Shadow economies at times of banking crisis. Empirics and Theory.

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Abstract

This paper investigates the response of the shadow economy to banking crises. Our empirical analysis, based on a large sample of countries, suggests that the informal sector is a powerful buffer, which expands at times of banking crises and absorbs a large proportion of the fall in official output. To rationalise our evidence, we build a dynamic stochastic general equilibrium model which accounts for financial frictions and nominal rigidities. In line with a large empirical literature on the shadow economy, we assume that in the informal sector access to external finance is limited and the production technology is relatively more labor intensive. The model predicts a large negative transmission effect to the unofficial economy following a banking shock in the official sector: about 60% of the official sector contraction is absorbed by the growth of the shadow economy.

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1 Introduction

Banking crises are typically associated with a prolonged decline in output and employment (Rajan and Zingales, 1998; Kroszner et al., 2007; Cerra and Saxena, 2008; Dell’Ariccia et al., 2008; Reinhart and Rogoff, 2009a). One less-investigated issue is the contemporaneous behavior of the informal economy during such episodes. Recent contributions, suggesting that the relative size of the shadow economy might be on the rise in the aftermath of the 2007 crisis (Schneider and Buehn, 2012; Elgin and Oztunali, 2012), have found considerable echo in the press.\footnote{“A Lengthening Shadow” The Economist, August 2010 ; “Europe: Hidden economy”, The Financial Times 8th June 2011; “Shadow economies all around the world: Model-based estimates”, Vox, available at \url{http://www.voxeu.org}}

Understanding the shadow economy adjustment to banking crises has important policy implications. On the one hand, the existence of an informal sector may add resilience to the economy when times are difficult. On the other hand, the erosion of the tax base greatly complicates the task of fiscal policy makers at a time of ballooning public deficits. In fact, revenue losses seem to be the main cause behind the dramatic increase in debt-to-GDP ratios that typically follows the outburst of a banking crisis (Reinhart and Rogoff, 2009b).

In this paper we study the empirical response of the shadow economy to the banking crisis. For our purposes, shadow economy data sets based on the MIMIC method, such as Schneider et al. (2010) are too limited in the time-series dimension of the sample. We therefore take an alternative route which is partly related to the electricity consumption approach to measuring the shadow economy. Assuming that the rate of change in electricity consumption is a proxy for the growth rate of total economic activity (TEA henceforth), we compute differential responses of TEA and official output growth rates to banking crises as a broad indicator of shadow economy dynamics. Standard measures of the relative size of the shadow economy based on the electricity consumption method are probably biased down because informal activities are less electricity intensive than formal activities and because it is difficult to take into account energy-saving technological change (Schneider and Enste, 2000; Porta and Shleifer, 2008). Neither criticism seem to apply here. In fact our methodology does not require any information about the size of the shadow economy and the obtained responses to crisis episodes are unlikely to be affected by long run technical change.
We find that the TEA fall is very small relative to what we observe for the official economy growth rate. Further, the TEA drop is never statistically significant if we control for variations in the relative price of electricity and for sectoral output composition in the official economy, as in (Eilat and Zinnes, 2002; Onnis and Tirelli, 2010). By contrast, we show that financial crises are followed by a deep contraction in the rate of gross fixed capital formation. The apparent increase in the ratio of electricity consumption to GDP (and to the stock of fixed capital) is particularly striking in light of the huge literature on the procyclical pattern of electricity consumption. In fact electricity consumption is used as a proxy for capacity utilization in business cycle models (Bils and Cho, 1994; Comin and Gertler, 2006) and as a leading indicator for business cycle conditions (Marchetti and Parigi, 2000; Kamada and Masuda, 2001).

Obviously, the cyclical pattern of the unofficial economy cannot be directly observed, but this evidence suggests a large shadow economy increase in response to financial crises. To rationalise our empirical results, we build a two-sector DSGE model which accounts for price stickiness and for credit market frictions. Our characterisation of the shadow economy sector is consistent with two “stylised facts”. The first is that firms operating in the shadow economy have access to a relatively more labor intensive production technology (Amaral and Quintin, 2006; Koreshkova, 2006). The second is that the financial market are segmented, formal and informal financial sectors coexist and access of informal firms to outside finance is typically limited (Banerjee and Duflo, 2007; Madestam, 2008; Batini et al., 2011). The bulk of existing empirical evidence emphasises that informal financing is based on relationships lending ("mafia protection", informal credit by money lenders, etc.) which is less transparent and efficient in credit provision with respect to official credit markets but is also relatively more efficiently in monitoring and enforcing repayment from small firms. Contrary to the common wisdom informal credit markets should not be considered necessarily as a constraint for the economy; according to Besley and Levenson (1996) Taiwan’s fast growth greatly benefited from informal financial institutions. More recently several studies have investigated the apparently very important role of Chinese informal financial institutions (Allen et al., 2005, 2012; Ayyagari et al., 2010). Finally, some studies have documented the critical role played by informal financial networks even in developed countries such as Italy (Guiso et al., 2004) and the US (Garmaise and Moskowitz, 2003).

Following a banking shock in the official sector, our model predicts a
disruption of capital formation in the official economy. This, in turn, is associated to a fall in official output and employment. The ensuing real wage fall favours an increase in output and employment in the unofficial economy. The sectoral reallocation of employment causes a persistent increase in the marginal productivity of capital in the unofficial sector, and triggers a surge in unofficial investment. We obtain a large negative transmission effect: about 60% of the official sector contraction is absorbed by the growth of the shadow sector.

Previous empirical evidence on the cyclical pattern of the shadow economy is mixed. Bajada (2003) and Giles (1997) find a procyclical relationship in Australia and New Zealand, respectively. A number of studies based on the MIMIC approach support the view that the shadow economy acts as a buffer, increasing its size in periods of recession (Bajada and Schneider, 2005; Schneider and Enste, 2000; Feld and Schneider, 2010). Russo (2008 obtains a similar result for the US using the electricity consumption approach. Our theoretical results do not fully confirm this view. In fact we find that technology shocks induce a positive correlation between sectoral outputs, even if the relative size of the unofficial sector exhibits a countercyclical pattern. Thus our model does not merely produces a countercyclical behaviour of the underground economy, it points to specific role of the credit market in the propagation of the shock between the two sectors of the economy.

Existing business cycle models of the informal economy basically fall into two categories. In the first one there are real business cycle models that incorporate an informal sector (Conesa Roca et al., 2001; Busato and Chiarini, 2004; Granda-Carvajal, 2010). In the second one there are models that focus on the labor market, assuming either search frictions Bosch and Esteban-Pretel (2012) or the dual labor markets hypothesis Fiess et al. (2010). These two alternative assumptions about the functioning of the labor market are central in the DSGE models respectively presented in Castillo and Montoro (2010) and Mattesini and Rossi (2009). None of these contributions incorporates financial frictions as important elements of business cycle fluctuations and of the inter-sectoral transmission of shocks as we do in the present work. The only exception is Batini et al. (2011); our paper differs in two key aspects: the first is the modeling strategy of financial frictions that we derive endogenously while they assume an exogenous external finance premium; the second is the focus on banking shocks, which are neglected in their contribution.

The remainder of the paper is structured as follows: section 2 describes
the data and the methodology used in the empirical analysis section 3 presents
the empirical results, section 4 describes the theoretical model and illustrates
the results. Finally section 5 concludes.

2 Data and methodology

2.1 Measuring banking crises
Defining banking crises is often controversial, due to the lack of a consensus
definition and to the need of a certain degree of judgement. We adopt the
well known classification by Laeven and Valencia (2008), Laeven and Valencia
(2010) who focus on systemic banking crises excluding distress events that
the starting year of the crises by a) deposit runs, defined as a monthly
percentage decline in deposits in excess of 5 percent, b) the introduction of
deposit freezes or blanket guarantees; c) liquidity support or central bank
interventions, defined as the ratio of monetary authorities’ claims on banks
as a fraction of total deposits of “at least 5% and at least double the ratio
compared to the previous year”.

2.2 Methodology
We consider two approximations to the growth of total economic activity.
The first one is the growth rate of electricity consumption $TEAg$. The second
one $MTEAg$ is obtained filtering $TEAg$ to remove the effects variations in
the relative price of electricity and of sectoral output composition in the
official economy. 2

Our methodology for identifying the effects of banking crises follows
Cerra and Saxena (2008) who in turn draw on the influential work by Romer
and Romer (1989).3 We estimate the following autoregressive model.

$$\Delta Y_{i,t} = \alpha_i + \sum_{s=1}^{4} \beta \Delta Y_{i,t-s} + \sum_{s=0}^{3} \gamma DFC_{i,t-s} + \epsilon_{i,t}$$  \hspace{1cm} (1)

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2Methods used for computing TEA are explained in the Appendix.

3See also Romer and Romer (2010) for a more recent analysis on the impact of fiscal shocks.
Where $\Delta Y = TEAg$, $MTEAg$ is the growth rate of the total economy, $DFC$ is a dummy variable for the presence of a financial crisis, $i$ is a country index. The number of lags of both the dependent variable and the crisis dummy have been chosen to maximise the informativeness of the model.\(^4\)

We estimate (1) using panel data that control for the presence of fixed effects and allow for heteroskedasticity of the error term and for autocorrelation within groups (countries).

The impact of the crisis has been estimated calculating impulse response function constructed using 1000 Monte Carlo simulations. The significance of the results is computed by calculating 95% confidence bands. The Appendix describes the procedure with greater details.

### 3 Empirical results

Figure 1 shows the effect of banking crises on official GDP and on TEA. The measured impact of banking crises on GDP growth confirms the results of the literature and in particular the findings of Cerra and Saxena (2008). Crisis episodes have a long lasting and permanent effect on GDP, and the fall in official investment is even stronger. Results are strikingly different when the analysis is replicated for the growth of total economic activity. The $TEA$ drop is very limited, suggesting a potentially strong role of the shadow economy as a shock absorber in response to the crisis. Further, the response of $MTEA$ is never significant (figure 2). These results are confirmed when we split the sample into different subgroups following a per-capita income criterion.

We checked whether crises reflect some other global shock common to all countries. The first row of Figure 3 shows impulse responses of equation (1) adding time dummies. Results are unchanged. In addition we tested whether our results are driven by some extreme values. The first row of 3 shows impulse responses excluding outliers in growth rates of total economic activity ($MTEA$). Even in this case our estimates are confirmed.

We based our analysis on the implicit assumption that financial crises are exogenous to the growth of total economic activity. To control for potential endogeneity we implemented two tests. First, we estimated the model excluding the contemporaneous effect of the crisis on total output growth. The

\[^4\]The four-year time lag is consistent with the literature estimating the output responses at the same or at higher frequencies (Cerra and Saxena, 2008).
second line of Figure 3 illustrates the result showing that nothing changes. Second, we predicted financial crises with current and lagged values of the growth rate of the relevant variable, using a logit model. Table 3 shows that neither TEA nor MTEA help to forecast financial crises. Repeating the same exercise without the contemporaneous effect yields identical results.

4 Financial frictions and the shadow economy in a DSGE model

Models of dual credit markets assume that the informal financial sector is relatively less efficient, thus generating in the shadow economy a relative scarcity of capital and greater reliance of entrepreneurs on self-financing Straub (2005); Pratap and Quintin (2006); Antunes et al. (2008). Incorporating these contributions within the framework of DSGE models is not straightforward. We extend the RBC cum agency costs model developed in Carlstrom and Fuerst (1997), C-F henceforth) to account for both the official (o) and the underground (s) sectors of the economy.

Assumptions about the working of financial markets closely follow C-F. A fraction $\eta$ of the population - the entrepreneurs - have the ability to transform $i$ consumption goods into $i\omega$ capital goods, where $\omega$ is a random variable subject to idiosyncratic shocks. $\omega$ realizations are entrepreneur’s private information, generating standard agency issues. A fraction $\eta^o$ of entrepreneurs operates in the official sector, the rest is in the shadow economy. In each sector of the economy, a capital mutual fund or "bank" (CMF$j$, $j = o, s$) collects funds from the common pool of households' savings and lends them to entrepreneurs. $^5$ We depart from C-F in assuming that financial operations are costly, i.e. they entail a dissipation of resources, and are relatively larger in the informal sector. This allows to characterize a situation where the financial sector of the shadow economy is relatively inefficient, i.e. unofficial economy entrepreneurs earn a relatively smaller proportion of investment proceedings, and the proportion of external financing in this sector is relatively limited.$^6$

$^5$Several studies document that funds originating in the official economy are then intermediated in the informal financial market and channeled into the unofficial sector of the economy (Conning and Udry (2007); also see Madestam (2008) and the studies cited therein).

$^6$De Soto (2000) forcefully argues that informal assets are much more difficult to leverage into loans than assets belonging to entrepreneurs who operate in the official sector.
Relative to recent contributions such as Gertler et al. (2011), our characterization of the financial market is admittedly crude to provide a satisfactory description of the banking sector in a general equilibrium framework. Nevertheless, it captures a key mechanism that drives the transmission of bank crises to the shadow economy, i.e. the large and persistent disruption in the process of capital accumulation in the official economy documented in section 3.

In both sectors perfectly competitive firms produce wholesale goods which are then sold to monopolistically competitive retail firms. Retail prices are sticky. We assume a perfectly competitive labour market, in line with Amaral and Quintin (2006); Pratap and Quintin (2006).\footnote{Maloney (2004, 99) and Pratap and Quintin (2006) provide evidence against the labor market segmentation assumption. Another strand of literature emphasises the role of search frictions in shaping occupational choices between the two sectors (see Batini et al., 2011 for a survey). We retain the competitive labor market hypothesis in order to sharpen our focus on credit frictions.}

The sequence of events is standard from C-F.

- Wholesale firms hire labor and capital from households and entrepreneurs to produce consumption goods. Retail firms differentiate goods and adjust consumption prices. Firms choices are conditional to productivity shocks and to the monetary policy rule.
- Households choose consumption and the allocation of investment between the official and the unofficial financial intermediaries.
- CMFs lends consumption goods to entrepreneurs.
- Entrepreneurs use borrowed and own resources to create capital.
- The idiosyncratic shock of each entrepreneur is realised and the debt contract is enforced. Proceedings from creation of new investment goods are then split between households and entrepreneurs.
- Entrepreneurs choose their consumption.

In the model the inter-sectoral transmission of shocks typically occurs through flows in the factor services across the two sectors, driven by arbitrage conditions in the capital and labor markets.
4.1 Firms

In each sector \( j (o, s) \), perfectly competitive (flex-price) firms produce wholesale goods \( I^j \) and sell them to retail producers \( R^j \) that introduce product differentiation and are subject to price rigidity.

Households preferences over the goods produced in the economy are defined as follows. The CES consumption bundle, \( c_t \), is

\[
c_t = \left[ (1 - \alpha_c)^{\frac{1}{\varepsilon}} (c_t^o)^{\frac{1}{\varepsilon}} + (\alpha_c)^{\frac{1}{\varepsilon}} (c_t^s)^{\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{2}
\]

Further, each \( c^j_t \) is also defined as a CES bundle.

\[
c^j_t = \left( \int_0^1 c^j_t(z^j)^{\frac{\sigma^j}{\sigma^j - 1}} dz^j \right)^{\frac{\sigma^j}{\sigma^j - 1}} \tag{3}
\]

The parameter \( \varepsilon > 1 \) measures the elasticity of substitution between the official and the shadow consumption bundles, \( c^o_t \) and \( c^s_t \), whereas \( \sigma^j > 1 \) measures the elasticity of substitution among the differentiated goods that form \( c^j_t \).

Demand functions for individual goods within each consumption bundle are

\[
c^j_t(z^j) = \left( \frac{P^R_{tj}(z^j)}{P^R_t} \right)^{-\sigma^j} c^j_t
\]

where

\[
P^R_t = \left( \int_0^1 \left( \frac{P^R_{tj}(z)}{P^R_t} \right)^{1-\sigma^j} dz \right)^{-\frac{1}{1-\sigma^j}}
\]

\[
c^o_t = (1 - \alpha_c) \left( \frac{P^R_{t0}}{P^R_t} \right)^{-\varepsilon} c
\]

\[
c^s_t = \alpha_c \left( \frac{P^R_{ts}}{P^R_t} \right)^{-\varepsilon} c
\]

\footnote{Assumptions about the steady state size of entrepreneurs and firms in the two sectors are just innocuous normalisations as long as we allow for capital and labor intersectoral flows For sake of simplicity we do not consider flows of firms, and entrepreneurs, between the two sectors over the business cycle.}
\[ P_t = \left( (1 - \alpha c) \left( P_t^{Ro} \right)^{(1-\varepsilon)} + \alpha c \left( P_t^{Rs} \right)^{(1-\varepsilon)} \right)^{\frac{1}{1-\varepsilon}} \]  \hspace{1cm} (4)

respectively define the sectoral retail price index, the demand functions for the sectoral bundles and the consumption price index.

4.1.1 Wholesale producers

Wholesale producers have access to the production technology
\[ y_{jt} = \exp \left( \theta_{jt} \right) \left( k_{jt} \right)^{\alpha_j} \left( h_{jt} \right)^{1-\alpha_j} \]
where \( y_{jt}, k_{jt}, h_{jt} \) respectively define sector-specific output, capital and labour and \( \theta_{jt} \) captures sectoral productivity shocks. In the following we shall only consider shocks hitting the official sector:
\[ \theta_o = \rho \theta_t + \xi_t \]
Factor demands are defined as follows
\[ \frac{P_t}{P_{tj}^{ij}} w_t = (1 - \alpha_j) \theta_t \left( \frac{k_{jt}}{h_{jt}} \right)^{\alpha_j} \]  \hspace{1cm} (5)
\[ r_{jt} = \alpha_j \theta_t \left( \frac{k_{jt}}{h_{jt}} \right)^{-(1-\alpha_j)} \]  \hspace{1cm} (6)
where \( w_t, P_t, P_{tj}^{ij}, r_{jt} \) respectively define the real consumption wage rate, the consumption price index the sectoral price index for intermediate goods and the sectoral real return on capital. Intermediate sector real marginal costs, \( mc_{tj}^{ij} \), are
\[ mc_{tj}^{ij} = \left( \frac{r_{jt}}{\alpha_j} \right)^{\alpha_j} \left( \frac{P_t}{P_{tj}^{ij} w_t} \right)^{1-\alpha_j} \]  \hspace{1cm} (7)
Sectoral intermediate prices equal nominal marginal costs
\[ P_{tj}^{ij} = \left( \frac{P_{tj}^{ij} r_{jt}}{\alpha_j} \right)^{\alpha_j} \left( \frac{P_t w_t}{(1 - \alpha_j)} \right)^{1-\alpha_j} \]  \hspace{1cm} (8)
\[ P_t^{ij} = \left( \frac{r_{jt}}{\alpha_j} \right)^{1-\alpha_j} \left( \frac{P_t w_t}{(1 - \alpha_j)} \right) \]  \hspace{1cm} (9)
4.1.2 Retail producers

We assume a sticky price specification based on Rotemberg (1982) quadratic cost of nominal price adjustment:

\[
\frac{\varphi}{2} \left( \frac{P^R_j(z)/P^R_{j-1}(z)}{\pi^R_j(\delta_n)} - 1 \right)^2
\]

where \( \varphi \geq 0 \) is a measure of price stickiness, \( \pi^R_j = \frac{P^R_j}{P^R_{j-1}} \) denotes the sectoral gross inflation rate and \( \delta_n \) is a price indexation parameter.

In a symmetrical equilibrium the price adjustment rule satisfies:

\[
\left( \frac{(1 - \sigma^j)}{\sigma^j} + \frac{P_{i,t}^j}{P_t^R} \right) \frac{\sigma^j}{\varphi} + \beta \left( \frac{\lambda_{t+s}}{\lambda_t} \right) \frac{y^{j+1}}{y^j} \left( \frac{\pi^R_j}{\pi^R_{t+1}} \delta_n - 1 \right) \left( \frac{\pi^R_j}{\pi^R_{t+1}} \delta_n \right) = \left( \frac{\pi^R_j}{\pi^R_{t-1}} \delta_n - 1 \right) \left( \frac{\pi^R_j}{\pi^R_{t-1}} \delta_n \right)
\]

where \( \frac{P_{i,t}^j}{P_t^R} \) defines real marginal costs in terms of the sectoral retail price. Consumption price inflation is

\[
\pi_t = \frac{P^i_t}{P^R_{t-1}}
\]

4.2 Households

Households are characterized by a standard utility function

\[
U_i^t = E_t \sum_{k=0}^{\infty} \delta^k \left\{ \ln \left( c_{t+k}^i - bc_{t+k-1} \right) - \frac{\psi}{1 + \phi} (h_{t+k})^{1+\phi} \right\}
\]

which accounts for external habits. The intertemporal Euler equation is 9

\[
\lambda_t = \beta \lambda_{t+1} \frac{R_t}{\pi^R_{t+1}}
\]

9For sake of simplicity we drop superscripts \( i \)
where $R_t$ is the interest rate paid on a nominally riskless bond and

$$
\lambda_t = \frac{1}{c_t - bc_{t-1}}
$$

is the marginal utility of the consumption bundle.

In the competitive labour market, the standard labour supply condition is

$$
\frac{\psi(h_t)^\phi}{\lambda_t} = w_t
$$

where $w_t$ is the wage rate in units of the consumption bundle. For each sector $j$, households capital demand is driven by

$$
q^j_t = \beta \frac{\lambda_{t+1}}{\lambda_t} [q^j_{t+1}(1 - \delta) + r^j_{t+1}]
$$

and $r^j_{t+1}$ defines the return that firms pay on borrowed capital goods.

### 4.3 The financial contract

In each sector the financial contract is stipulated between a risk-neutral entrepreneur endowed with financial wealth $n^j_t$, and the sectoral CMF$^j$. Information about entrepreneur-specific $\omega$ realisations becomes available to CMF$^j$ at the monitoring cost $\mu^j$. Here we make the additional assumptions that: $i)$ for every funding project, CMF$^j$ is subject to a cost $i^j \chi^j$; $ii)$ when entering the borrowing relationship, the entrepreneur is subject to a cost $\kappa^j n^j_t$. These two assumptions will be used to characterise differences in the use of external finance across sectors.

The entrepreneur borrows $(i^j - (1 - \kappa^j) n^j)$ and agrees to repay the loan at the gross rate $(1 + r^j)$; it is assumed for simplicity that $r^j = 0$ since CMF funding is intra-period. This implies that all rents accrue to entrepreneurs, net of the CMF's operational costs. To preserve the simplicity and tractability of the model, we assume that an entrepreneur's "union" is assigned the task of collecting individual contributions necessary to finance $\chi^j$.

The contract is also defined by the pair $(\tilde{\omega}^j, \omega^j)$ that maximises the entrepreneur's expected income subject to the CMF being indifferent between lending or retaining the funds. Note that $\tilde{\omega}^j$ defines both the default threshold and the payment rate accruing to the lender from non-defaulting entrepreneurs. When $\omega < \tilde{\omega}^j$ default occurs and the lender monitors. We assume that idiosyncratic productivity shocks are uniformly distributed with
support \([\bar{\omega}, \omega]\), and that defaulting entrepreneurs retain the fixed amount \(\omega\), so that in the next period they can borrow again. Therefore CMF\(_j\) proceedings from shock realisations \(\omega < \bar{\omega}^j\) amount to \(\omega - (\mu^j + \omega)\).

Finally, we define the entrepreneur contribution scheme necessary to ensure that CMFs operational costs are covered. i) only non-defaulting entrepreneurs contribute a fraction of the proceedings from their investment \(i^j \omega\); ii) the individual contribution of non-defaulting entrepreneurs is \(\bar{\chi}^j\) unless the entrepreneur is characterised by shock realisations \(\bar{\omega}^j < \omega < \bar{\omega}^j + \bar{\chi}^j + \omega\). In this latter case the contribution amounts to \(\max\{0, \omega - (\omega + \bar{\omega}^j)\}\) and the entrepreneur retains the amount \(\omega\) which allows him to borrow again in the next period.\(^{11}\)

The optimal contract maximises \(q^j \cdot i^j f(\bar{\omega}^j)\) subject to \(q^j i^j g(\bar{\omega}^j) \geq (i^j - (1 - \kappa^j) n^j)\), where \(q^j\) defines sector \(j\) consumption price of capital, and

\[
f(\bar{\omega}^j) = \phi \left( \frac{\bar{\omega}^2}{2} - \frac{\bar{\omega}^j}{2} \right) - \phi (\bar{\omega} - \bar{\omega}^j) \omega^j + \phi (\bar{\omega}^j - \omega) - \chi^j; \phi = (\bar{\omega} - \omega)^{-1}
\]

\[
g(\bar{\omega}^j) = \phi (\bar{\omega} - \bar{\omega}^j) \omega^j + \phi \left( \frac{\bar{\omega}^j}{2} - \frac{\omega^2}{2} \right) - \phi (\bar{\omega}^j - \omega) (\mu^j + \omega)
\]

respectively define the investment shares\(^{12}\) accruing to the entrepreneur and to the CMF\(^j\). Note that we set

\[
\chi^s = \chi^s
\]

\[
\chi^o = \chi^o \exp (v^X_t)
\]

where

\[
v^X_t = \rho^X v^X_{t-1} + \xi^X_t; \xi^X_t \text{ i.i.d.}
\]

defines the financial shock to the official sector.

Formally, the contract is defined by the following first order conditions

\(^{10}\)In fact it is well known that the financial contract is not well defined when the borrower’s net worth is zero. To get around this problem C-F opt for the alternative assumption that entrepreneurs supply their labor to firms.

\(^{11}\)It would be straightforward to show that this characterisation of the contribution scheme allows to preserve in our context the optimality of the simple contract defined in C-F.

\(^{12}\)The fixed cost \(\chi^j\) reduces rents, which are entirely appropriated by the entrepreneur.
\[ i_t^j = \frac{(1 - \kappa^j) n_t^j}{1 - q_t^j g(\tilde{\omega}_t^j)} \]  \hspace{1cm} (21)

\[ q_t^j \left\{ 1 - \phi \left( \tilde{\omega}_t^j - \omega \right) \left( \mu^j \right) + \phi \mu^j \frac{f(\tilde{\omega}_t^j)}{f'(\tilde{\omega}_t^j)} \right\} = 1 \]  \hspace{1cm} (22)

where

\[ f'(\tilde{\omega}_t^j) = -\phi (\bar{\omega} - \tilde{\omega}_t^j) + \phi \omega \]  \hspace{1cm} (23)

For any given \( q_t^j \), conditions (21), (22) identify the default threshold and the amount of investment,\(^{13}\) where term \( \frac{(1 - \kappa^j)}{1 - q_t^j g(\tilde{\omega}_t^j)} \) is the leverage ratio.

### 4.4 Entrepreneurs

Following C-F, risk-neutral entrepreneurs, characterised by superscript \( e \), maximise

\[ U^e,j_t = E_t \sum_{k=0}^{\infty} (\beta \gamma)^k c_{t+k}^e \]

where \( c^e,j \) is defined over the consumption bundle In each sector, entrepreneurs demand for firms capital is

\[ q_t^j = \beta \gamma [q_{t+1}^j (1 - \delta) + r_{t+1}^j] \frac{(1 - \kappa^j) q_{t+1}^j f(\tilde{\omega}_{t+1}^j)}{1 - q_{t+1}^j g(\tilde{\omega}_{t+1}^j)} \]  \hspace{1cm} (24)

Note that the CMFs operating in the two sectors borrow from the same pool of households, and are therefore constrained to guarantee the same return:

\[ q_t^s g(\tilde{\omega}_t^s) = q_t^o g(\tilde{\omega}_t^o) \]  \hspace{1cm} (25)

From (21) and (25) it is easy to see that \( \kappa^s > \kappa^o \) allows to obtain a less leveraged shadow economy. Further, from (15), (24) we obtain

\[ \frac{q_{t+1}^s (1 - \kappa^s) f(\tilde{\omega}_{t+1}^s)}{1 - q_{t+1}^s g(\tilde{\omega}_{t+1}^s)} = \frac{q_{t+1}^o (1 - \kappa^o) f(\tilde{\omega}_{t+1}^o)}{1 - q_{t+1}^o g(\tilde{\omega}_{t+1}^o)}. \]  \hspace{1cm} (26)

Due to the identical preferences of the entrepreneurs operating in the two sectors and to the common pool of households’ funds, the expected rate of

\(^{13}\)As shown in C-F condition () denotes both individual and aggregate investment decisions.
return on the entrepreneurs’ internal funds is unique. Given the structural
differences between the two financial sectors, the adjustment of the relative
capital price, $q_s^o$, is crucial to obtain this result.

4.5 Capital accumulation

Sectoral capital accumulation is driven by

$$k_t^j = (1 - \delta)k_{t-1}^j + \eta \eta^j i_t^j \left( 1 - \phi (\omega_t^j - \omega) \mu^j - \chi_t^j \right) \quad (27)$$

where monitoring costs and CMFs operational costs $\chi^j$ provide a rational-
ization of investment adjustment costs. Our results will show that shocks
to $\chi^o$ have powerful effects on capital accumulation and generate a large
transmission effect to the unofficial sector.

Dynamics of average wealth of entrepreneurs in sector $j$ are

$$n_t^j = (q_t^j (1 - \delta) + r_t^j) \left( n_{t-1}^j \frac{(1 - \kappa^j) f (\omega_{t-1}^j) \mu_{t-1}^j}{1 - q_{t-1}^j g(\omega_{t-1}^j)} - \left( \frac{c_t^j}{P_t} \right) \right) \quad (28)$$

4.6 Resource constraints

In each sector the resource constraint is

$$\theta_t^j \left( k_t^j \right)^{\alpha_j} \left( h_t^j \right)^{1-\alpha_j} = \left( 1 - \alpha_c \right) \left( \frac{P_t^j}{P_t} \right)^{-\varepsilon} C_t^* + \eta \eta^j (i_t^j) \quad (29)$$

$$C_t^* = (1 - \eta) c_t + \eta \left( \eta^s \right) c_t^es + \eta^o c_t^eo \quad (30)$$

Finally, the labor resource constraint is

$$h_t^o + h_t^s = (1 - \eta) h_t \quad (31)$$
4.7 Monetary policy

Monetary policy reacts to the official consumption inflation rate following a standard Taylor rule, augmented for nominal interest rate smoothing.\(^ {14}\)

\[
\frac{R_t}{R} = (\pi_t^{Ro})^{\phi^\pi} \left( \frac{R_{t-1}}{R} \right)^{(1-\rho^R)}
\]  

(32)

4.8 Calibration

Parameters characterising the official economy and households preferences are fairly standard. The values chosen for the household subjective discount factor, \(\beta\), the capital income share \(\alpha^o\), the capital depreciation rate, \(\delta\), the entrepreneurs fraction \(\eta\) and the entrepreneurs subjective discount factor, \(\gamma\), are as in C-F. From Wouters and Smets (2005) we take the consumption habit parameter \(b = 0.7\), the inverse of the Fritsch elasticity \(\psi = 2\), the degree of inflation indexation \(\delta_\pi = 1\),\(^ {15}\) the Taylor rule parameters \(\phi^\pi = 1.5\) and \(\rho^R = 0.9\). The degree of price stickiness, \(\varphi^o = 4.37\), and the price-elasticity parameter \(\sigma^o = 1.2\) are taken from Schmitt-Grohe and Uribe (2004). The elasticity of substitution between official and shadow consumption bundles, is set at 1.5 as in Batini et al. (2011). Turning to firms operating in the shadow economy, to capture the relatively low capital intensity in their production function we have chosen the capital share parameter, \(\alpha^s = 0.28\), as in Koreshkova (2006); we have also assumed that firms operating in the unofficial retail sector have limited market power, \(\sigma^s = 20\). To the best of our knowledge, there is no evidence about nominal rigidities in the unofficial sector, we therefore take as benchmarks the values adopted for the degree of price stickiness and for inflation indexation in the official sector.\(^ {16}\)

Turning to the financial sectors parameters, we set \(E(\omega) = 1\), \(\sigma^2_\omega = \omega = 0.01\). To capture the well known features that leverage is smaller in

\(^{14}\)In our simulations we also experimented with a monetary policy feedback on the current official output gap finding that our key results are entirely confirmed. For reasons of space we do not report here the impulse response functions obtained under this policy rule.

\(^{15}\)As a matter of fact Smets and Wouters adopt a more complex indexation scheme, but we replicate their assumption that the long-run Phillips curve is vertical.

\(^{16}\)On theoretical grounds it is not obvious that the proportional output cost associated to price revisions should be different across the two sectors. We also experimented with \(\varphi^s = 2.18\), and our results were entirely confirmed.
the unofficial sector we set $\kappa^s = 0.1$ and $\kappa^o = 0$, implying that leverage is $X\%$ lower in the shadow economy sector. The relative inefficiency of the informal financial sector is obtained by setting $\chi^o = 0.01$, and $\chi^s = 0.15$.\footnote{Our simulations show that neither the sign nor the amplitude of the transmission mechanism in response to banking shocks depends on the relative size of $\kappa^s$ and $\chi^s$. In fact these parameters contribute to determine the relative size of the shadow economy in steady state. Proof available upon request.} The monitoring cost in the official sector, $\mu^o$ is calibrated to obtain a bankruptcy rate $\frac{(\tilde{\omega}^o_t - \omega)}{(\omega - \tilde{\omega})} = 0.03$. Obtaining direct estimates of monitoring costs or turnover rates in informal economies is very difficult. Straub (2005) argues that monitoring costs are probably lower in informal sectors, but contract enforcement is probably easier. Since we have already modeled the relative inefficiency of the shadow economy financial sector, we opted for $\mu^s = \mu^o$. Finally we close the model by calibrating the preference parameters $\psi$ and $\alpha_c$. The former is set to obtain $h = 1$ in steady state. The latter is assigned two alternative values, that allow to obtain in steady state a high (low) share of the unofficial economy $SH = \frac{y^s}{y^o}$, $(SH = 0.44; 0.11)$.\footnote{Derivation of the steady state is reported in Appendix II.}

4.8.1 Model dynamics and intersectoral transmission\footnote{Derivation of the steady state is reported in Appendix II.}

In Figures 4-5 we plot the IRFs to the shock defined in (19) and (20), in a steady state characterised by respectively a low (11%) and a high (44%) share of the shadow economy. It is easy to see that the shock is followed by a sharp reduction in official investment, which drives the official output and employment fall. The model predicts a strong reduction in the nominal and real interest rates and a sharp increase in the risk premium, $-\frac{R_t}{\pi_{t+1}}$, a phenomenon typically associated to financial crises. The informal sector benefits from the reduction in real interest rates and wages. In fact informal investment output and employment increase in spite of the lower relative price of official sector goods. The informal sector expansion appears very effective in cushioning the negative impact of the financial shock: total output reduction is 60% lower than the official output fall. This result obtains irrespective of the steady state share of the informal sector. For a better understanding of the transmission mechanism, we implemented a sensitivity analysis on some key parameters. We found that the total output loss has
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Household subjective discount factor, consistent with a 4% real interest rate</td>
</tr>
<tr>
<td>$\sigma^o$</td>
<td>6</td>
<td>price-elasticity of demand for a differentiated good, consistent with a 20% price mark-up</td>
</tr>
<tr>
<td>$\sigma^s$</td>
<td>20</td>
<td>price-elasticity of demand for a differentiated good, consistent with a 5% price mark-up</td>
</tr>
<tr>
<td>$\varphi^o$</td>
<td>4.37</td>
<td>degree of price stickiness</td>
</tr>
<tr>
<td>$\varphi^s$</td>
<td>4.37-2.18</td>
<td>degree of price stickiness</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2</td>
<td>Inverse of Frish elasticity</td>
</tr>
<tr>
<td>$\delta^\pi$</td>
<td>1</td>
<td>inflation indexation parameter</td>
</tr>
<tr>
<td>$\alpha^o$</td>
<td>0.36</td>
<td>capital income share</td>
</tr>
<tr>
<td>$\alpha^s$</td>
<td>0.28</td>
<td>capital income share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.02</td>
<td>depreciation rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.94</td>
<td>entrepreneur subjective discount factor</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>1.5</td>
<td>elasticity of substitution between official and shadow consumption bundles</td>
</tr>
<tr>
<td>$\rho^\chi$</td>
<td>0.8</td>
<td>financial shock autocorrelation</td>
</tr>
<tr>
<td>$\rho^\theta$</td>
<td>0.95</td>
<td>productivity shock autocorrelation</td>
</tr>
<tr>
<td>$\rho^R$</td>
<td>0.1</td>
<td>Interest rate smoothing parameter</td>
</tr>
<tr>
<td>$\phi^\pi$</td>
<td>1.5</td>
<td>Taylor parameter</td>
</tr>
<tr>
<td>$b$</td>
<td>0.7</td>
<td>External habit parameter</td>
</tr>
<tr>
<td>$\kappa^o$</td>
<td>0</td>
<td>entrepreneur wealth dissipation parameter</td>
</tr>
<tr>
<td>$\kappa^s$</td>
<td>0.1</td>
<td>entrepreneur wealth dissipation parameter</td>
</tr>
<tr>
<td>$\chi^o$</td>
<td>0.015</td>
<td>financial efficiency parameter</td>
</tr>
<tr>
<td>$\chi^s$</td>
<td>0.15</td>
<td>financial efficiency parameter</td>
</tr>
</tbody>
</table>
Table 2: Sectoral output correlations

<table>
<thead>
<tr>
<th></th>
<th>Financial shock</th>
<th>Productivity shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>High SH =</td>
<td>-0.9935</td>
<td>0.9410</td>
</tr>
<tr>
<td>Low SH =</td>
<td>-0.9618</td>
<td>0.9690</td>
</tr>
</tbody>
</table>

a moderate increase if official and unofficial goods are close substitutes ($\varepsilon = 100$) and if retail prices are flexible. This is intuitively plausible, because price flexibility and goods substitutability tilt demand towards official sector goods. By contrast, when we raised the capital income share in the unofficial sector ($\alpha^s = \alpha^o$) we observed a dramatic increase in the response of unofficial labor demand after the initial wage fall. As a result the total output loss became negligible.\(^{19}\)

It is interesting to compare these results with the dynamic responses to a standard productivity shock in the official economy, described in Figure 6. In this case we obtain an increase in official output and a fall in official employment. This well-known effect is in line with a large body of theoretical and empirical literature (Gali and Rabanal, n.d.; Fernald, 2007; Canova et al., 2010). The fall in the real interest rate stimulates demand for unofficial goods, and the lower real wage allows the informal sector to increase employment. The the transmission of this shock is therefore positive even if we observe a countercyclical movement of the unofficial output share. Table 2 presents sectoral correlations caused by the two different shocks.

## 5 Conclusions

We produced empirical evidence suggesting that the size of the shadow economy increases in the aftermath of banking crises. Our theoretical model allows to highlight the transmission channels that might generate this result. Basically, the banking shock disrupt the formation of capital in the official economy, causing a reduction in official employment and a fall in the real wage that allows the unofficial sector to absorb more labor, thereby increasing the expected returns from unofficial capital formation. Further research should relax the hypothesis of complete labor market integration, introduc-

\(^{19}\)The transmission of the shock appears largely independent from the choice of the unofficial economy financial parameters $\chi^s$ and $\kappa^s$. Details of relevant simulations available upon request.
ing, for instance a distinction between a formal labor market characterized by search frictions and a competitive informal labor market, as in Zenou (2008).

Another important development would be to investigate the role of fiscal policy. Empirical research should highlight whether official and unofficial outputs were affected by different choices about public debt accumulation in response to crisis episodes. Developing a fiscal sector in our theoretical model could identify which policies should be implemented in response to banking crises.
References


Batini, Nicoletta, Paul Levine, Emanuela Lotti, and Bo Yang, “Informality, Frictions and Monetary Policy,” School of Economics Discussion Papers 0711, School of Economics, University of Surrey July 2011.


Table 3: Predicting banking crises

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>MTEA</th>
<th>TEM</th>
<th>Shadow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_0$</td>
<td>-0.060***</td>
<td>-0.005</td>
<td>-0.010</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$L_1$</td>
<td>-0.010</td>
<td>-0.006</td>
<td>-0.015</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$L_2$</td>
<td>-0.015</td>
<td>-0.006</td>
<td>-0.010</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$L_3$</td>
<td>0.012</td>
<td>-0.007</td>
<td>-0.005</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$L_4$</td>
<td>0.009</td>
<td>-0.004</td>
<td>-0.003</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>N. Obs</td>
<td>2788</td>
<td>2541</td>
<td>2541</td>
<td>2541</td>
</tr>
<tr>
<td>N. of Count.</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

Note: Dependent variable is banking crisis, regressors are official GDP growth (col 1), Total Economic Activity growth (col 2), electric consumption growth (Col 3), shadow growth (col 4). $L_0$...$L_4$ denote regressors at lag 0 to 4. Estimation is panel logic random effect. Robust standard errors reported in parentheses. ***,**,* denote significance at the 1, 5 and 10 percent levels respectively.
Figure 1: Impulse responses of financial crises (total electricity method): overall and country groups
Figure 2: Impulse responses of financial crises (modified total electricity approach): overall and country groups
Figure 3: Impulse responses of financial crises (modified total electricity approach): robustness checks and shadow economy
Figure 4: Impulse responses of financial shock. Low share of the shadow economy
Figure 5: Impulse responses of financial shock. High share of the shadow economy

- Official Y
- Shadow Y
- Total Y
- Official k
- Shadow k
- Total k
- Official i
- Shadow i
- Total i
- Official labour
- Shadow labour
- Total labour
- Cushoning effect

32
Figure 6: Impulse responses of productivity shock.
6 Appendix

6.1 Estimates for Total Economic Activity

Data on electricity consumption, real price of electricity, share of industrial income and official GDP have been obtained from Energy Information Administration, International Energy Agency, World Bank and United Nations, respectively.

Approximating the growth of total economic activity by means of electricity consumption growth is a straightforward exercise. in the following we outline the method used to obtain $MTEAg$, which is akin to \cite{Eilat-Zinnes-02,Onnis-Tirelli-10}. Our analysis is based on the assumption that changes in the domestic real price of electricity capture the effects of supply shocks and of long term efficiency gains caused by technical change, whereas changes in the industry share of GDP affect the component of electricity consumption which is directly related to the country-specific evolution in the composition of domestic output. The first stage of our application of the MTE procedure is therefore based on the following equation:

$$\Delta Elec_{i,t} = \alpha_i + \beta_1 \Delta Eprice_{i,t} + \beta_2 \Delta IndGdp_{i,t} + \varepsilon_{i,t} \quad (33)$$

where subscripts $t, i$ are time and country indexes, $\Delta Elec$, $\Delta Eprice$ and $\Delta IndGdp$ respectively describe annual percentage changes in electricity consumption, in the real price of electricity and in the industry share of GDP.

Once the relative-price and demand-composition effects have been identified, the residual changes in electricity consumption, $\Delta Elec^{res}_{i,t}$, may be used as a proxy for the growth rate in total economic activity ($MTEAg$):

$$\Delta Elec^{res}_{i,t} = \Delta Elec_{i,t} - [\beta_1 \Delta Eprice_{i,t} + \beta_2 \Delta IndGdp_{i,t}] \quad (34)$$

Since the time series dimension of the panel is relatively long, the econometric methodology is based on a preliminary stationarity and cointegration analysis of the relevant variables. Variables $\Delta Elec$, $\Delta Eprice$, $\Delta IndGdp$ exhibit non stationarity, tested using Im et al. (2003), Pesaran (2007), Hadri (2000), Kwiatkowski et al. (1992), ADF and Phillips-Perron unit root tests. A cointegrating relationships between $\Delta Elec$, $\Delta Eprice$ and $\Delta IndGdp$ has been, therefore, detected using the residual-based procedure developed by Pedroni (1999), Pedroni (2004).
Due to the presence of cointegrated time series, in our estimate of equation (33) we use the group-mean panel Fully Modified Ordinary Least Squares (FMOLS) method proposed by Pedroni (2000), Pedroni (2001). The group-FMOLS estimates reported in Onnis and Tirelli (2010) suggest that a positive and statistically significant relationship exists between the changes in electric consumption and those in the share of industry. On the contrary, a negative and statistically significant relationship exists between the changes in electric consumption and those in electricity price.  

6.2 Computation of impulse response functions

Impulse responses have been calculated as follows. First we have estimated equation (1) by GLS with fixed effects and time dummies. Obtained the estimated coefficients we have assumed that they are drawn from a multivariate normal distribution with mean the estimated vector of coefficients and as variance the estimated variance covariance matrix.

We have drawn a sample of 1000 coefficients from the distribution and we have simulated the cumulative effect of a financial crises. Confidence intervals have been calculated from the 2.5 and 97.5 percentiles.

6.3 Steady state derivation

From (15) (22) (24) we obtain

\[ \tilde{\omega}^j = (\omega - \bar{\omega}) - \frac{\mu^j}{1 - (1 - \kappa^j) \gamma} \] (35)

this from (16) (17) (18) (19) allows to obtain \( f(\tilde{\omega}^j) \), \( g(\tilde{\omega}^j) \). Then from (15) (24) we get the relative price of capital in the official economy.

\[20\text{To use changes in country-specific electricity price as an explanatory variable for changes in electricity consumption may generate problems of endogeneity. Firstly, we have re-estimated equation (33) adopting an alternative more exogenous real price of energy for 26 OECD countries and a global index of energy price for the remaining 23 countries. Second, we have used the global price of energy for the entire panel. In both situations we have obtained the same result. There is a positive and statistically significant relationship between changes in electricity consumption and changes in industry share of GDP. There is a negative and statistically significant relationship between changes in electricity usage and changes in the price of energy.} \]
\[ q^o = \frac{1}{(f(\tilde{\omega}^o)\gamma + g(\tilde{\omega}^o))} \] (36)

From (25) and (26) we get the relative price of capital in the unofficial economy

\[ \frac{q^s}{q^o} = \frac{(1 - \kappa^o) f(\tilde{\omega}^o)}{(1 - \kappa^s) f(\tilde{\omega}^s)} \]

Note that \( \frac{q^s}{q^o} \) is entirely determined by those parameters that characterize the relative financial markets imperfections in the shadow economy, \( \kappa^j \chi^j \mu^j \). In our benchmark calibrations we obtain \( \frac{q^s}{q^o} = \). When we set \( \chi^s = \chi^o \) and \( \kappa^s = \kappa^o \) we obtain that relative sectoral capital price amounts to and to respectively.

From (15) we obtain the sectoral real return on capital.

\[ r^j = q^j \left( \frac{1}{\beta} - (1 - \delta) \right) \]

This also shows that the relative sectoral marginal productivity of capital is equal to the relative price of capital and is therefore explained by the same factors.

From (6) we get the implied capital-labor ratios in the two sectors which obviously increase in \( \alpha^j \).

\[ \left( \frac{k^j}{h^j} \right) = \left( \frac{r^j}{\alpha^j} \right)^{-\frac{1}{1 - \alpha^j}} \]

From (32) we obtain that \( \pi^o = 1 \). Steady state equilibrium therefore implies that \( \pi^s = 1 \). From (8) and (11) we get the solution for relative prices, which are determined by financial factors, relative markups and the technology parameters.

\[ \frac{P^R_o}{P^R_s} = \frac{\sigma^o}{\sigma^s - 1} \frac{(\frac{r^o}{\alpha^o})^{\frac{1}{1 - \alpha^o}}}{(\frac{r^s}{\alpha^s})^{\frac{1}{1 - \alpha^s}}} \left( \frac{(1 - \alpha^s)}{(1 - \alpha^o)} \right) \]

(37)

Using (4) we obtain

\[ \left( \frac{P}{P^R_s} \right) = \left( (1 - \alpha) \left( \frac{P^R_o}{P^R_s} \right)^{(1 - \varepsilon)} + \alpha \right) \]

\[ \frac{1}{(1 - \varepsilon)} \]
Table 4: Sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>( \sigma^s = \sigma^o )</th>
<th>( \alpha^s = \alpha^o )</th>
<th>( \chi^s = \chi^o )</th>
<th>( k^s = k^o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>High SH</td>
<td>0.436</td>
<td>0.332</td>
<td>0.801</td>
<td>0.437</td>
<td>0.4753</td>
</tr>
<tr>
<td>Low SH</td>
<td>0.115</td>
<td>0.088</td>
<td>0.211</td>
<td>0.116</td>
<td>0.1259</td>
</tr>
</tbody>
</table>

Then from (5) we get the solution for the real wage.

\[
w = (1 - \alpha^s) \left( \frac{k^s}{h^s} \right)^{\alpha^s} \left( \frac{P^{Rs}}{P} \right)
\]

and for the relative retail price of official goods

\[
\left( \frac{P^{Ro}}{P} \right) = \frac{w}{(1 - \alpha^o)} \left( \frac{k^o}{h^o} \right)^{-\alpha^o}
\]

Now from (27) we get

\[
k^j_i = (1 - \delta)k^j_{i-1} + \eta^j_i \eta^j_i (1 - \phi (\tilde{\omega}^j_i - \omega)) \left( \frac{k^j_i}{h^j_i} \right)^{1-\alpha^j}
\]

Thus, from (29) we get the solution for the unofficial economy relative size, \( SH = \frac{y^u}{y^o} \).

Now derive \( SH \)

\[
SH = \frac{\alpha_c}{(1 - \alpha_c)} \left( \frac{P^{Rs}}{P^{Ro}} \right)^{-\epsilon} \left( 1 - \frac{\nu^o}{y^o} \right) \left( 1 - \frac{\nu^o}{y^o} \right) = \frac{\alpha_c}{(1 - \alpha_c)} \left( \frac{\sigma^o}{\sigma^{o-1}} \left( \frac{q^o(1-(1-\delta))}{\alpha^o} \right)^{1-\sigma^o} \left( 1 - \frac{\nu^o}{y^o} \right) \right)^{\epsilon} \left( 1 - \frac{\delta}{(1-\phi(\tilde{\omega}^o - \omega))\mu^o - \chi^o} \right) \left( \frac{q^o(1-(1-\delta))}{\alpha^o} \right) \left( \frac{\sigma^s}{\sigma^{s-1}} \left( \frac{q^s(1-(1-\delta))}{\alpha^s} \right)^{1-\sigma^s} \left( 1 - \frac{\nu^o}{y^o} \right) \right)
\]

Our solution for \( SH \) highlights the importance of the substitutability between official and unofficial goods, i.e. parameter \( \epsilon \), technology parameters, relative markups and the sectoral investment-to-output share. In Table 4 we report the sensitivity of \( SH \) to changes in these parameters starting from the
two parameterizations for $\alpha_c$ that deliver high and low values for $SH$. An increase in unofficial retail price markups unabiguously reduces the unofficial sector supply. In fact, raising $\sigma^s$ to the official economy level lowers $SH$ by about $\frac{1}{3}$. An almost identical result obtains if we impose strong substitutability between the official and the shadow consumption bundles, $\varepsilon = 100$. Note that after the increase in $\varepsilon$ we observe a much stronger effect of relative prices. These, in turn, are entirely determined by supply side effects, namely financial frictions, markups and technology parameters. In our benchmark calibration, for the high and low values of $SH$ we get $\frac{p^{Rs}}{p^{Ro}}$ and $\frac{p^{Rs}}{p^{Ro}}$ respectively. In spite of larger markups the price of unofficial goods is always relatively high, due to the higher cost of capital, determined by financial frictions, and to technology parameters. From (37) we know that consumption real wages do not matter for the determination of relative prices, but technology parameter $\alpha^s$ greatly strengthens the adverse effect of the relatively higher price of capital in the unofficial sector. In fact, when we set $\alpha^s = \alpha^o = 0.36$ the value of $SH$ nearly doubles. To assess the consequences of financial factors, we lower both $\chi^s$ and $\kappa^s$ to the values chosen for the official economy.  

We find that only the fall in $\kappa^s$ seem to matter, determining a 10% increase in $SH$. Finally, these results are obtained for both parameterizations of $\alpha_c$, confirming that this is just a scale parameter.

To conclude the derivation of the steady state bear in mind that

$$h^s = \left(\frac{k^s}{h^s}\right)^{-\alpha^s} y^s = \left(\frac{k^s}{h^s}\right)^{-(SH)} y^o, h^o = \left(\frac{k^o}{h^o}\right)^{-\alpha^o} y^o$$

From the aggregate labour resource constraint , where we have imposed that total labor supply $h = 1$ (we calibrate $\Gamma$ to obtain this) we obtain the solution for $y^o, y^s, i^j, n^j, h^j, c^j, c$

$$\left(\left(\frac{k^s}{h^s}\right)^{-\alpha^s} (SH) + \left(\frac{k^o}{h^o}\right)^{-\alpha^o}\right) y^o = 1 - \eta$$

$$y^s = (SH) y^o$$

we can now complete the steady state solution

$$i^j = \frac{\delta}{\eta y^j \left(1 - \phi \left(\tilde{\omega}^j - \omega\right) \mu^j - \mu^j\right)} \left(\frac{k^j}{h^j}\right)^{-\alpha^j} y^j$$

\footnote{From (35) it is easy to see that $\tilde{\omega}^S \equiv \omega^o$ when $\kappa^s = \kappa^o$.}
\[ c^j = y^j - \hat{v}^j \]
\[ n^j = i^j \left( \frac{1 - q^j g(\tilde{w}^j)}{1 - \hat{k}^j} \right) \]
\[ h^j = \left( \frac{k^j}{h^j} \right)^{-\alpha^j} y^j \]
\[ c^{\epsilon j} = \left( \frac{1}{\beta \gamma} - 1 \right) n^j \]
\[ y^s = \alpha_c \left( \frac{P^s}{P} \right)^{-\varepsilon} ((1 - \eta) c + \eta ((1 - \eta^o) c^{\epsilon s} + \eta^o c^{\epsilon o})) + \eta (1 - \eta^o) (i^s) \]