Knowing Finance or Knowing Those Who Know It?  
The Interrelation Between Financial Literacy and Investment Choices

Riccardo Calcagno* and Chiara Monticone†

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Abstract

This paper analyzes a static portfolio choice where the investor can choose to delegate her portfolio management to a professional intermediary, who can give biased advice due to a conflict of interests. The choice of delegation depends both on advisors’ and investors’ characteristics. The latter include the cost of information search and the level of “sophistication”, meaning investor’s ability to correctly account for the bias in her decisions. The model studies which investors are more likely to delegate, and what are the consequences of this choice on the ex-post portfolio performance. We find that not only more “naïve” investors are more likely to delegate, but they tend delegate to “worse” advisors, i.e. with higher bias. This results in a higher probability to suffer ex-post losses.

1 Introduction

Financial advisors can have an important role in helping non-professional investors to take their portfolio decisions. For instance, financial advisory services can exploit economies of scale in information acquisition, and help correcting investors’ cognitive errors (Bluethgen et al. (2008)).

We argue that in this context the degree of financial literacy of an individual investor plays a crucial role in her investment choices not only directly, i.e. through the selection of the preferred financial portfolio, but also indirectly through the decision to rely on the expertise of financial professionals or not, and if so, on which one. Indeed, the empirical literature suggests

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*VU University Amsterdam, Tinbergen Institute and CeRP (Collegio Carlo Alberto, Turin).
†University of Torino and CeRP (Collegio Carlo Alberto, Turin).
that financial literacy affects the choice of financial advisors (or information sources). For instance, more financially literate individuals are more likely to chose “better” advice, i.e. they prefer professionals to informal sources (Bernheim (1998); Lusardi and Mitchell (2006); van Rooij et al. (2007)).

Our basic argument is the following. Even if advisors had an informational advantage and were fully able to correct their customers’ behavioral errors, their advice is likely to be biased because of possible conflicts of interests. Recognizing this agency conflict between an investor-client and a professional advisor, we propose to disentangle two aspects of the financial literacy of an investor. We consider separately (i) her ability to understand basic financial principles and to know the characteristics of the most common financial products, from (ii) the ability to understand advisors’ conflicts of interests and to account for the possibility to receive a biased advise. Starting from this recognition, we study which investors are more likely to delegate their portfolio decision to advisors, what are the consequences of this choice in terms of ex-post performance, and which aspect of financial literacy – knowledge vs. sophistication – has the greatest impact on financial outcomes.

This paper proposes a model where the investor can carry out portfolio management on her own or she can delegate it to a financial advisor. In choosing between these two possibilities, she trades off the effort she needs to collect and understand the relevant information on her own and the bias that the advisor introduces in her portfolio as a result of his conflict of interest. Given the level of deception that the financial advisor includes in his recommendations, the decision to delegate portfolio management depends not only on the cost of information search, but also on the investor’s degree of financial sophistication, represented by the ability to understand and take into account the broker’s incentives.

Using this framework we show that investors who bear a high cost of investing without the help of an advisor – those with little financial knowledge or high opportunity cost of time – are more likely to delegate their portfolio management. In addition, very “naïf” investors, who are relatively unaware of the agency conflict between them and the professional advisors, are more prone to entrust their portfolio to advisors than sophisticated individuals. Due to the failure to recognize advisors’ bias, they are more likely not only to delegate, but also to choose relatively more biased intermediaries. As a

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1To put the model in relation to reality, it is important to keep in mind that the availability of different sources of financial advice is not the same in every country. For instance, in the US independent financial advice is available while in Italy it is almost non-existent. This is important because, for example, the non-availability of professional and unbiased advice might have an impact on the likelihood of consulting informal (friends, relatives) or biased (banks) sources, or no source at all.

2By now the advisor’s bias is assumed to be exogenous with respect to the investor’s financial literacy.
result, “naïf” investors have a higher probability to suffer ex-post losses.

Our results suggest then that lacking the ability to understand the agency issues between clients and financial intermediaries may be even more harmful than having a low knowledge of basic financial principles.

Many studies try to assess whether advisors have an information advantage with respect to non-professional investors (see e.g. Cowles (1933); Desai and Jain (1995); Barber et al. (2001); Metrick (1999) among others), and whether advisors can correct behavioral biases such as disposition effect, overtrading and under-diversification (Shapira and Venezia (2001); Barber and Odean (2000, 2001)). Even though results on these points are at best mixed, at least advisors’ mistakes appear to be less serious than investors’ ones, justifying our assumption that individual investors turn to financial experts for meaningful advice.

The agency conflict between advisors and their client has received a large attention in the microeconomic literature. For instance, Ottaviani (2000) studies incentives for truthful information disclosure by an informed financial adviser – who is assumed to have both a professional and a partisan objective – to an uninformed investor with an uncertain degree of strategic sophistication. Similarly, Inderst and Ottaviani (2009) analyze the equilibrium amount of “misselling” arising from the inherent conflict between two tasks performed by direct marketing agents, i.e. prospecting for customers and advising on the product’s suitability for their specific needs.

If we look closer to the specific relation between financial experts and non-professional investors, Krausz and Paroush (2002) study the optimal amount of deception advisors put in their advice as a function of the investor’s characteristics, represented by wealth and attitude towards risk, and by the characteristics of assets, such as riskiness and degree of market competition. Krausz and Paroush (2002) then provide a compelling argument for the assumption that the advice of professionals can be biased. Moreover, Carlin (2009) shows that the degree of complexity in retail financial markets is endogenous and arises from the fact that consumers’ ignorance is a source of market power for firms, since it allows them to increase the horizontal differentiation of products, hence their prices.

The rest of the paper is organized as follows. Section 2 presents the setup of the model. Section 3 develops the choice between delegation and private signal acquisition as a function of advisors’ and investors’ characteristics, and the impact of this choice on portfolio ex-post performance. Section 4 concludes.
2 The model

We present a one period model of portfolio selection in which an investor $i$ chooses to invest his initial wealth $W_0$ in either of two assets: a riskless asset with return $R_f$ or one risky asset with return: $\tilde{R} \sim N(\mu, \frac{1}{\sigma^2})$ with $\mu \geq 0$. For analytic tractability we assume $R_f = 0$.

The investor $i$ does not know the realization of $\tilde{R}$ but can obtain an informative signal (which differ in terms of bias and precision) from different sources (self, friends, financial advisors...).\(^3\) We denote the different sources with the index $j$, where $j = 0$ indicates that investor $i$ collects the signal by himself. For each investor $i$ a set $J_i = \{0, 1, \ldots, N_i\}$ of sources is available. We assume that $i$ chooses one source $j \in J_i$ among the ones available to him\(^4\).

Before investing in the risky asset investor $i$ may either collect an informative signal by herself or rely on the advisor $j$ which provides her with an informative signal $s_j = R + \varepsilon_j$ with $\varepsilon_j \sim N(h_j, \frac{1}{p_j})$. Previously to the revelation of $s_j$ however, $i$ has to delegate to $j$ the portfolio choice\(^5\).

For notational convenience, we will denote with $j = 0$ the case in which the investor collects the information signal by herself. For $j = 0$ the precision of the signal $p_{0,i}$ is chosen optimally by $i$ taking into account the relative costs and benefits of information. The precision $p_{0,i}$ is an increasing function of the effort that the investor exerts in collecting information. The cost of exerting such an effort increases in the individual parameter $\varphi_i$ that can be interpreted in two different ways: (i) it may measure the opportunity cost of time, or (ii) the individual prior knowledge and experience in handling stocks (broadly speaking his financial literacy). We assume that the cost of collecting a signal with precision $p$ is increasing in $\varphi_i$: $c(p) = \varphi_i p^2$. Wealthier investors with a higher opportunity cost of time exhibit a higher $\varphi_i$; alternatively, less literate investors have higher $\varphi_i$.

Any advisor $j$ potentially introduces a bias $h_j \geq 0$ in the signal $s_j$ he provides to the investor: This bias can be a function of various exogenous determinants, e.g., the commission earned by the financial advisors (Ottaviani (2000); Inderst and Ottaviani (2009)), the degree of competition in the market (Bolton et al. (2007)), the characteristics of the investor (Krausz and Paroush (2002)), the degree of sophistication in the market (Carlin (2009)), etc. The parameter $h_j$ can also be interpreted as the “honest” mistake an investor $i$ makes in collecting information.

\(^3\)To keep things simple, we restrict the advice not to be about which product to buy but about how much of the risky asset to include in the portfolio. We also restrict our analysis to information and advice about stocks, even though people might seek advice also concerning mortgages and other loans, insurance, etc.

\(^4\)This is w.l.o.g. if one considers that each element $j \in J_i$ can be considered as the union of potential many advisors.

\(^5\)Or, in other words, a broker $j$ offers his advice to $i$ only after $i$ has delegated to him the choice of investment through a binding contract. See Ottaviani (2000) for the conditions under which delegation is optimal.
formal advisor (friends, relatives) makes in suggesting the risky asset return (i.e., the mistake arises just by the advisor being poorly skilled/uniformed, not by a conflict of interests). Hence, for the moment the parameters \((h_j, p_j)\) are considered as given by the investor, while we provide in the Appendix a model which microfounds the choice of the optimal \((h^*_j, p^*_j)\) for an intermediate \(j\) given the demand conditions. The signal \(s_j\) can also be written as:

\[ s_j = R + h_j + \epsilon_j \]

in order to identify an unbiased error term \(\epsilon_j \sim N(0, \frac{1}{p_j})\).

Once the investor has obtained a signal (either by own effort or through an advisor), she updates her beliefs about the distribution of the risky asset return:

\[ \tilde{R}|s_j \sim N(\mu + s_j \sigma_p, \frac{1}{\sigma_p}) \]

and then she chooses her optimal portfolio in the case \(j = 0\); otherwise, for all \(j \in J, j \neq 0\) she receives the portfolio chosen by the source \(j\). The portfolio bought by \(j\) on behalf of the investor \(i\) is the optimal one given investor’s preferences, but considering the signal \(s_j\) (which may be biased upward). Hence, we assume that the cost of delegation here consists in an overexposure to the risky asset.

Each investor is also characterized by an individual level of financial experience \(t_i\). The more the investor is experienced in her relation with financial advisors, the more she can correct for the bias \(h_j\) introduced by \(j\) in the signal \(s_j\): when \(t_i = 1\) we say the investor is “sophisticated” since she is able to correctly solve the agency problem with their financial advisors and perfectly anticipate the bias in the signal she receives; when \(t_i = 0\) the investor is “naïf” because she does not recognize the bias included in the advice she receives, hence she does not correct for it. Alternatively, the parameter \(t_i\) can be interpreted as trust towards the advisors, with large \(t_i\) indicating low trust and small \(t_i\) indicating large trust. In the following we will consider how any level of \(t_i \in [0, 1]\) affects the choice by investor \(i\) to delegate or not her investment choice and if so, to which source \(j\).

3 The choice between delegation and self-enquiring

We determine which source of information \(j\) the investor \(i\) chooses given her characteristics \((\varphi_i, t_i)\) and the possible sources of advice \(j \in J_i\), the market offers her, where each advisor \(j\) is characterized by a \((h_j, p_j)\) considered as given by the investor.

The portfolio choice problem of investor \(i\) with negative exponential utility with a given information set \(H\) can be stated as:

\[
\max_v E[U(W_1) | H] = E[W_1 | H] - \frac{\gamma}{2} Var[W_1 | H]
\]

s.t. \(W_1 = (W_0 - v) + v(1 + \tilde{R}) = W_0 + v\tilde{R}\)

\[6\]This new notation is introduced in order to identify an unbiased error term \(\epsilon_j\).

\[7\]We normalize the risk-free rate to zero for simplicity.
or equivalently
\[
\max_v W_0 + v E[\hat{R} | H] - \frac{\gamma}{2} v^2 \text{Var}[\hat{R} | H]
\]
where \(v\) is the amount of wealth invested in the risky asset and \(\gamma\) is the parameter of absolute risk aversion.

The optimal investment in the risky asset given information \(H\) is standard:
\[
v^* = \frac{E[\hat{R} | H]}{\gamma \text{Var}[\hat{R} | H]}
\]
and substituting for \(v^*\) in the objective function we obtain
\[
E[U(W_1) | H] = W_0 + \frac{1}{2} \left( \frac{E[\hat{R} | H]}{\gamma \text{Var}[\hat{R} | H]} \right)^2
\]

We start considering the case in which \(i\) collects an informative signal by herself (i.e., \(j = 0\)).

### 3.1 Private acquisition of the signal by the investor

We assume that the signal the investor obtains when acquiring information by herself is unbiased, that is \(h_0 = 0\) and \(\epsilon_0 \sim N(0, \frac{1}{p_0})\).

Once \(i\) has obtained the signal \(s_0\) with precision \(p_0\) she chooses her optimal portfolio \(v^*_i,0 = \frac{E[\hat{R} | s_0]}{\gamma \text{Var}[\hat{R} | s_0]} = \frac{\mu \sigma + s_0 p_0}{\sigma + p_0} \) and her utility is equal to
\[
V_i | s_0 : = V_{i,0} = W_0 + \frac{1}{2} \left( \frac{E[\hat{R} | s_0]}{\gamma \text{Var}[\hat{R} | s_0]} \right)^2 = W_0 + \frac{1}{2\gamma} \left( \frac{\mu \sigma + s_0 p_0}{\sigma + p_0} \right)^2
\]

Investor \(i\) decides the optimal precision of the signal before obtaining it (i.e. with an empty information set \(H_0\)); in doing this, she optimally trades off the benefits and the costs of getting a higher precision signal. She solves then \(^{8}\)

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\(^{8}\)Remember that
\[
E[\hat{R} + \bar{\epsilon}_o | H_0] = 0
\]
\[
E[(\bar{\epsilon} + \bar{\epsilon}_0)^2 | H_0] = \text{Var}[(\bar{\epsilon} + \bar{\epsilon}_0) | H_0] + (E[(\bar{\epsilon} + \bar{\epsilon}_0) | H_0])^2 = \frac{1}{\sigma + \frac{1}{p_0}}
\]
\[
\text{Var}[(\bar{\epsilon} + \bar{\epsilon}_0)^2 | H_0] = 2\left( \frac{1}{\sigma + \frac{1}{p_0}} \right)^2
\]
\[
\text{Cov}[(\bar{\epsilon} + \bar{\epsilon}_0)^2, (\bar{\epsilon} + \bar{\epsilon}_0) | H_0] = E[(\bar{\epsilon} + \bar{\epsilon}_0)^3 | H_0] = 0
\]
\[
\max_{p_0} \quad EU_{self,0} = E[V_{i,0} | H_0] - \frac{\gamma}{2} Var[V_{i,0} | H_0] - \varphi_i p_0^2 \\
= W_0 + \frac{2 \mu^2 \sigma (\sigma + p_0)(\sigma - p_0) + p_0(2\sigma - p_0)}{4 \gamma \sigma^2} - \varphi_i p_0^2
\] (2)

Taking the f.o.c. of problem (2) one obtains

\[
p_{i,0}^* = \frac{\sigma}{1 + 2\sigma(\mu^2 + 2\gamma \varphi_i)}
\] (3)

**Lemma 1:** The optimal precision \(p_{i,0}^*\) of the signal acquired by the investor by herself is decreasing in \(\gamma, \varphi_i, \mu\) and the expected utility from acquiring an informative signal privately is decreasing in \(\varphi_i\). If \(h_j = 0\) for any \(j \in J_i\), then \(i\) chooses to delegate to an advisor \(j\) when \(p_j \geq p_{i,0}^*\).

**Proof:** The comparative statics on \(p_{i,0}^*\) are immediate from (3). When \(h_j = 0\) delegation is costless and if the advisor provides an information \(p_j\) at least as precise as \(p_{i,0}^*\) the investor is better off since she saves the costs of information collection \(\varphi_i p_0^2\). ■

From (2) we can see that when collecting the information signal by herself, the investor never chooses at the optimum a signal with precision higher than \(\sigma\). To see the intuition of this result, consider again (1): the absolute value of the optimal portfolio \(v^* = \frac{E[\tilde{R}|s]}{\gamma Var[\tilde{R}|s]} = \frac{\mu \sigma + sp}{\gamma}\) is increasing in \(p\). Then, coeteris paribus, the higher the precision of the information received, the more aggressive the exposure on the risky asset of the investor portfolio. This implies that both

\[
E[V_{i,0} | H_0] = \frac{\mu^2 \sigma}{2\gamma} + p_0 \frac{\mu^2 \sigma + 1}{2\gamma \sigma}
\]

and

\[
Var[V_{i,0} | H_0] = p_0 \frac{p_0 + 2\sigma \mu^2 (\sigma + p_0)}{2\gamma \sigma^2}
\]

are increasing in \(p_0\) since the portfolio \(v^*\) contains more of the risky asset. For this reason the expected utility \(E[V_{i,0} | H_0] - \frac{\gamma}{2} Var[V_{i,0} | H_0]\) has an interior maximum at

\[
\hat{p}_0 = \frac{\sigma}{2\mu^2 \sigma + 1} \leq \sigma
\]

even when the signal precision is costless for the investor. Hence, \(p_{0,i}^* < \hat{p}_0 \leq \sigma\). Moreover, \(\sigma\) and \(p_{i,0}^*\) are likely to be substitutes since

\[
\frac{\partial p_{i,0}^*}{\partial \sigma} = \frac{1 - 4\varphi_i \gamma \sigma^2}{[1 + 2\sigma(\mu^2 + 2\gamma \varphi_i)]^2}
\]
is negative for a sufficiently large cost of collecting a precise signal (it is surely negative for \( \phi_i \geq 1 \), \( \sigma \geq 1 \). In the following we can assume then w.l.o.g. that \( p_{i,0}^* \leq \sigma \).

If we assume that the cost of information collection (\( \phi_i \)) also (inversely) depends on the degree of financial literacy of investor \( i \) we can reinterpret Lemma 1 as follows: an investor with low literacy obtains at optimum a signal with lower precision than the one obtained by an otherwise identical investor with high literacy; the lower the degree of financial literacy, the lower the expected utility of acquiring a costly private signal for the investor.

Finally, plug in the expression for \( p_{0,i}^* \) contained in equation (3) into \( EU_{i,\text{self}} \) in equation (2) to obtain the maximum ex-ante utility the investor can reach not delegating, obtaining:

\[
EU_{i,\text{self}} = W_0 + \frac{2\mu^2\sigma(4\phi\gamma\sigma^2 + 2\mu^2\sigma + 1) + 1}{4\gamma(4\phi\gamma\sigma^2 + 2\mu^2\sigma + 1)} = W_0 + \frac{\mu^2\sigma + 1}{2\gamma[1 + 2\sigma(\mu^2 + 2\gamma\phi\sigma_i)]} = W_0 + \frac{2\mu^2\sigma + 1}{4\gamma[1 + 2\mu^2\sigma + 4\gamma\phi_i\sigma^2]} \tag{4}
\]

### 3.2 The choice of the delegated manager

We assume that the signal \( s_j \) the investor obtains from the source \( j \) can be biased, that is \( h_j \geq 0 \). Recall that \( s_j \) can be written as \( s_j = R + h_j + \epsilon_j \) with \( \epsilon_j \sim N(0, \frac{1}{p_j}) \) where the noise term \( \epsilon_j \) is uncorrelated with all other random variables of the model. The effect of a positive (resp. negative) bias in \( s_j \) is to induce the investor to buy a share of the risky asset which is higher (resp. lower) than the one she would have bought she had received an unbiased signal. Similar ways of modeling the bias are present also in Krausz and Paroush (2002), Ottaviani (2000), Inderst and Ottaviani (2009). This is consistent with the idea that portfolio managers earn commissions on the amount of risky assets sold and therefore have an incentive to encourage more aggressive portfolios.

In this section we consider the bias \( h_j \) and the precision of the signal \( p_j \) as exogenous. In the Appendix B we develop a simple model similar to the one in Krausz and Paroush (2002) explaining the choice of the optimal \((h_j, p_j)\) by the financial advisor \( j \).

We characterize here the expected utility of investor \( i \) who decides to delegate to \( j \) the choice of her portfolio, obtaining in exchange the information contained in his signal \( s_j \). The portfolio bought by \( j \) on behalf of the investor \( i \) is denoted by \( v_i(h_j, p_j) = \frac{\mu\sigma + h_j p_j}{\gamma} = \frac{\mu\sigma + (R + h_j + \epsilon_j)p_j}{\gamma} \).

We assume that the awareness of the bias by an investor depends on her
expertise in dealing with financial advisors, or “sophistication”, which we denote by \( t_i \). In the following \( t_i = 0 \) indicates “naïf” investors, and \( t_i = 1 \) “sophisticated” ones\(^9\) while \( t_i > 1 \) indicates investors very skeptical towards financial advisors. The individual level of sophistication \( t_i \) can be thought as a dimension of the investor’s financial literacy, or at least it may depend on it. However, this is not necessarily true: an investor may be highly sophisticated in our definition if she is aware of the agency problem between her and her portfolio manager. This awareness may also originate from an element of (dis)trust towards intermediaries, or by the past experience of the investor: both these elements are not necessarily correlated with the investor’s financial education.

Naïf investors buy the biased portfolio \( v_i(h_j, p_j) \) thinking that \( h_j = 0 \) when they evaluate their ex-ante utility, whereas investors with \( t_i > 0 \) are at least partially aware of the bias introduced by \( j \) in the choice of \( v_i(h_j, p_j) \) so that they correct for the bias by a degree \( t_i h_j \).

The utility of an investor \( i \) who has delegated his investment to \( j \) obtaining the portfolio \( v_i(h_j, p_j) = \frac{\mu \sigma + s_j p_j}{\gamma} \) is equal to:

\[
V^t_i|s_j = V^t_{i,j} = W_{0,i} + \frac{\mu \sigma + s_j p_j}{\gamma} \frac{\mu \sigma + (s_j - t_i h_j)p_j}{\sigma + p_j} - \frac{\gamma}{2} \frac{\left(\frac{\mu \sigma + s_j p_j}{\sigma + p_j}\right)^2}{\sigma + p_j} \frac{1}{\sigma + p_j} = W_{0,i} + \frac{1}{2 \gamma} \frac{(\mu \sigma + s_j p_j)^2}{(\sigma + p_j)} - \frac{h_j p_j t_i}{\gamma} \left(\frac{\mu \sigma + s_j p_j}{\sigma + p_j}\right)
\]

In order to assess the choice of the information source \( j \) by the investor, we need to evaluate this utility conditional to an information set \( H_0 \) which does not contain the realization of the signal:

\[
E[U_{i,j}|H_0] = E[V^t_{i,j}|H_0] - \frac{\gamma}{2} Var[V^t_{i,j}|H_0]
\]

\[
E[V^t_{i,j}|H_0] = W_{0,i} + \frac{1}{2 \gamma (\sigma + p_j)} \left(\frac{\mu^2 \sigma^2 + p_j^2}{2 \gamma} \left(\frac{1}{\sigma + p_j} + (\mu + h_j)^2\right) + 2 \mu \sigma p_j \left(\mu + h_j\right) + \frac{h_j p_j t_i}{\gamma} \frac{\mu \sigma + p_j (\mu + h_j)}{\sigma + p_j}\right) - \frac{h_j p_j t_i}{\gamma} \frac{\mu \sigma + p_j (\mu + h_j)}{\sigma + p_j}
\]

\[
= W_{0,i} + \frac{\mu^2 \sigma^2 + p_j^2}{2 \gamma (\sigma + p_j)} - \frac{h_j^2 p_j^2 (2 t_i - 1)}{\gamma (\sigma + p_j)} - \frac{h_j p_j \mu (t_i - 1)}{\gamma}
\]

where the first term is the expected value obtained in case the investor acquires a signal of precision \( p^t_{i,0} = p_j \) privately (i.e. by herself). The

\(^9\)These investors are perfectly able to solve for the optimal \((h_j, p_j)\) of \( j \) and can the correct the signal \( s_j \) from the bias \( h_j \) correctly.
negative term is a function of the bias \( h_j \) and of the degree of investor sophistication \( t_i \).

For the variance term:

\[
\text{Var}[V_{i,j}^t | H_0] = \frac{p_j}{4(\mu + h_j)^2} \left( \frac{1}{\sigma} + \frac{1}{p_j} \right)^2 + 2 \left( \frac{1}{\sigma} + \frac{1}{p_j} \right)^2 + 4\mu^2\sigma^2 p_j^2 \left( \frac{1}{\sigma} + \frac{1}{p_j} \right) + 8\mu\sigma p_j^3 (\mu + h_j) \left( \frac{1}{\sigma} + \frac{1}{p_j} \right) + \\
\frac{h_j^2 p_j^2 t_i^2}{\gamma^2(\sigma + p_j)^2} \left( \frac{1}{\sigma} + \frac{1}{p_j} \right) + \\
- \frac{h_j p_j t_i}{\gamma^2(\sigma + p_j)^2} \left( 2p_j^3 (\mu + h_j) \left( \frac{1}{\sigma} + \frac{1}{p_j} \right) + 2\mu p_j^2 \left( \frac{1}{\sigma} + \frac{1}{p_j} \right) \right)
\]

\[
= p_j(2\mu^2\sigma^2 + 2\mu^2 p_j \sigma + p_j) + h_j p_j^2 [2\mu(\sigma + p_j) + h_j p_j] - h_j p_j^2 t_i [2\mu(\sigma + p_j) + h_j p_j(2 - t_i)]
\]

\[
= \text{Var}[V_{i,0}^t | H_0] + h_j p_j^2 [2\mu(\sigma + p_j) + h_j p_j] - h_j p_j^2 t_i [2\mu(\sigma + p_j) + h_j p_j(2 - t_i)]
\]

\[
= \frac{\gamma^2(\sigma + p_j)}{\gamma^2(\sigma + p_j)}
\]

(6)

Define the overall utility as

\[
EU_{i,j}^t = E[V_{i,j}^t | H_0] - \frac{\gamma}{2} \text{Var}[V_{i,j}^t | H_0]
\]

\[
= W_0 + 2\mu^2\sigma(\sigma + p_j)(\sigma - p_j) + p_j(2\sigma - p_j) + \\
\frac{h_j p_j}{2\gamma\sigma(\sigma + p_j)} \left[ -h_j t_i p_j^2 (\sigma - p_j) [2\mu(\sigma + p_j)(1 - t_i) + h_j p_j(1 - 2t_i)] \right]
\]

(7)

as the sum of two elements in equations (5) and (6): the first term is the expected utility investor \( i \) obtains acquiring the signal with the same precision \( p_{i,0} = p_j \) privately (net of information collection costs \( \varphi_i p_{i,0}^2 \)), while the second term depends on the bias \( h_j \) introduced by the portfolio manager and on the degree of awareness \( t_i \) of the investor.

We can then think of \( EU_{i,j}^t \) as a function of the fundamentals \( (\mu, \sigma, \gamma) \) and of the parameters characterizing the individual \( i \) and the advisor \( j \), i.e. \( (t_i; h_j, p_j) \).

### 3.2.1 The effect of the advisors’ parameters \( (h_j, p_j) \) on the choice between delegation and self-enquiring

In this section we characterize some comparative statics of \( EU_{i,j}^t \) as in (7) with respect to \( h_j \) and \( p_j \) considering the case of an investor \( i \) with a given awareness \( t_i \).

**Proposition 1:** Let \( \sigma > p_j \). Consider two advisors \( j \) and \( j' \) with \( h_j > h_j' \) and \( p_j = p_j' \). Then:

(i) All investors \( i \) with \( t_i < t_0 \) prefer to delegate their portfolio choice to \( j \) rather than to \( j' \);
(ii) All investors \( i \) with \( t_i \in [t_0, 1] \) prefer to delegate to \( j \) rather than to \( j' \) as long as \( h_j < h_j^* \) where
\[
h_j^* = \frac{\mu(\sigma^2 - p_j^2)(1 - t_i)}{p_j \sigma(2t_i - 1) + p_j^2(t_i - 1)^2}
\] (8)
and
\[
t_0 = \frac{p_j - \sigma + \sqrt{\sigma - p_j^2 + 4p_j(\sigma - p_j)}}{2p_j}
\] (9)

(iii) Investors with \( t_i > 1 \) prefer to delegate to \( j' \).

**Proof:** (i)-(ii) The first derivative of function \( EU_{i,j}^t \) as in (7) with respect to \( h_j \) is nil at \( h_j^* \) while
\[
\frac{\partial^2 EU_{i,j}^t}{\partial h_j^2} = \frac{p_j}{\gamma \sigma + p_j} \left( p_j \sigma(1 - 2t_i) - p_j^2(t_i - 1)^2 \right)
\] (10)
Notice that \( \frac{\partial^2 EU_{i,j}^t}{\partial h_j^2} > 0 \) when \( p_j \sigma(1 - 2t_i) - p_j^2(t_i - 1)^2 > 0 \): in such a case, the function \( EU_{i,j}^t \) is convex, and the minimum \( h_j^* < 0 \) for \( \sigma > p_j \) and \( t_i < 1 \). Thus \( EU_{i,j}^t \) is increasing in \( h_j \) for all positive \( h_j \).

Considering again \( \sigma > p_j \) and \( t_i < 1 \), we have that \( EU_{i,j}^t \) is concave when \( p_j \sigma(1 - 2t_i) - p_j^2(t_i - 1)^2 < 0 \): in such a case the maximum \( h_j^* > 0 \). Then, when \( p_j \sigma(1 - 2t_i) - p_j^2(t_i - 1)^2 < 0 \) the investor \( i \) prefers an advisor \( j \) with higher \( h_j \) as long as \( h_j \leq h_j^* \).

Finally, studying the function \( f(t_i) = p_j \sigma(2t_i - 1) + p_j^2(t_i - 1)^2 \), \( sgn(f) = -sgn(\frac{\partial^2 EU_{i,j}^t}{\partial h_j^2}) \). It is easy to verify that \( f(t_i) \geq 0 \) for \( t_i \leq t_0 \) and \( t_i \geq t_0 \) where \( t_0 < 0 \) and \( t_0 \) is given by (9), where \( t_0 \in [0, 1] \) for all \( \sigma > 0 \), \( p_j > 0 \), \( \sigma > p_j \). Thus, when \( t_i \in [0, t_0] \) we obtain result (i), while for \( t_i \in [t_0, 1] \) we have result (ii).

(iii) For \( t_i > 1 \) and \( \sigma > p_j \), \( h_j^* \) is certainly negative, as well as \( \frac{\partial^2 EU_{i,j}^t}{\partial h_j^2} \). Thus the function \( EU_{i,j}^t \) is concave and its maximum \( h_j^* < 0 \). This shows that \( EU_{i,j}^t \) is always decreasing for positive \( h_j \). □

Very naif investors, i.e. investors with \( t_i < t_0 \) prefer to delegate their portfolio choice to advisors who introduce more bias, once we fix the precision of their information \( p_j \). This effect is weaker for investors with higher sophistication, who prefer to receive a higher bias \( h_j \) only when the level of such a bias is relatively low.

The reason for this apparently surprising result is that relatively naif investors do not take fully into account the bias introduced by their portfolio manager when evaluating their expected utility: in such a way they expect to earn a return on their delegated portfolio which is (partly) upward biased by the advisor’s signal.
3.2.2 The effect of the investor awareness \( t_i \)

In this section we study the function \( \text{EU}^t_{i,j} \) in (7) as dependent on the individual sophistication \( t_i \) in order to determine which individuals are more likely to acquire information by themselves and which ones prefer to delegate their portfolio choice. Notice that \( \text{EU}^t_{i,j} \) is concave with respect to \( t_i \), with a maximum at 

\[
t^*_i = -\frac{(\sigma - p_j)[\mu(\sigma + p_j) + h_jp_j]}{h_jp_j^2}
\]

where \( t^*_i < 0 \) for \( \sigma > p_j \). This implies that for \( t_i > 0 \), \( \text{EU}^t_{i,j} \) is everywhere decreasing in \( t_i \).

Now substitute the expression for \( p^*_0,i \) in equation (3) into \( \text{EU}^t_{i,j} \) in equation (7) and define as \( \text{EU}^{st}_{i,j} \) the overall utility obtainable when the precision of the signal offered by advisor \( j \) is \( p_j = p^*_0,i \):

\[
\text{EU}^{st}_{i,j} = -\frac{h^2_j - 4\lambda^2 h_j t_i(2\varphi,\gamma\sigma + \mu^2)(4\varphi,\gamma\mu\sigma^2 + 4\mu^2\sigma + 2\mu + h_j)}{4\gamma(2\varphi,\gamma\sigma + \mu^2\sigma + 1)(4\varphi,\gamma\sigma + 2\mu^2\sigma + 1)} + \frac{8h_j\mu^{\ast\ast}(2\varphi,\gamma\sigma + \mu^2)(4\varphi,\gamma\mu^2\sigma + 1) + 2h_j^2\sigma^2(2\varphi,\gamma\sigma + \mu^2)}{4\gamma(2\varphi,\gamma\sigma + 2\mu^2\sigma + 1)^2} + \frac{4\mu^2(8\varphi_i^2\gamma^2\sigma^2 + 1) + 8\mu^2\sigma^2(4\varphi_i^2\gamma^2\sigma^2 + 1) + 8\mu^2\sigma^3 + 8\varphi_i^2\gamma^2\sigma^2 + 1}{4\gamma(4\varphi_i^2\gamma^2\sigma^2 + 2\mu^2\sigma + 1)^2}
\]

(11)

**Proposition 2:** Suppose \( \sigma = 1 \) and consider an investor \( i \) characterized by \( (t_i, \varphi_i) \) who would privately collect a signal with optimal precision \( p^*_0,i < 1 \) defined as in (3), and an advisor \( j \) characterized by \( h_j \geq 0 \) and \( p_j = p^*_0,i \). Then:

(i) if \( t_i = 0 \) investor \( i \) always delegates her portfolio choice to \( j \), for any \( h_j \geq 0 \);

(ii) for any given \( h_j > 0 \) it exists a \( \bar{t}(h_j) \) such that \( i \) delegates her choice to \( j \) if \( t_i < \bar{t}(h_j) \) otherwise she invests by herself; moreover, \( \bar{t}(h_j) \) is decreasing in \( h_j \);

(iii) \( \bar{t}(h_j) \) is increasing in \( \varphi_i \);

**Proofs:**

(i) Start substituting \( \sigma = 1 \) in \( U^*_{i,self} \) (see equation (4))

\[
U^*_{i,self} = \frac{2\mu^2 + 1 + 4\varphi + 4\gamma\varphi}{4\gamma} > 0
\]

Then substitute \( \sigma = 1 \) in \( \text{EU}^{st}_{i,j} \) (see equation (11)). Some algebra shows that for \( t_i = 0 \)

\[
\text{EU}^*_t = 0 - U^*_{i,self} = \frac{2\gamma\varphi + (\mu^2 + 2\varphi\gamma)(\mu^2 + 2\varphi\gamma + 4h(\mu^2 + 2\gamma\mu))}{2\gamma(1 + \mu^2 + 2\gamma\varphi)(1 + 2\mu^2 + 4\gamma\varphi)^2} > 0
\]

(ii) Denote \( \Delta \text{EU}^t_{i,j} = \text{EU}^*_t - U^*_{i,self} \) where \( \Delta \text{EU}^t_{i,j} \) can be interpreted as a function of \( (t_i, h_j) \):

\[
\Delta \text{EU}^t_{i,j} = \frac{4\gamma\varphi(1 + \mu^2 + 2\gamma\varphi) - 8h(l - 1)(\mu^2 + 2\varphi\gamma)(1 + \mu^2 + 2\gamma\varphi) - h^2[-2\mu^2 + 2\varphi\gamma + (l + 4\mu^2 + 8\gamma\varphi)]}{4\gamma(1 + 2\mu^2 + 4\gamma\varphi)(1 + 2\mu^2 + 4\gamma\varphi)^2}
\]

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which is continuous and differentiable everywhere for \( t_i > 0 \) and \( h_j > 0 \). The equation

\[
\Delta U_{i,j}^t = 0
\]

defines an indifference curve \( \mathcal{I}(h_j) \) in the space \((t_i, h_j) \subseteq R_+^2\). Unfortunately it is not possible to specify analytically \( \mathcal{I}(h_j) \) in the whole \( R_+^2 \), but we can characterize it pointwise. In particular, \( \mathcal{I}'(h') = 1 \), with \( h' = \frac{2\sqrt{\gamma \phi(1+\mu^2+2\gamma \phi)}}{\sqrt{1+2\mu^2+4\gamma \phi}} \) indicating that when \( h_j = h' \) investors with \( t_i = 1 \) are indifferent between investing by themselves and delegating. Moreover, since \( \frac{\partial (\Delta U_{i,j}^t)}{\partial t} < 0 \) we have that if \( t_i > 1 \) \( i \) prefers to invest by themselves, while for \( t_i < 1 \) \( i \) delegates her choice to \( j \).

We use then the implicit function theorem to study the slope of \( \mathcal{I}(h_j) \):

\[
\frac{d\mathcal{I}(h_j)}{dh_j} = -\frac{\frac{\partial (\Delta U_{i,j}^t)}{\partial h}}{\frac{\partial (\Delta U_{i,j}^t)}{\partial t}} \bigg|_{(h, \mathcal{I}(h))} = 1 - \frac{t}{\mu} - \frac{t(1+2\mu^2+4\gamma \phi)}{4\mu(\mu^2+2\gamma \phi)(1+\mu^2+2\gamma \phi)+h(t+2\mu^2+4\gamma \phi)}
\]

to find that \( \frac{d\mathcal{I}(h_i)}{dh_i} < 0 \) at any point \((h, \mathcal{I}(h)) \) with \( \bar{t} \geq 1 \): there the indifference curve \( \mathcal{I}(h_j) \) is decreasing in \((t_i, h_j) \subseteq R_+^2\). Studying the sign of \( \frac{d\mathcal{I}(h_i)}{dh_i} \) when \( \bar{t} < 1 \) we obtain that this is negative also for \( t_0 < \bar{t} < 1 \) and \( h \geq h_0 \).

This means that for \( t \) and \( h \) sufficiently far from zero the indifference curve \( \mathcal{I}(h_j) \) is decreasing.

Finally, it is tedious but easy to check that \( \frac{\partial (\Delta U_{i,j}^t)}{\partial t} < 0 \) everywhere: this shows that if we move locally away from \( \mathcal{I}(h_j) \), we obtain that for \( t_i > \mathcal{I}(h_j) \) \( i \) prefers to invest by themselves, while for \( t_i < \mathcal{I}(h_j) \) \( i \) delegates her choice to \( j \). Figure 1 shows this numerically.

(iii) For this statement we give a proof that holds locally at the point \((h', 1) \) of \((h, \mathcal{I}(h)) \). The expression of \( h' \) increases when \( \phi \) increases, meaning that an investor \( i \) with \( t_i = 1 \) is indifferent between investing herself or delegating for values of \( h_j \) higher the higher is \( \phi_i \). Hence \((h, \mathcal{I}(h)) \) at least locally in \((h', 1) \) moves upward-right when \( \phi_i \) increases. It can be shown numerically (see Figure 1) that this is true for all values of \( t_i \) and \( h_j \).

Naïf investors delegate their portfolio choice even if advisors do not know more than what they would optimally learn by themselves. Notice that by choosing to delegate, the investor increases the probability of making a loss ex-post (compared to the case where she would have invested by herself, see Proposition 2).\(^{11}\)

\[^{10}\text{The precise bounds are } t_0 = -\left(\mu^2 + 4\gamma \phi\right) + \sqrt{\left(2\mu^2 + 4\gamma \phi\right)\left(2\mu^2 + 4\gamma \phi + 1\right) - \mu^2} \text{ and } h_0 = \frac{4\mu(\mu^2+2\gamma \phi)(1+\mu^2+2\gamma \phi)(1-t)}{2(\mu^2+2\gamma \phi)-4(\mu^2+2\gamma \phi)-t^2}.
\]

\[^{11}\text{In a static model, that’s the end of the story. But how does she “learn” from this if we repeat the investment choice twice (}\times t\text{)?}\]
The above result has an immediate implication for the financial industry: financial advisors do not need to provide naif investors with information additional to what they could collect by themselves in order to obtain their delegation. This results can be seen as an indirect confirmation of Carlin (2009) who shows that the monopoly power of the financial industry depends on the degree of sophistication of investors.

For any given bias $h_j$, Proposition 2 (ii) shows that coeteris paribus the relatively less aware investors (i.e. lower $t_i$) chose to delegate their investment choice, while the more sophisticated invested independently. The higher the $t_i$, the lower the bias that makes $i$ indifferent between delegation and independent investment. Alternatively, relatively unaware investors delegate to advisors who bias their recommendation quite seriously. See this in Figure 1.

Figure 1: The locus $(h, t(h))$ such that $\Delta U_{i,j} = 0$ moves upward-right as $\varphi_i$ increases.

Point (iii) shows that if an investor suffers a high cost of self-information collection (i.e. high $\varphi_i$), whatever her degree of awareness $t_i$ she is more likely to delegate to the same type of broker than an investor with lower $\varphi_i$. If we interpret high $\varphi_i$ with low financial literacy, then relatively illiterate investors$^{12}$ would delegate more (if we think $t_i$ does not depend on literacy). If $\varphi_i$ measures the opportunity cost of time for $i$, then rich investors tend to delegate more. This is in line with the empirical results of Hackethal et al. (2009).

---

$^{12}$This holds if we assume that financial literacy does not affect the level of $t_i$. 
Finally, one may want to consider the individual sophistication $t_i$ as dependent on financial literacy. If high literacy causes high sophistication $t_i$ then more literate agents delegate less, or if they do it, they do it to better (i.e. less biased) advisors. Notice however that the link between financial literacy and individual sophistication as we interpret it here may not be clear cut: it is easy to point a class of individuals with relatively low financial literacy but with a good comprehension of the agency problem between themselves and their financial advisors/manager/private banker (especially in the case of relatively wealthy individuals).

3.2.3 The choice of the investor and the portfolio performance

When the investor delegates her investment choice to a financial intermediary, she obtains a portfolio which is biased towards the risky asset. Proposition 1 and 2 show that this effect is likely to affect more the relatively naif investors because they are more willing to choose delegation even for potentially high levels of the bias.

Allowing a naif investor to trade in such a risky asset may then cause her to be worse off compared to the case she had invested all her wealth in the riskless asset. In order for this to occur, it is sufficient that the following two conditions are satisfied:

\[
\begin{align*}
\text{ex-ante:} & \quad EU_{t_i}^j > W_{0,i} \\
\text{ex-post:} & \quad W_0 + v_i(h_j, p_j)R < W_0 \\
& \quad \Leftrightarrow v_i(h_j, p_j)R < 0
\end{align*}
\]

where $R$ is the realization of $\tilde{R}$ and $v_i(h_j, p_j) = \frac{\mu\sigma + h_j p_i}{\gamma}$ is the (biased) portfolio bought by $i$ via the advisor $j$\textsuperscript{13}.

In order to study how likely it is that an investor who has delegated her portfolio choice is worse off than in the case she had invested only in the riskless asset, we have to study the random variable $\tilde{W}_{1,i} = \tilde{v}_i\tilde{R}$ whose distribution cannot be expressed analytically: hence we proceed with numerical simulations.

Table 1 shows the percentage of investors choosing to delegate their portfolio allocation to an advisor as a function of the advisor’s bias $h_j$ and precision $p_j$ (and assuming a uniform distribution of “sophistication” $t_i$ between zero and one). In this table three values of $p_j$ are shown: $p_j$ can be lower, equal or greater to the precision the investor could collect by herself (at optimum) $p_{t_i}^*$. When $p_j$ is sufficiently lower than $p_{t_i}^*$, investors always

\textsuperscript{13}Notice that the first is always satisfied for $t_i = 0$. 

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chose not to delegate, because the optimal precision they can obtain by themselves yields higher ex-ante utility. On the contrary, for \( h_j = 0 \) and \( p_j \geq p_{i,0}^* \) investors prefer to delegate to an advisor (as shown in Lemma 1 above).

Table 1 also shows the percentage of investors suffering monetary losses as a function of \( h_j, p_j \). If investors delegate they are more likely to suffer ex-post losses with respect to the self-enquiring case. Moreover, the share of investors experiencing ex-post losses in the case of delegation is increasing with respect to \( h_j \) and decreasing in \( p_j \), because the bias induces an overexposure to the risky asset (by increasing \( v_i(h_j, p_j) = \mu + (R + h_j + \varepsilon_j)p_j \gamma \)), while precision at the same time increases the mean of \( v_i(h_j, p_j) \) and reduces its variance. Instead, in the case of self-enquiring the share of investors experiencing ex-post losses does not depend either on \( h_j \) or \( p_j \) because it is a function of \( p_{i,0}^* \) which remains constant across the table.

### Table 1: Choice between delegation and self-enquiring, and ex-post losses

<table>
<thead>
<tr>
<th>( h_j )</th>
<th>( p_j )</th>
<th>% delegating</th>
<th>Probability of ex-post losses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Delegation</td>
</tr>
<tr>
<td>0</td>
<td>0.7</td>
<td>0%</td>
<td>–</td>
</tr>
<tr>
<td>0</td>
<td>0.985</td>
<td>100%</td>
<td>24.98%</td>
</tr>
<tr>
<td>0</td>
<td>0.99</td>
<td>100%</td>
<td>24.94%</td>
</tr>
<tr>
<td>0.15</td>
<td>0.7</td>
<td>0%</td>
<td>–</td>
</tr>
<tr>
<td>0.15</td>
<td>0.985</td>
<td>68%</td>
<td>25.06%</td>
</tr>
<tr>
<td>0.15</td>
<td>0.99</td>
<td>68%</td>
<td>25.02%</td>
</tr>
<tr>
<td>0.20</td>
<td>0.7</td>
<td>0%</td>
<td>–</td>
</tr>
<tr>
<td>0.20</td>
<td>0.985</td>
<td>51%</td>
<td>25.19%</td>
</tr>
<tr>
<td>0.20</td>
<td>0.99</td>
<td>51%</td>
<td>25.14%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.7</td>
<td>0%</td>
<td>–</td>
</tr>
<tr>
<td>0.25</td>
<td>0.985</td>
<td>42%</td>
<td>25.37%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.99</td>
<td>41%</td>
<td>25.33%</td>
</tr>
</tbody>
</table>

Notes: 100,000 observations; \( \mu = 0.05, \sigma = 1, \gamma = 2, p_{i,0}^* = 0.985, \varphi_i = \frac{1}{4\pi p_{i,0}^* \sigma^2}, t_i \sim U(0, 1) \).

Table 2 shows the percentage of investors suffering monetary losses as a function of their individual sophistication \( t_i \) and of the advisors bias \( h_j \). In this table \( p_j = p_{i,0}^* \) so the advisor is providing an information signal whose quality is equal to the one the investor could collect by herself (at optimum) paying a cost: hence, delegation in this case only saves information collection costs.
Table 2: Probability of ex-post losses

<table>
<thead>
<tr>
<th></th>
<th>$h_j = 0$</th>
<th>$h_j = 0.025$</th>
<th>$h_j = 0.1$</th>
<th>$h_j = 0.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_i = 0$</td>
<td>25.104%</td>
<td>25.107%</td>
<td>25.184%</td>
<td>25.534%</td>
</tr>
<tr>
<td>$t_i = 0.25$</td>
<td>25.104%</td>
<td>25.107%</td>
<td>25.184%</td>
<td>25.104%</td>
</tr>
<tr>
<td>$t_i = 0.5$</td>
<td>25.104%</td>
<td>25.107%</td>
<td>25.104%</td>
<td>25.104%</td>
</tr>
<tr>
<td>$t_i = 0.75$</td>
<td>25.104%</td>
<td>25.107%</td>
<td>25.104%</td>
<td>25.104%</td>
</tr>
<tr>
<td>$t_i = 1$</td>
<td>25.104%</td>
<td>25.107%</td>
<td>25.104%</td>
<td>25.104%</td>
</tr>
</tbody>
</table>

Notes: 100,000 observations; $\mu = 0.05$, $\sigma = 1$, $\gamma = 2$, $\varphi_i = 0.0001$, $p_j = p_j^0$.
Grey area: private signal; White area: delegation.

Table 2 illustrates which investors delegate and which do not under various scenarios ($t_i, h_j$): for $h_j = 0$ (and in general for low values of $h_j$) the investor decides to delegate (white area). For higher values of $h_j$ ($h_j = 0.1$) investors with higher $t_i$ decide to acquire the information signal by themselves (grey area). When $h_j = 0.25$ all investors except the naif ones self select their own portfolio collecting information by themselves.

The percentage of investors suffering monetary losses ex-post is independent of the level of investor sophistication $t_i$ as long as this does not affect the choice of $i$ whether to invest by herself or to delegate. Obviously, if the investor chooses to self select her portfolio, the loss is independent of the bias of the advisor.

Intuitively, the portfolio loss increases in the bias $h_j$. Not only, but the loss suffered by the investors who invest without advisors is always (weakly) lower than the one suffered by investors with advisors. These two observations together imply that naif investors are exposed to higher losses, especially for high levels of $h_j$ (at which they still delegate).

4 Conclusions

This paper analyzes a static portfolio allocation choice between a riskless and a risky asset where the investor has the choice between investing by herself or delegating the portfolio management to a professional intermediary. Due to a possible conflict of interests between the intermediary and the investor, the intermediary’s advice can be biased towards an overexposure on the risky asset.

In addition to a given level of absolute risk aversion, the investor is characterized by (i) a given cost of choosing the optimal portfolio by herself – that can be interpreted as a function of financial knowledge, opportunity
cost of time, etc. – and (ii) by a given level of “sophistication”: the latter indicates whether the investor is aware of the advisor’s incentives and whether she is able to correctly discount for the advisor bias in the portfolio and delegation decisions.

The choice of delegation depends both on investors’ and advisors’ characteristics. In particular, lower sophistication and higher information costs lead to a higher probability of relying on the advisor. Very naif investors prefer to delegate because they fail to take into account the bias imposed by the advisor on their portfolios. Not only more naif investors are more likely to delegate, but they delegate to “worse” advisors, i.e. advisors with higher bias. On the contrary, relatively sophisticated investors delegate only if they have a high information cost or if the bias is comparatively low.

The choice to delegate the portfolio selection has consequences on the ex-post portfolio performance. The size of the bias increases the probability to suffer ex-post losses, while the degree of sophistication indirectly reduces it, by reducing the likelihood of delegation. These results suggest that lacking the ability to process financial information may have less serious consequences than not understanding advisors’ conflict of interest.

References


5 Appendix: the broker’s problem

This is similar to the problem studied in Krausz and Paroush (2002) (with some modifications).\(^\text{14}\)

The broker \(j\) wants to find the optimal \(h_j\) and \(p_j\) to impose on investor \(i\). The broker’s profits increase with the bias he can “sell” to the investor, but decrease in the probability he has to pay a penalty (this can be interpreted as a reputation cost). The penalty depends on the difference between the “biased” and “unbiased” portfolios.

Investor \(i\) going to advisor \(j\) chooses the portfolio

\[
v_i^* = \frac{E[\hat{R}|s_j]}{\gamma \text{Var}[R|s_j]} = \frac{\mu \sigma + s_j p_j}{\gamma}
\]

where \(s_j\) contains the bias: \(s_j = \hat{R} + h_j + \epsilon_j\) with \(\epsilon_j \sim N(0, \frac{1}{p_j})\).

An unbiased portfolio would be

\[
\hat{v}_i^* = \frac{\mu \sigma + \hat{s}_j p_j}{\gamma}
\]

where \(\hat{s}_j = \hat{R} + \epsilon_j = s_j - h_j\). The advisor also pays an information collection cost \(c(p_j) = \varphi_j p_j^2\) and obtains a commission \(\beta\) (from ?) for every unit of asset sold.

The broker maximizes his profits:\(^\text{15}\) \(^\text{16}\)

\[
\max_{p_j, h_j} E[\pi_j|H_0] = \beta E[v_i^*|H_0] - \varphi_j p_j^2 - k E[(v_i^* - \hat{v}_i^*)^2|H_0]
\]

\[
= \frac{\beta}{\gamma} [\mu \sigma + p_j(\mu + h_j)] - \varphi_j p_j^2 - \frac{k p_j^2 h_j^2}{\gamma^2}
\]

and the optimal values are:

\(^\text{14}\)In Inderst and Ottaviani (2009) it is not specified whether the payment of the penalty depends on the ex-ante or ex-post “true” rate of return.

\(^\text{15}\)Note that this is taken at \(H_0\) because \(j\) maximizes his profits before the realization of \(\epsilon_j\) is known.

\(^\text{16}\)The penalty can also be computed with respect to the optimal portfolio based on the ex-post realization of \(R\):\[
v_u = \frac{R}{\sigma + p_j} = \frac{R(\sigma + p_j)}{\gamma}
\]
\[
\begin{align*}
p_j^* &= \frac{\beta \mu}{2 \varphi_j \gamma} \\
h_j^* &= \frac{\varphi_j \gamma^2}{k \mu}
\end{align*}
\]

The optimal precision is decreasing in the cost of collecting information \( \varphi_j \). The optimal bias is increasing in the investor’s risk aversion (when the investor is risk averse, the advisor tries to compensate his profit loss by selling a more biased portfolio). The bias is also increasing in \( \varphi_j \), again as a compensation for the fact that collecting information is costly. The bias is decreasing in the probability \( k \) of having to pay a penalty.