partial (1 - n)} = \frac{\partial \sigma}{\partial g} \frac{\partial g}{\partial (1 - n)}\) we can derive, again after some algebra, the expression:

\[
\frac{\partial \sigma}{\partial (1 - n)} = -\Pi \frac{(\psi^2 - \theta^2)}{[(1 - \theta + (\theta - \psi)g]^2}, \tag{A.6}
\]

where \(\Pi > 0\) is a function of share parameters:

\[
\Pi = \frac{\pi_Z(1 - \pi_Z)}{(1 - n)(1 - \pi_X) \left[\frac{1 - \pi_Z}{1 - \pi_X + \pi_Z}\right]^2}. \tag{A.7}
\]

Given that \(\Pi > 0\) and that the denominator of (A.6) is positive, the effect of a change in \(1 - n\) will be positive if \(\theta^2 > \psi^2\). Hence, an increase in the share of input \(M\) will increase aggregate \(\sigma\) if \(|\theta| > |\psi|\).