Are There Bubbles in the Sterling-dollar Exchange Rate?
New Evidence from Sequential ADF Tests

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Abstract
There has been mixed evidence regarding the existence of rational bubbles in the foreign exchange markets. This paper introduces recently developed sequential unit root tests into the analysis of exchange rates bubbles. We find strong evidence of explosive behavior in the nominal Sterling-dollar exchange rate. However, this explosive behavior should not be simply interpreted as evidence of rational bubbles, as we show that it might be driven by the relative prices of traded goods.

Keywords: exchange rates, rational bubbles, sequential unit root test
JEL classifications: C1, F3

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1 Introduction

Following the breakdown of the Bretton-Woods system of fixed exchange rates in the early 1970s, major developed countries switched from fixed into a floating exchange rate regime. History has witnessed many episodes of crises in the Sterling-dollar exchange market, such as the 1976 Sterling crisis, the strong depreciation in the mid-1980s, the 1992 Black Wednesday UK currency crisis, and the recent 2008 financial crisis. Dramatic depreciation of the Sterling-dollar rate during these crisis periods has puzzled practitioners as well as researchers. Some economists conjecture that speculative bubbles were driving the market during these periods. For example, Evans (1986) finds significant evidence of bubbles in the Sterling-dollar exchange rate in the early 1980s, while Meese (1986), West (1987) and Wu (1995) yield mixed results.

Recently, various new tests have been developed to detect speculative bubbles in asset prices, including Al-Anaswah and Wilfling (2011), Lammerding et al. (2013), Phillips et al. (2011b) and Phillips et al. (2011a). We employ the sequential unit root tests proposed by Phillips et al. (2011b) and Phillips et al. (2011a), which are based on the type of indirect stationarity tests initiated by Diba and Grossman (1984) and Hamilton and Whiteman (1985). These indirect tests have the advantage of detecting speculative bubbles despite a potential misspecification of the market fundamental process.

This paper applies the sequential unit root tests so as to shed new light on the debate on the existence of rational bubbles in exchange rates. We find strong evidence for explosive behavior in the nominal Sterling-dollar exchange rates. In order to shed light on the causes of the explosiveness, we also test for explosive behavior in the underlying fundamentals. Engel (1999) points out that movements in the US exchange rate are mainly driven by the relative prices of traded goods and not those of nontraded goods. Following Engel (1999), we construct the relative prices of traded and nontraded goods as fundamentals for exchange rates. Results show that the traded goods fundamental may explain the explosiveness in the Sterling-dollar exchange rate. Our findings thus shed doubt on claims that the Sterling-dollar exchange has been driven by speculative bubbles.

The remainder of the article is organized as follows: Section 2 describes the rational bubble model of the foreign exchange rate. Section 3 briefly introduces the econometric methods that we have applied. Section 4 presents the evidence on the explosiveness of the Sterling-dollar exchange rate and Section 5 concludes.

2 Rational Bubbles in Exchange Rate Dynamics

As stated by Obstfeld and Rogoff (1996, p. 529), “the nominal exchange rate must be viewed as an asset price”, which implies that it is determined by current and expected values of fundamentals. We thus assume the following present value model of exchange rate in line

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1 Another interesting application of the sequential unit root tests is the recent study by Pavlidis et al. (2012) who test the Efficient Market Hypothesis with forward exchange rates. The tests have also been applied to study the existence of speculative bubbles in commodity price and housing prices by Gutierrez (forthcoming), Phillips and Yu (2011) and Bohl et al. (forthcoming).
with Engel and West (2005) and León-Ledesma and Mihailov (forthcoming):

\[ s_t = (1 - \gamma) \sum_{j=0}^{k} \gamma^j E_t[f_{t+j}] + \gamma^{k+1} E_t[s_{t+k+1}], \]  

(1)

where \( s_t \) is the nominal exchange rate, and \( f_t \) is the market fundamental at time period \( t \). \( \gamma \) denotes the discount factor. By imposing the transversality condition

\[ \lim_{k \to \infty} \gamma^k E_t[s_{t+1}] = 0, \]

we assure that the exchange rate will only depend on future expected fundamentals in the long run. However, if the transversality condition does not hold, the exchange rate may be subject to an explosive rational bubble. Assuming that the bubble follows an AR(1) process, it can be written as

\[ b_t = \frac{1}{\gamma} b_{t-1} + \varepsilon_t, \]

(2)

where the first-order autoregressive coefficient \( \frac{1}{\gamma} \) is greater than 1, as the bubble is an explosive process. Errors are captured by \( \varepsilon_t \sim NID(0, \sigma^2) \). Therefore, we can write the exchange rate as

\[ s_t = s_t^f + b_t \quad \text{or} \quad s_t - s_t^f = b_t, \]

(3)

where \( s_t^f \) denotes the discounted sum of all future economic fundamentals and \( b_t \) the bubble component. We assume that \( s_t^f \) is linearly dependent on the economic fundamental \( f_t \). In accordance to Engel and West (2005) we also assume that \( f_t \) is I(1). According to the Purchasing Power Parity model, the economic fundamental for the nominal exchange rate is the price differential:

\[ f_t = p_t - p_t^*, \]

(4)

where \( p_t \) denotes the log level of the domestic price index. Asterisks denote foreign counterparts. For decomposing the price index into indexes of nontraded and traded goods, Engel (1999) considers a price index for a country as a weighted average of traded- and nontraded-goods prices \( p_t = (1 - \alpha)p_T^T + \alpha p_N^N \). \( p_T^T \) denotes the log of the traded goods price index, \( p_N^N \) the log of the nontraded goods price index and \( \alpha \) the share of the nontraded goods component. For the foreign country, one can also write \( p_t^* = (1 - \beta)p_T^T* + \beta p_N^N* \). It follows that the price differential \( (f_t) \) can be decomposed into two components, the traded goods component \( (f_T^T) \) and the nontraded goods component \( (f_N^N) \).

\[ \left( \frac{p_t - p_t^*}{f_t} \right) = \left( \frac{p_T^T - p_T^T*}{f_T^T} \right) + \alpha \left( \frac{p_N^N - p_T^T}{f_T^T} \right) - \beta \left( \frac{p_N^N* - p_T^T*}{f_T^T} \right). \]

(5)

The producer price index (PPI) is the most broadly available and frequently used index to represent the price level of traded goods. Though there are some producer goods that are
not traded, PPI is measured at the production site and thus exclude marketing and other nontraded consumer services. Thus we construct the traded goods component using the PPI following Engel (1999):

\[ f_t^T = \ln(PPI_t) - \ln(PPI_t^*), \]  

(6)

The relative nontraded goods component is constructed from the aggregate consumer price indexes (CPI) relative to aggregate PPI:

\[ f_t^N = \ln(CPI_t) - \ln(PPI_t) - (\ln(CPI_t^*) - \ln(PPI_t^*)), \]  

(7)

In the following section, we demonstrate how explosiveness can be detected in the nominal Sterling-dollar exchange rates \( s_t \), and the ratio of the exchange rate relative to the two types of economic fundamentals, using recursive right-tailed unit root tests by Phillips et al. (2011b) and Phillips et al. (2011a).

### 3 The Sequential ADF Tests

Phillips et al. (2011b) provide a new framework to test for bubble phenomena in asset prices. Homm and Breitung (2012) show that this sup ADF (SADF) test is capable of detecting periodically collapsing bubbles and is robust against multiple breaks due to a possible burst of the bubble. The test procedure is based on the autoregressive process

\[ x_t = \mu + \delta x_{t-1} + \sum_{j=1}^{J} \phi_j \Delta x_{t-j} + \varepsilon_t, \]  

(8)

where \( x_t \) is the time series of interest, \( E(\varepsilon_t) = 0 \) and \( E(\varepsilon_t^2) = \sigma^2 \). The unit root null hypothesis is \( H_0 : \delta = 1 \) and the right-tailed alternative hypothesis is \( H_1 : \delta > 1 \).

Given a fraction \( r_0 \) of the total sample as an initial window size, Equation (8) is estimated recursively fixing the first observation as the starting the point, and using the subsets of sample data increased by one observation stepwise.

For a subsample starting from the first observation and at a fractional size of the full sample \( r_2 \), where \( r_0 < r_2 \leq 1 \), the corresponding ADF test statistic can be denoted by \( ADF_{r_2} \). Hence \( ADF_1 \) corresponds to the ADF test statistic of the full sample. The SADF test statistic is thus the supremum value of \( ADF_{r_2} \), for \( r_0 < r_2 \leq 1 \).

\[ \text{SADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \{ ADF_{r_2} \}, \]  

(9)

Evidence of explosive behavior is obtained on certain time series if the SADF statistic is larger than the right-side critical values for a chosen nominal size.

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2Note that no assumption is made about \( \alpha \) or \( \beta \). Through transformation it is easy to show that \( f_t^N = \alpha(p_t^N - p_t^T) - \beta(p_t^N* - p_t^T*) = (p_t - p_t^T) - (p_t^* - p_t^T*). \)
One limitation of the SADF test is that the starting point is fixed as the first observation of the sample. This implies that in the presence of two bubbles, the second bubble may not be detected if it is dominated by the first bubble. Therefore, Phillips et al. (2011b) also apply a rolling version of the SADF test, where the starting window moves over the sample. However, the size of the starting window is still fixed, which limits the power of the test. Phillips et al. (2011a) extend the SADF test by nested it in a loop, which increments the starting point \((r_1 \in [0, r_2 - r_0])\) each run. The generalized SADF test (GSADF) is able to detect potential multiple bubbles in the data and thus overcomes the weakness of the SADF test:

\[
GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} \{ADF_{r_2}\}.
\]  

Consequently, both the SADF test and the rolling SADF test are nested in the GSADF test. It is important to note that the tests may fail to detect an early bubble if the starting window size is too large.

4 Explosive Behavior in the Sterling-dollar Exchange Rates

Our study focuses on the bilateral exchange rates between the United States and Great Britain. We obtained time series of the British Pound/ US dollar exchange rate from the OECD database. The time series of the consumer price index (US) and retailer price index (UK) as well as the producer price index (PPI) are obtain from the IMF International Financial Statistics and used for constructing the fundamentals of the exchange rates. All times series are transformed into logarithm. We work with monthly data, because a higher frequency of price data is not available. The data sample ranges from 1972 M1 to 2012 M6 and covers 486 monthly observations. Hence, our sample covers the period after the breakdown of the Bretton-Woods system of fixed exchange rates. We set the lag order to zero for all time series, because Phillips et al. (2011a) demonstrate with Monte-Carlo simulations that lag selection criteria such as Campbell and Perron (1991) result in significant size distortion and lower power of both the SADF and the GSADF tests.

Results for the nominal Sterling-dollar exchange rate \(s_t\) are shown at the third row of Table 1. The standard right-sided ADF test statistic seems to suggest no explosive behavior in the nominal exchange rate. However, this result could be misleading if periodically collapsing bubbles occur during the given period (see Evans (1991)). The SADF and the GSADF tests are capable of overcoming this shortcoming. The null hypothesis that there is no explosive behavior in the nominal Sterling-dollar exchange rate is rejected at the 1% significance level for the SADF test. Non-explosiveness is also rejected at the 5% significance level according to the GSADF test. Figure 1 shows the time series of the log nominal exchange rate and the corresponding sequence of \(ADF_t\) statistics. The \(ADF_t\) sequence displays clear evidence of multiple periods of explosiveness. First, the test reports explosiveness in 1976, which corresponds to the 1976 Sterling crisis. Secondly, we find explosiveness in 1985. At that time, the US dollar appreciated heavily against several currencies.
Table 1: Tests for Explosive Behavior in the Sterling-dollar Exchange Rate

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF</th>
<th>SADF</th>
<th>GSADF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_t$</td>
<td>-2.478</td>
<td>2.128**</td>
<td>2.416*</td>
</tr>
<tr>
<td>$s_t - f_t^N$</td>
<td>-1.934</td>
<td>2.630**</td>
<td>2.794*</td>
</tr>
<tr>
<td>$s_t - f_t^T$</td>
<td>-1.827</td>
<td>0.374</td>
<td>1.623</td>
</tr>
<tr>
<td>CV 1%</td>
<td>0.614</td>
<td>1.984</td>
<td>2.860</td>
</tr>
<tr>
<td>CV 5%</td>
<td>-0.091</td>
<td>1.490</td>
<td>2.340</td>
</tr>
<tr>
<td>CV 10%</td>
<td>-0.451</td>
<td>1.218</td>
<td>2.106</td>
</tr>
</tbody>
</table>

This table shows the various test statistics of the nominal exchange rates $s_t$, the ratio of the exchange rate to the nontraded goods fundamental $s_t - f_t^N$, and the ratio of the exchange rate to the traded goods fundamental $s_t - f_t^T$ (see Equation (6) and Equation (7)). The initial window size $r_0$ is set as three years (36 observations) for the SADF and GSADF tests. Critical Values are obtained from Monte-Carlo simulations with 5000 replications for the ADF, SADF and GSADF tests. The items marked with * are significant at 5% significance level, and the items marked with ** are significant at 1% significance level.

The explosiveness in the nominal exchange rate could be driven either by rational bubbles or explosive fundamentals. The fourth row of Table 1 shows the test results for the ratio of the exchange rate to the nontraded goods fundamental $s_t - f_t^N$. The exchange rate remains explosive after the relative prices of nontraded goods are accounted for. Figure 2(a) displays the sequence of the $ADF_t$ statistics for the exchange rate to the nontraded goods fundamental ratio, which behaves very similar to those of the nominal exchange rate $s_t$ in Figure 1. Thus the relative prices of nontraded goods $f_t^N$ play no role in explaining the explosiveness in the nominal exchange rate.

In contrast, no evidence of explosive behavior is found in the relative ratio of the exchange rate to the traded goods fundamental $s_t - f_t^T$. The null hypothesis that the series is nonexplosive can not be rejected at the 10% significance level for either the SADF or the GSADF test. Figure 2(b) displays the result of the SADF test graphically. The GSADF statistics show exactly the same pattern (see appendix). Therefore, the explosive behavior in the nominal Sterling-dollar exchange rate may be driven by the relative prices of traded goods between the US and Great Britain.\(^3\) The two periods where the explosiveness diminishes are characterised by large commodity shocks. Moreover, manufacturing and mining, two large sectors in the UK until the mid-1980s, were heavily unionised, creating large wage-price spirals. Both effects may have driven up UK PPI inflation causing the observed pattern.

These findings are not in favor of the speculative bubble hypothesis in the nominal Sterling-dollar exchange rate, because the explosive behavior in the exchange rate may be

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\(^3\) As a robustness check, we test the price ratios separately. The series $f_t^T$ exhibits explosiveness during the two periods where the explosiveness in the ratio of the exchange rate to the traded goods fundamental diminishes. Results are available on request.
driven by the relative prices of trades goods. Our results are in accordance with those of Engel (1999) and Betts and Kehoe (2005) who show that the relative prices of traded goods explain most of the movements in exchange rates.

5 Conclusion

In this paper we provide new evidence casting doubt on the bubble hypothesis in the nominal Sterling-dollar exchange rate by employing recent sequential ADF tests developed by Phillips et al. (2011b) and Phillips et al. (2011a). Though we find explosive behavior in the nominal exchange rate, the explosiveness coincides with explosive behavior in the relative prices of traded goods. Hence, our findings are not in favor of the bubble hypothesis. In line with Engel (1999) and Betts and Kehoe (2005), our results demonstrate that the relative prices of nontraded goods play little role in the movements of exchange rates, while the relative prices of traded goods seem to be an important determinant. Consequently, we show that it is crucial to take the underlying fundamentals into account when identifying rational bubbles in asset prices, because explosiveness in the asset price alone is not a sufficient condition. This is an important insight for policy makers and practitioners as well.

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References


A Figures

Figure 1: The Nominal Sterling-dollar Exchange Rate (SADF Test)

Note: This graph shows the series of the nominal Sterling dollar exchange rate \( s_t \) (right, dotted) and its corresponding sequence of ADF statistics (left, solid). The dashed line represents the 5% critical values of the SADF test.
Figure 2: The Sterling-dollar Exchange Rate to Fundamental Ratios (SADF Test)

(a) The Ratio of the Exchange Rate to the Nontraded Goods Fundamental

(b) The Ratio of the Exchange Rate to the Traded Goods Fundamental

Note: The upper panel shows the series of the ratio of the exchange rate to the nontraded goods fundamental $s_t - f_t^N$ (right, dotted) and its corresponding sequence of ADF statistics (left, solid). The lower panel shows the series of the ratio of the exchange rate to the traded goods fundamental $s_t - f_t^T$ (right, dotted) and its corresponding sequence of ADF statistics (left, solid). The dashed line represents the 5% critical values of the SADF test.
Figure 3: The Nominal Sterling-dollar Exchange Rate (GSADF Test)

Note: This graph shows the series of the nominal Sterling dollar exchange rate $s_t$ (right, dotted) and its corresponding sequence of ADF statistics (left, solid). The dashed line represents the 5% GSADF critical values.
Figure 4: The Sterling-dollar Exchange Rate to Fundamental Ratios (GSADF Test)

(a) The Ratio of the Exchange Rate to the Nontraded Goods Fundamental

(b) The Ratio of the Exchange Rate to the Traded Goods Fundamental

Note: The upper panel shows the series of the ratio of the exchange rate to the nontraded goods fundamental $s_t - f_t^N$ (right, dotted) and its corresponding sequence of ADF statistics (left, solid). The lower panel shows the series of the ratio of the exchange rate to the traded goods fundamental $s_t - f_t^T$ and its corresponding sequence of ADF statistics. The dashed line represents the 5% GSADF critical values.