Strong comovements of exchange rates: Theoretical and empirical cases when currencies become the same asset

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The aim of this paper is to detect periods in which two currencies can be classified as being the "same" asset. Two currencies can be treated as the same asset if their exchange rates vis-à-vis the same base currency are cointegrated with a cointegration vector that is consistent with the triangular arbitrage condition. In a first step, it is theoretically derived under what conditions, with respect to the process of the fundamentals, the exchange rates are cointegrated. By applying the rational expectations asset approach to exchange rates and a behavioural oriented model, we can show that bivariate cointegration between exchange rates can arise if and only if either the processes of fundamentals in country 2 and 3 are stationary or the fundamentals in country 2 and 3 are cointegrated.

The empirical results yield that periods of comovements between the US dollar and Pound sterling based upon the Euro prevail during the 1990s and periods of comovements between Euro and Pound sterling denominated in US dollar prevail since the introduction of the Euro. Furthermore, no long-run relationships can be discovered.

This paper gives four major innovations to the literature: firstly, it is theoretically shown under what conditions exchange rates can be bivariately cointegrated, secondly, it is tested for cointegration by using the cross-rate identity, i.e. deducing recursively, thirdly, from this point of view, the cointegration methodology is applied within a tri-lateral framework, i.e. detecting cointegration between exchange rates that are not only denominated in U.S. dollars, and fourthly, it is shown that comovements between two exchange rates in a narrower sense exist but only in small periods whereas the periods are classified.

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1 Introduction

In past decades much attention was directed to the explanation of bilateral exchange rates’ movements. A big field of research surrounds the question: are future exchange rates predictable? A vast strand in literature is concerned with this topic and ultimately started with the seminal papers by Meese and Rogoff (1983a, 1983b), who tested fundamental theories of exchange rate determination like the monetary model and variations empirically and used them to make out-of-sample forecasts.

Meese and Rogoff found that none of the fundamental models can outperform the results of a random walk forecast. Only for long horizons do fundamental models perform better.1 Many subsequent papers broadly confirm the results of Meese and Rogoff by using fundamental models (Mark, 1995; Chinn and Meese, 1995 or Cheung and Chinn, 1998). On the other hand some researchers can show predictability even at shorter horizons (MacDonald and Taylor, 1994; MacDonald and Marsh, 1997 or Cheung et al., 2005).2

Another strand departs from the use of fundamental models and uses different information like that drawn from forward rates or from past prices. Clarida and Sarno (1997) and Clarida et al. (2003) focus on the information contained in the forward rate to forecast exchange rates and can show forecastability at various horizons. Engel and Hamilton (1990) apply a regime switching approach developed by Hamilton (1989) to three foreign exchange rates to the U.S. Dollar. They find that the U.S. Dollar exchange rates show longer periods of appreciations and depreciations (“long swings”) that counteract the random walk view.3 What all mentioned contributions have in common is that they investigate bilateral exchange rates and stress more or less concretely the linkage to fundamentals. Although most of the papers have investigated different exchange rates simultaneously the linkages between them are mostly neglected.

Baillie and Bollerslev (1990) investigate the volatility in foreign exchange rates and can detect specific patterns for intra-daily data but no volatility spillovers between currencies and markets, while in Baillie and Bollerslev (1989b) the parameters for leptokurtosis and conditional heteroskedasticity are very similar across countries. In addition, Engle et al. (1990) and Ito et al. (1992) do find volatility linkages across markets. Common volatility movements between exchange rates are detected first by Diebold and Nerlove (1989). By applying a multivariate framework for time dependent volatility on the U.S. Dollar exchange rates of currencies that participated in the European Monetary System (EMS), Bollerslev (1990) gives also evidence for comovements in volatility between exchange rates. Nikkinen et al. (2006) examine currency options on the Euro, British Pound and Swiss franc exchange rates to the U.S. dollar for volatility linkages. Currency options reflect markets’ volatility expectations and for this reason it can be said that the Euro’s volatility expectations have

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1 As Rogoff (2002) writes, these results were questioned by both the referees and his thesis advisor Rudiger Dornbusch.
2 MacDonald/Taylor (1994) apply a monetary model within an error correction framework and can outperform the random walk even at shorter horizons. MacDonald/Marsh (1997) use an expanded version of the purchasing power parity. Cheung et al. (2005) investigate not only monetary models and also test newer fundamental models that take, for example, productivity developments into account. Abhyankar et al. (2005) take explicitly account of the economic value of exchange rate forecasts and show that the results of a monetary model can be more economically valuable for professional forecasters.
3 Many subsequent papers can also show the non-linear nature or foreign exchange rates (Kaminsky, 1993; Engel, 1994; Evans and Lewis, 1995; Dewachter, 1997; Klausen, 2005). From this point of view, the exchange rates convey a departure from the random walk hypothesis. Dewachter (2001) and Dueker and Neely (2007) use a Markov switching model to evaluate the profitability of technical trading rules. Marsh (2000) applies a Markov switching model to daily exchange rates and can show that a Markov approach fits the data well but is not able to produce reliable forecasts.
a significant impact on the currencies GBP and CHF. Volatility spillovers between the two most traded exchange rates, namely the DeutscheMark-U.S. dollar and the Japanese yen-U.S. dollar, can be discovered by Hong (2001). Here, the causality runs from the DEM-USD to the JPY-USD rate. A similar result is obtained by Brooks and Hinich (1999) and Inagaki (2007) but with respect to the British pound-U.S. dollar exchange rate and the corresponding Euro rate, whereas the Euro rate is more volatile than the British Pound rate, as Malik (2005) figures out. In a trivariate framework Perez-Rodriguez (2006) can also show volatility spillover effects for the exchange rates of the Euro, Pound Sterling and Japanese yen in terms of U.S. dollar. Black and McMillan (2004) investigate the post Bretton Woods era before the introduction of the Euro for volatility comovements and spillover effects. They can show significant volatility spillovers and long-run volatility trends for various exchange rates but only few significant spillover effects from volatility to the mean. Despite the fact that comovements in volatility are directed to exchange rates’ changes, little is said about common movements in the return series and in this respect about common movements in the levels. Parallel movements between exchange rates are predominantly tested in the literature within the market efficiency framework. For this reason, the cointegration analysis is applied. The evidence of cointegration on the foreign exchange market is not clear. Some papers can reject the hypothesis of no cointegration (Baillie and Bollerslev, 1989; Sephton and Larsen, 1991; Norrbib, 1994; Lajaunie and Naka, 1997; Woo, 1999; Haug et al., 2000) while some others cannot (MacDonald and Taylor, 1989; Hakkio and Rush, 1989; Coleman, 1990; Copeland, 1991; Rapp and Sharma, 1999). The results are very sensitive to the period of observation and the used model (Sephton and Larsen, 1991; Jeon and Seo, 2003; Phengpis, 2006). Cointegration can mostly be considered between exchange rates that participate in the EMS before the introduction of the Euro. Kühl (2007) is the first to apply the methodology to the most traded exchange rates in terms of U.S. dollars since the introduction of the Euro. In this most recent period there exists bivariate cointegration between the Euro-U.S. dollar rate and the British pound-U.S. dollar rate. Dwyer and Wallace (1992), Baffes (1994), and Ferré and Hall (2002) show that two currencies can be treated as the same asset if their exchange rates vis-à-vis the same base currency are cointegrated with a cointegration vector that is consistent with the triangular arbitrage condition (Frenkel and Levich, 1975; Levich, 1985). Departing from the assumption of obtaining stationarity by differencing one time, i.e. the levels are integrated of order 1, Dwyer and Wallace (1992) point out that if two exchange rates in terms of the same base currency are cointegrated the cross rate is required to be stationary. Baillie and Bollerslev (1994) argue by using the methodology of Granger and Joyeux (1980) that exchange rates (in a multivariate framework) are fractionally cointegrated, i.e. the integration parameter of the residuals is a fraction. In a bivariate cointegration analysis the cross rate is the residual and hence must be either stationary or mean-reverting fractionally integrated. Cheung (1993) demonstrates for five exchange rates in terms of U.S. dollars by using the Geweke and Porter-Hudak (1983) (GPH) estimator that the observed exchange rates are fractionally inte-

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4Granger (1986) states that prices of different assets on a speculative market cannot be cointegrated because cointegration would mean forecastability of at least one asset price. Referring to Fama (1970, 1976) the market is not efficient regarding the weak form of market efficiency.

5In addition, cointegration can be detected for the British pound – Swedish Krona and Euro – Swedish Krona pairs in sub-samples.

6If the hypothesis of no cointegration is rejected and the cointegration vector is not consistent with the no-arbitrage condition, the market is seen to be (informationally) inefficient because the currencies expressed in the same currency are different asset but their exchange rates comove over the long-run.
grated, i.e. following a long-memory process. In contrast, Barkouklas et al. (2004) do not find long memory in exchange rates. The most recent contribution in this respect of Jin et al. (2006) explores 19 exchange rates denominated in U.S. dollars and finds fractionally integration for most of them. Granger (1986) introduces the concept of fractional cointegration that is applied by Cheung and Lai (1993) to the purchasing power parity. In a triangular context, fractional integration of one exchange rate would be evidence for fractional cointegration if the other two involved exchange rates are integrated of the same order.

Barberis et al. (2005) argue that comovements in asset prices should be linked to comovements in fundamental values (factors that influence the cash flow). Using their methodology these are “fundamental-based comovements”. Pindyck and Rotemberg (1990) label comovements of different commodities’ prices that are not covered by changes in fundamentals with “excess co-movement”. Referring to that point, Barberis et al. (2005) speak about “friction-based” and “sentiment based” theories of comovements. They distinguish here between three categories: simplify portfolio decision view, habit view and information diffusion view. A similar argumentation can be applied to exchange rates.

The aim of this paper is to detect periods in which two currencies can be classified as being the “same” asset. This requires that their exchange rates expressed in the same base currency are cointegrated. As given by Stock and Watson (1990) bivariate cointegration is equivalent to the formulation of one common stochastic trend. Two exchange rates are cointegrated if they share a common stochastic trend. This requires the cross rate to be stationary and hence results if the other two exchange rates are non-stationary and, as a consequence, cointegrated. In a first step, it is theoretically derived under what conditions, with respect to the process of the fundamentals, the exchange rates are cointegrated. By applying the rational expectations asset approach by Frenkel and Mussa (1980, 1985) and Mussa (1976, 1979) we can show that bivariate cointegration between exchange rates can arise if and only if either the processes of fundamentals in country 2 and 3 are stationary or the fundamentals in country 2 and 3 are cointegrated. The evaluation of cointegration relationships’ sources applying a behavioural oriented model in the vein of Barberis et al. (2005) is quite difficult. For the further scrutiny, we assume that the exchange rate forms as a weighted process that contains a rational expectation and a behavioural part, in which the behavioural variable is orthogonal to each fundamental process. If all fundamental processes are non-stationary all three processes of fundamental variables must be cointegrated and the behavioural variable must be stationary. The only case in which the behavioural variable generates a long-run co-movement is when the weights of the fundamental and behavioural part of the exchange rates formation process are equal to both exchange rates. In this case the solution is equivalent with the rational expectation solution through the asset approach. The empirical results yield that periods of comovements between the U.S. dollar and the Pound sterling based upon the Euro prevail during the 1990s and periods of comovements between Euro and Pound sterling denominated in U.S. dollar prevail since the introduction of the Euro. Furthermore, no long-run relationships can be discovered.

The remainder is organised as follows. After the introduction, section 2 provides theoretical considerations when currencies can be classified as being the same asset. Firstly, a general discussion

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7 Cheung (1993) uses weekly data while Jin et al. (2006) focus on monthly data.
8 This concept traces back to Frankel and Froot (1986) and is still used as in De Grauwe and Grimaldi (2006), for example.
is provides whereas the methodology is extended to a rational expectation and a behavioural approach. In section 3 the econometric methodology is outlined. It is introduced tests for stationarity, regression of the integration’s parameter and cointegration techniques. In the following section 4 the empirical results are presented. An economic interpretation of the empirical results is carried out in section 5. Section 6 finally concludes.

2 Economic framework
2.1 Currencies as the same asset

The question to answer is under which conditions different currencies can be seen as the same asset. From Baffes (1994) follows that two currencies are not different assets if their exchange rates vis-à-vis the same denomination currency are cointegrated. Two currencies become the same asset if a set of fundamentals is common to both exchange rates. In equations (1) and (2) the general price formation process is given in the context of a three country case with base currency 1 of country 1 and the two other currencies of country 2 and 3. Here, it is assumed that each exchange rate between countries \(i\) and \(j\) \((s_{ij}^t)\) at time \(t\) builds upon the information about fundamentals available at time \(t-1\) and contained in the information set \(\Phi_{i-1}^t\) with \(i\) for each country. In line with Baffes (1994), it is straightforward to argue that common movements are expected if variations in the fundamentals of country 1, i.e. the country in which currency the other two are denominated, occur without changes in the fundamentals of country two and three.

\[
E(s_{12}^t|\Phi_{1-1}^t, \Phi_{2-1}^t) = s_{12}^{t-1} \tag{1}
\]

\[
E(s_{13}^t|\Phi_{1-1}^t, \Phi_{3-1}^t) = s_{13}^{t-1} \tag{2}
\]

Cointegration can arise if the variations in the common fundamentals prevail over the long run. Following the cointegration literature, cointegration between two variables means automatically that the system is driven by one common stochastic trend (Stock and Watson, 1990). Based on this argument, the two processes determining the exchange rates can be formalised. It shall be assumed that both exchange rates are driven by a set of fundamentals \(z_t\) and an individual white noise error term \((\epsilon_{12}^t)\) whereas the process of fundamentals should follow a simple random walk (with \(\nu_t^1\) as the white noise error term) as given in equ. (3), (4) and (5). The impact from the process of the fundamentals is not equal to both exchange rates and given by \(\alpha_{12}^{1j}\).

\[
s_{12}^t = \alpha_{12}^{12} \cdot z_t^1 + \epsilon_{12}^t \tag{3}
\]

and

\[
s_{13}^t = \alpha_{13}^{13} \cdot z_t^1 + \epsilon_{13}^t \tag{4}
\]

with

\[
z_t^1 = z_{t-1}^1 + \nu_t^1 \tag{5}
\]

\(^9\)Further, Baffes (1994) writes that this can be the case if the two countries whose currencies are expressed in the same currency fix their bilateral exchange rate or the economic structure and policy completely coincide. C.f. p. 275.

\(^{10}\)It is assumed \(0 < \alpha_{1j}^{1j} < 1\).
In a triangular framework the market is efficient if no cross market arbitrage opportunities exist. Frenkel and Levich (1975) and Levich (1985) speak about triangular arbitrage in this context meaning that if the exchange rates are expressed in logs the difference between two exchange rates must be equal to the third one (eq. (6)).

\[ s_{12}^t - s_{13}^t = s_{32}^t. \] (6)

Following Granger (1981, 1983) and Engle and Granger (1987), two time series are cointegrated if a linear combination between them exists that is stationary. In order to be consistent with the triangular arbitrage condition, cointegration between \( s_{12}^t \) and \( s_{13}^t \) has two implications, i.e. requirements. Firstly, the cointegration vector must have the form \( (1, -1) \) and secondly, the cross rate is stationary (Dwyer and Wallace, 1992). Using this knowledge and assuming explicitly the validity of the triangular arbitrage condition, eq. (5) can be set in eq. (3) and (4). After taking (6) into account and making use of the infinite moving average representation of a random walk, it results:

\[ s_{12}^t - s_{13}^t = (\alpha_{12} - \alpha_{13}) \left( z_0 + \sum_{i=1}^{t} \nu_i \right) + \epsilon_{12}^t - \epsilon_{13}^t \] (7)

The exchange rates under observation from eq. (7) are cointegrated if and only if the right-hand side is stationary. As can be seen, this requires the coefficients of \( \alpha_{12} \) and \( \alpha_{13} \) to be equal, so that the effect over the cumulative errors drops out of the equation.

From these considerations, it can be seen that currencies denominated in the same currency are the same asset if the impact of the common fundamental’s process (the only process that is allowed to be non-stationary) has the same magnitude.

### 2.2 Exchange rate forming with rational expectations

In the previous section, the requirements under which currencies expressed in the same currency can be seen as the same asset are derived. The point of departure was a very simple representation of the fundamentals. It was assumed that only the fundamentals that are common to both information sets are allowed to vary and exhibit non-stationary behaviour. In reality, the assumption cannot be longer held because macroeconomic fundamentals that determined exchange rates’ behaviour are mostly integrated of one (Nelson and Plosser, 1982). Hence, the conditions are derived under which a cointegration relationship between two exchange rates denominated in the same currency is established if all three relevant fundamentals’ processes are allowed to be non-stationary. In order to assess the impact of fundamentals on exchange rates, a very general framework is applied. Since the underlying question is concerned with the inspection of exchange rates as same assets, the “asset approach” by Mussa (1976, 1977) and Frenkel and Mussa (1980, 1985) is used. Thus, we maintain the consistency to investigate exchange rates as same assets.

Exchange rates should be seen as relative prices of national monies instead of relative prices of national outputs.\(^{11} \) For this reason, the determination of the exchange rates does not take place

\(^{11}\)If transaction costs are present in the market the equality is to be adjusted by a constant term that drives a wedge between the difference and the cross rate.
under the condition of equilibrium in the markets for flows of funds. Since relative prices of domestic monies are concerned, the conditions for equilibrium in the markets for stocks of assets are relevant. The monetary approach to exchange rates is an example of that kind of models. Here, the equilibrium results from the demand for and the supply of domestic money, which means the stocks of both counterparts are relevant. In the face of disequilibrium on the money market, subsequent flows of funds are generated to restore the equilibrium but these flows are not important for determining the exchange rate (Mussa, 1976, pp. 233-234). As in the asset approach changes in exchange rates are largely unpredictable. Thus, occurrences of new information have an impact on expectations regarding the underlying fundamentals and hence are the driving forces in the exchange rate determination process (Mussa, 1977). From this point of view, rather than the magnitude of flows it is the willingness to hold stocks at each market condition that is important for the pricing process (Mussa, 1976, p. 235; Mussa, 1979, p. 91).

It is assumed that the exchange rate at time \( t \) (\( s_t \)) depends on the initial environmental conditions contained in the vector \( x_t \) (all expressions in logarithms) and on the expected change between the current exchange rate and that of the consecutive period based on the information set \( \Phi_t \) available at the current period.

\[
s_t = x_t + b \cdot [E(s_{t+1}|\Phi_t) - s_t] \tag{8}
\]

The initial environmental conditions describe the fundamentals in period \( t \) and hence also the supply and demand coming from the resulting market conditions. The second part of the exchange rate equation arise from the assumption that the market players form expectations regarding the exchange rate they expect in the next period (\( E(s_{t+1}|\Phi_t) \)). To be complete, the parameter \( b \) weights the impact of the expected change (Frenkel and Mussa, 1980, 1985). The greater the value of \( b \), the more weight lays on the expected change, i.e. the more sensitive the exchange rate will be to expected changes.

At this point, it is assumed that the expectations of the market players are rational.\(^{14}\) Thus, the market players know the validity of equation (8) in each subsequent period. To be more precise, the expected exchange rate in \( t + 1 \) depends on the basic conditions in time \( t + 1 \) (if already known or expected) and the expected change in the exchange rate from period \( t + 1 \) to \( t + 2 \) conditioned on the information set in \( t \), whereas the weight \( b \) is held constant. For all consecutive periods the same consideration can be applied. Through forward iteration and resolving for \( s_t \) the exchange rate formation equation can be derived. Assuming the validity of the transversality condition\(^{15}\), i.e. ruling out rational bubbles, equation (9) is obtained.

\[
s_t = \frac{1}{1 + b} \sum_{k=0}^{\infty} \left( \frac{b}{1 + b} \right)^k \cdot E(x_{t+k}|\Phi_t) \tag{9}
\]

From equ. (9) it results that the exchange rate in period \( t \) is the discounted sum of fundamentals’ expected values conditioned on information set \( \Phi_t \), i.e. on the information available at time \( t \), the

\(^{13}\)At this, the market volume is not important for the extent the prices change. It is more important the desirability to hold stocks at each relevant market price, i.e. exchange rate. For this reason, Mussa (1976) argues about the importance of sentiments’ structure. The trade volume only depends on the degree of accordance in sentiments.

\(^{14}\)Expectations are rational in the sense that the market players know the relevant equilibrium pricing model and they also are able to use all available information correctly.

current period (Mussa, 1977; Frenkel and Mussa, 1980, 1985). The general solution of the asset approach is closely linked with the efficient market theory. A market is seen to be efficient if all available and for the pricing process relevant information is contained in the current exchange rate (Fama, 1970; LeRoy, 1989). If the market forms rational expectations regarding future exchange rates (or based upon expected fundamentals), i.e. the validity of equation (9), all information is already included in the exchange rate and no excess profits can be earned by exploiting the current information set (Jensen, 1978). Changes in expected fundamentals are already included in the actual exchange rates.\textsuperscript{16} Hence, only new information about future fundamentals can alter the exchange rate (Mussa, 1977, pp. 135-138). The extent to which the exchange rate changes in the face of new information about future fundamentals depends on the discount factor in equation (9).

The above outlined theory of exchange rate determination shall be used to evaluate the conditions under which exchange rates move closely together in the long-run. Referring to the three country case, the corresponding exchange rate equation is provided in equation (10) for country 2 and in equation (11) for country 3, whereas the currencies are denominated in terms of currency 1.\textsuperscript{17}

\[
s_{12}^t = \frac{1}{1 + a} \sum_{k=0}^{\infty} \left( \frac{a}{1 + a} \right)^k \cdot E(x_{t+k}^{12} | \Phi_t^{12}) \tag{10}
\]

and

\[
s_{13}^t = \frac{1}{1 + b} \sum_{k=0}^{\infty} \left( \frac{b}{1 + b} \right)^k \cdot E(x_{t+k}^{13} | \Phi_t^{13}) \tag{11}
\]

Since the asset approach treats the exchange rate as the relative price of domestic assets, two destinations have an impact on the exchange rate. Thus, the processes of fundamentals in (10) and (11) must be split up in each case into two processes covering the fundamentals of the first and the second country $x_i^t$ and $x_j^t$.

\[
x^{ij} = x_i^t - x_j^t \tag{12}
\]

with $j = [2, 3]$. By using (12) and inserting in (10) it results equation (13)

\[
s_{ij}^t = \frac{1}{1 + a} \sum_{k=0}^{\infty} \left( \frac{a}{1 + a} \right)^k \cdot \left[ E(x_{t+k}^{1i} | \Phi_t^{1}) - E(x_{t+k}^{1j} | \Phi_t^{j}) \right] = \frac{1}{1 + a} \sum_{k=0}^{\infty} \left( \frac{a}{1 + a} \right)^k \cdot E(x_{t+k}^{1i} | \Phi_t^{1}) - \frac{1}{1 + a} \sum_{k=0}^{\infty} \left( \frac{a}{1 + a} \right)^k \cdot E(x_{t+k}^{1j} | \Phi_t^{j}). \tag{13}
\]

For the sake of simplicity, the two processes of discounted expected fundamentals can be abbreviated by $F_t^i$ for the first country and $F_t^j$ for country $j$. In general, the time series properties of the fundamental series are important for the evaluation of the time series properties of the exchange rate series. As Engel and West (2004, 2005) show, one case claims more attention. If the discount factor in (8) approaches zero, i.e. only a small weight lies on the expected change, and if the fundamentals are first-difference stationary, then the exchange rate exhibits near-random walk behaviour.\textsuperscript{18} This

\textsuperscript{16}The expected change can be expressed as the discounted difference between the expected exchange rate in the next period based upon the information in $t$ and the initial environmental fundamentals, $x_t$. C.f. Frenkel and Mussa (1985), p. 727.

\textsuperscript{17}The sensitivity parameter for $s_{12}^t$ is $a$ and for $s_{13}^t$ $b$.

\textsuperscript{18}In Engle and West (2004, 2005) eq. (8) is expressed differently. Hence, their discount factor approaches one.

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case is also interesting and should be borne in mind.

More attention should be directed to equation (14) and (15).

\[ s_{12}^t = F_1^t - F_2^t \]  \hspace{1cm} (14)

and

\[ s_{13}^t = F_1^t - F_3^t \]  \hspace{1cm} (15)

In the first case, the exchange rate is simply the difference between the process of fundamentals \( F_1^t \) and \( F_2^t \) and in the second case respectively. The two currencies under observation (currencies 2 and 3) are the same asset if their exchange rates denominated in the same currency (currency 1) are cointegrated with cointegration vector \((1, -1)\). In addition to equation (14) and (15), equation (6) is valid. Thus, equation (14) and (15) can be set in equation (6) and it results

\[ s_{12}^t - s_{13}^t = (F_1^t - F_2^t) - (F_1^t - F_3^t). \]  \hspace{1cm} (16)

In order to be cointegrated, it is required that a linear combination of \( s_{12}^t \) and \( s_{13}^t \) is stationary. The left-hand side of (16) is the linear combination if the no-arbitrage condition holds. Hence, the right-hand side must also be stationary. A short reformulation yields the result that the process of fundamentals with respect to country 1 cancels out. It is straightforward to argue that if the exchange rates are cointegrated and the no-arbitrage condition holds their difference is equal to the cross rate and therefore equal to the difference between the fundamentals of country 3 and 2.

\[ s_{12}^t - s_{13}^t = (F_3^t - F_2^t) = s_{32}^t \]  \hspace{1cm} (17)

Since cointegration requires the left- and right-handside of (17) to be stationary, it can be seen that stationarity only arises if either both processes of fundamentals are stationary or if the process of fundamentals of country 2 and 3 are cointegrated. The first case has already been derived in section 2.1. Departing from the more important second case in which all fundamental processes are first-difference stationary, it is shown that two exchange rates denominated in the same currency can only be cointegrated when the two non-common fundamentals are too. This result is derived under rational expectations.

It is consistent with the intuition as seen by the asset approach, the domestic currencies are expressed in the same currency, and hence, their fundamentals (country 2 and 3) are also converted into the same currency (of country 1). This result arises independently from the choice of the denomination currency. When currencies 2 and 3 are expressed in a fourth or fifth currency, these exchange rates are also cointegrated. Both currencies are the same asset if their fundamentals are linked in the long-run. In other words, neglecting short-run disturbances the process of the fundamentals has common stochastic trends.  

2.3 Exchange rate forming and behavioural aspects

In section 2.2, the conditions are derived under which two exchange rates are cointegrated if a rational expectations model is assumed. In this section, the same issue shall be investigated but with

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19 If more than one process of fundamentals is used - as usual, one can argue that the fundamentals have to be panel-cointegrated for cointegration between the exchange rates.
the constraint that expectations are not fully rational. From this point of view, the impact of non-rationality is concerned. Since irrationality comes into play it is assumed that comovements occur that are not covered by movements in fundamentals. Following Pindyck and Rotemberg (1990), such comovements can be labelled as 'excess comovements'. Before 'excess' comovements are investigated theoretically, a short discussion of major aspects in the behavioural finance literature shall be provided.

The beginning of the microstructure approach to finance and to the foreign exchange market distinguishes between two types of traders: informed and uninformed traders (exemplarily to notice Figlewski, 1978; Grossman, 1976, 1977, 1978, 1981; Grossman and Stiglitz, 1980). These types of models look for interactions on the market and for the resulting pricing process if information is heterogeneous. From this point of view they partly take account of private information, i.e. insider trading (Jaffe, 1974; Jaffe and Winkler, 1976). With the failure of market efficiency tests, in particular on the capital market (see e.g. for the stock market Shleifer, 1981, 1986; Summers, 1986; Poterba and Summers, 1988), models were introduced that takes account of non rational traders (De Long et al. 1989, 1990a,b, 1991). These series of papers use a term introduced by Black (1986). He stresses the term ”noise” for trading on useless information. Useless information can be seen as information that is not contained in the information set Φ_t that comprises the true information about future (expected) developments of fundamentals. Thus, market participants trade on noise because they believe that the information is useful (Black, 1986, p. 531). Shleifer and Summers (1990) provide a differentiation between the two types of traders. Traders that form rational expectations correctly are those that are called ”arbitrageurs”. In contrast to them, traders that trade on noise, as defined by Black (1985), are called noise traders (Shleifer and Summers, 1990, p. 20). The distinction between noise traders and arbitrageurs lies in the assumption that noise traders act independently from rational considerations. The general discussion by Black (1985) is extended in the way that systematic biases drive a wedge between fundamental market prices and those prices that are seen as being correct from the noise trader’s point of view. Hence, noise traders systematically overestimate or underestimate the current fundamental value. Friedman (1953) explicitly rules out the possibility that non-rational traders have a long-lasting impact on the market. He argues that arbitrageurs trading on fundamentals stabilize the market in terms of bringing the market price back to the fundamental value. But as shown by the aforementioned series of papers by De Long et al., noise traders can have a long-lasting impact on assets’ prices. Based on Friedman’s argument, market imperfections should be responsible for the non-negligible effect of noise traders’ actions. Shleifer and Summers (1990) outline verbally what Shleifer and Vishny (1997) show in more detail: market imperfections prevent the arbitrageurs to remove mispricing from assets. For this reason, prices do not reflect the fundamental value of the underlying asset. Barberis and Thaler (2003) summarize the factors that prevent arbitrageurs from erasing a mispricing. These are fundamental risk, noise trader risk and implementation costs. Rational traders are also faced with fundamental

20 Although Kyle (1985) draws more on rational expectation models he already allows for three different types of traders, whereas one of them is a trader who trades on noise. In this theoretical framework, the equilibrium solution is consistent with an efficient market.

21 In the literature, this is already frequently used for traders that exploit riskless profit opportunities. In this respect, the definition is not new.

22 A broad survey of concepts that can help explain noise traders’ behaviour can be found in Hirshleifer (2001).

23 In Friedman (1953) the terms arbitrageur and non-rational trader are not used. To be congruent with the argumentation these terms are adjusted in this text.
risk that each investor has to bear. The cash flow can be smaller than expected if unexpected news occur. Noise trader risk is associated with the actions of noise traders on the market. Since noise traders can significantly influence the market price fundamental traders have to keep in mind that prices can be mispriced if they want to sell the position, i.e. liquidity is required. The time horizon of fundamentalists is assumed to be shorter than that of noise traders. In addition, transactions costs such as commissions or, more relevant for the foreign exchange market, the bid-ask spread are summarized in the term implementation costs.

By evaluating a mailed questionnaire to foreign exchange professionals from banks and from fund management companies doing their business in Germany, Menkhoff (1998) tries to test empirically the evidence of noise traders on the foreign exchange market. Menkhoff (1998) follows the distinction made by Shleifer and Summers (1990) and defines noise traders as those traders that do not use fundamental analysis and draw a greater weight on technical analysis.

In connection with the basic classification, Menkhoff (1998) tries to assure the proposition by subsequent hypothesis, like the trading horizon or the information processing. The association of noise traders with technical analysis is the simplest way to distinguish between fundamental and non-fundamental trading strategies. Shiller (1984) argues that social movements, fashions, or fads have a significant impact on the determination of asset prices. For this reason, it is expected that beliefs and sentiments of market participants significantly affect the price on speculative markets, including the foreign exchange market. In general, Menkhoff (1998) can confirm the existence of noise traders on the foreign exchange market but they cannot automatically be addressed to chartists. The so-called fundamentalists are also subject to beliefs and sentiments to the same extent as the non-fundamentalists (p. 554-561). This result can be seen as evidence of non-rationality despite the fact that the trading strategies are predominantly based on fundamental analysis. Therefore, as Menkhoff argues, ‘it becomes a price-determining factor of its own’. Thus, it can be concluded that beliefs and sentiments drive the foreign exchange market to a great extent.

Chart chasing must not necessarily be based on irrationality. What Menkhoff (1998) suggests and Shleifer and Summers (1990) have already pointed out is that the behaviour of fundamentalists can be a response to the behaviour of non-fundamentalists and directly linked to noise trader risk. If the noise traders move the exchange rate away from fundamental value the fundamentalists act rationally by being sensitive to beliefs and sentiments when they have recognise the mood in the market and have expectations about the duration of that specific market movement. In that context, it is rational not to attribute much to the fundamental value and to act with the market in order not to miss profit opportunities (Lux, 1995). Hence, factors not relevant for the fundamental value become important in determining the market rate (Cass and Shell, 1983; Woodford, 1990).

In addition, the ignorance or a lower assessment of own information in favour of information coming
from an external signal can result in an information cascade (Bikhchandani et al., 1992; Hirshleifer, 1993). If a specific signal is observed, the market evaluates this signal without incorporating the own knowledge of fundamentals. An information cascade has a greater probability of occurring if the specific signal comes from fashion (or community) leaders (Bikhchandani et al., 1992, p. 1002-1004). In Menkhoff’s questionnaire a big majority of the traders agree on the hypothesis that big market players can have a significant impact on the exchange rate.

The aim of this part is to build up a framework in which comovements between exchange rates can be explained apart from the impact of fundamentals. It has been shown by Menkhoff (1998) that beliefs and sentiments are relevant pricing factors on the foreign exchange market. When comovements between exchange rates can be observed it could be possible that the same beliefs and sentiments attributed to one exchange rate can also be found in a different exchange rate (denominated in the same currency). The linkages between the exchange rates can be twofold. On the one hand information relevant for the observed exchange rate under observation can be taken from a second exchange rate. Here, the second exchange rate, or better to say the market sector, acts as a source of information. It can be imagined that market movements in that market play the role of a specific signal that initiates herding behaviour or more generally expressed: (horizontal) chart chasing. Information about the future development of one exchange rate are used to evaluate the performance of a second exchange rate. Thus, a development regarding one exchange rate is translated into another one, despite the fact that fundamentals have not significantly changed. The reason can be seen in framing effects and the second exchange rate would be seen as a reference point (Tversky and Kahneman, 1981). Such behaviour can either emerge if the uncertainty on the market is high or the information flow is considerably low, so that all on the market available information is exploited regardless of their usefulness.

On the other hand as discussed for the asset market, comovements are established if investors group investments into fixed categories (Barberis and Shleifer, 2003). Such categories are classified by a specific style or common characteristics. Barberis and Shleifer (2003) argue that such a style depends on the legal form, the relevant market, or the surrounding fundamentals (p. 162). In addition, Lee et al. (1991) show that a common factor in securities’ return series is induced if only a subset of all available assets is traded and changes in sentiments or in the risk aversion of the investors occur. Since only a small subset of available assets is traded, the change in sentiments only generates comovements in these groups of assets. This is called ‘habit view’ of comovements. Barberis et al. (2005) suggest a third way how comovements on the asset market can be explained by behavioural concepts and is directed to information processing (information diffusion view). They argue that the speed of information diffusion varies across different assets on the market. If information is similarly incorporated into assets, i.e. with the same speed, a common factor is generated (p. 285). From these three different views for excess comovements, the information diffusion view can fewest be transferred to the foreign exchange market. It must be based upon the assumption that exchange rates with a lower market turnover convey a different speed of information processing. The habit view can be applied under certain circumstances to the foreign exchange market. If a small amount

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27 Herding behaviour can be ascribed to information cascades. Bikhchandani et al. (1992) show under what circumstances information cascades can arise and break down.

28 This is labelled horizontal chart chasing because not own past prices but past realisations of a different exchange rate are exploited.

29 See also Barberis and Thaler (2003), p. 1100 for a discussion of that topic.
of exchange rates denominated in the same currency have a specific importance and are thus traded as a subset of available exchange rates, changes in sentiments should initiate common movements. Besides this aspect, a classification of exchange rates into groups seems to be more plausible. All exchange rates denominated in the same currency can build such a group. Information directed to the environment of the denomination currency can have the same impact on all exchange rates despite the fact that the relative performance is important for each exchange rate. Here it can be seen, that the denomination currency is a key feature for the establishment of comovements on the foreign exchange market.

On the other hand, exchange rates can be subject to the same beliefs and sentiments because of the fundamental domain. This issue is indirectly linked to the last point and is qualitatively similar. If one exchange rate is faced with specific sentiments with respect to the denomination currency, the same sentiments should also be dominant in all other exchange rates that are denominated in that currency. The theoretical difference to the aforementioned point is that the relative performance is warranted here. This statement shall be explained with the help of a simple example. It is assumed that the only factors that determine the exchange rates are interest rates. From the practitioner’s point of view, the currency of country 1 is appreciating vis-à-vis another currency if the interest rates in country 1 raise. Further, it is assumed that the interest rates in country 3 are greater than in country 1 and the interest rates in country 1 greater than in country 2. Departing from the underlying linkage of fundamentals to exchange rates, the currency of country 1 will appreciate vis-à-vis the currency of country 2 while depreciating vis-à-vis the currency of country 3. If the denomination currency is faced with the same sentiments and beliefs, either the appreciation vis-à-vis currency 2 or the depreciation vis-à-vis 3 will be smaller than it would otherwise be the case. If the first exchange rate is seen as a reference point for the second or if sentiments spill over the currency 1 appreciates also vis-à-vis currency 3. Such behaviour can easier be empirically observed than same sentiments under different relative performances of fundamentals. In the last example, arbitrage opportunities would increase and should cause fundamentalists to exploit them. If they do not the reason might be noise traders risk and rational chart chasing is into play.

To investigate the outlined arguments, a similar framework shall be used as in the case with rational expectations. Following Kyle (1985), De Long et al., and Shleifer and Summers (1990), the market consists of traders that bear on fundamental analysis (fundamentalists) and traders that trade on noise. It is assumed that fundamental factors and the noise are orthogonal. The exchange rate observed on the market is a weighted average of the impact of fundamentalists and noise traders via their expectations.\(^{31}\)

\[
s_t = \gamma s^r_t + (1 - \gamma) s^b_t
\]

The term \(s^r_t\) presents the fundamentalists’ expectations while \(s^b_t\) those of the noise traders. For the case of the fundamentalists their exchange rates’ expectations form in accordance with the process described in section 2.2 in response to the expectations concerning the fundamentals

\[
s^r_{t12} = F^r_{t12}
\]

\(^{30}\)From this point of view, it is assumed that a market equilibrium is achieved at specific interest rate differentials. Thus, changes in these differentials would generate market transactions.

\(^{31}\)This setting bases upon the one used in Frankel and Froot (1986, 1990a) and Cutler et al. (1990) and is similar to Altavilla and De Grauwe (2005) and De Grauwe and Grimaldi (2006). Noise traders are seen as technical analysts here. In our framework this is not necessary.
and

\[ s_{t}^{13} = F_{t}^{13}. \]  

(20)

The behavioural part of eq. (18) consists, similarly to Barberis et al. (2005), of a white noise error term \((\epsilon_{t}^{12})\) and the sentiment directed to country 1 \((u_{t}^{1})\) \[ s_{t}^{b12} = \epsilon_{t}^{12} + u_{t}^{1} \]  

(21)

for the first exchange rate and

\[ s_{t}^{b13} = \epsilon_{t}^{13} + u_{t}^{1} \]  

(22)

for the second.

By using equations (18), (19), (20), (21), and (22) we get the expressions

\[ s_{t}^{12} = \gamma F_{t}^{12} + (1 - \gamma)u_{t}^{1} \]  

(23)

and for the second exchange rate

\[ s_{t}^{13} = \lambda F_{t}^{13} + (1 - \lambda)u_{t}^{1}. \]  

(24)

As can be seen in eq. (23) and (24), \(u_{t}^{1}\), the representation of the sentiment, is common to both exchange rates.

Again, by assuming the validity of no arbitrage condition\(^{33}\), we can build a linear combination of \(s_{t}^{12}\) and \(s_{t}^{13}\), i.e. subtracting (24) from (23)

\[ s_{t}^{12} - s_{t}^{13} = \gamma (F_{t}^{1} - F_{t}^{2}) + (1 - \gamma)u_{t}^{1} - \lambda (F_{t}^{1} - F_{t}^{3}) - (1 - \lambda)u_{t}^{1}. \]  

(25)

Next, the fundamental processes in (25) can be split up into their determinants

\[ s_{t}^{12} - s_{t}^{13} = (\gamma - \lambda)F_{t}^{1} + \lambda F_{t}^{3} - \gamma F_{t}^{2} + (\lambda - \gamma)u_{t}^{1}. \]  

(26)

The currencies 2 and 3 are the same asset, i.e. their exchange rates \(s_{t}^{12}\) and \(s_{t}^{13}\) are cointegrated, if eq. (26) is stationary. If the weights in (23) and (24) are absolutely the same, \(\lambda\) equals \(\gamma\), all terms except \(F_{t}^{3} - F_{t}^{2}\) drop out of the right-hand side of equation (26). Qualitatively, the result is the same as in the rational expectation model. Cointegration can only be established if either the two processes are stationary or if cointegrated. Only the impact of the difference on the cointegration relationship is slightly different because of the sentiment term’s impact on the exchange rate determination process. A special case arises if fundamental traders have no impact and the weight is equal to zero.\(^{34}\) In that case the sentiment term is the only factor that drives the exchange rates. As long as the impact of the sentiment term on the exchange rates is equal in both exchange rate determination processes the result equals section 2.1. Again, the intuition of

\(^{32}\)The term \(u_{t}^{1}\) can also catch spill over effects. It is only important that the term is included in the determination process.

\(^{33}\)This assumption is also appropriate within a behavioural framework because arbitrage opportunities can be exploited very quickly without the knowledge of any fundamental value.

\(^{34}\)A stationary fundamental process can be seen equivalently because in that case only random disturbances are important.
these results is straightforward and equivalent to the rational expectation case. The only difference is that in addition to the fundamentals a common sentiment factor drives the system. If the weights are not the same, some cases need to be distinguished. Further on, the sentiment variable is orthogonal to each fundamental’s process:

1. both $F_1^1$ and $F_2^1$ as well as $F_1^2$ and $F_3^2$ are cointegrated while $u_1^1$ is stationary,
2. $F_1^1$, $F_2^2$ and $F_3^2$ are cointegrated while $u_1^1$ is stationary,
3. two fundamental processes are cointegrated while the third fundamental process and $u_1^1$ are stationary,
4. all processes are stationary.

In all these cases, the sentiment term is not relevant because it has no long lasting impact if it is stationary. But in the most probable case in which variations in all exchange rate determining factors occur, the solution is equivalent to the rational expectation solution. Currencies can be classified as the same asset if and only if the weights of the impact variable in the exchange rates determination process are equal and the fundamental processes are cointegrated. If fundamentalists are on the market comovements can only arise when fundamental processes also comove. More interesting cases arise if the assumption of orthogonality between the sentiment variable and each fundamental’s process is relaxed. Economically, no different results would be obtained. A correlation between the sentiment and a fundamental variable would mean a reinforcement of the impact of the process that describes the fundamentals of country 1.

In the light of Nelson and Plosser’s results, the last cases seem to be less supposable, even with the assumption of a stationary sentiment term. As already discussed above, the noise trader term can also be approximated with technical analysis. Many contributions show for various markets that technical trading, i.e. chartism, is important for foreign exchange traders.\(^{35}\) Technical trading is often attributed to the expectation building process and is seen as equivalent to extrapolative expectations (Frankel and Froot, 1986, pp. 140/141).\(^{36}\) Hence, $u_t$ can be replaced by a term that takes the deviation of the last period’s realisation from the value two periods ago of the exchange rate into account. The dependence is positive because past developments are extrapolated into the future. Again, stationarity is only obtained if the weights are similar.

In a framework with non-stationary impact factors and taking account of behavioural aspects, comovements can only occur if the weights or the fundamental processes are equal.

3 **Econometric methodology**

3.1 **Tests for stationarity**

It is assumed that the time series $y_t$ can be expressed with the help of the following $ARIMA$ representation:

$$
(1 - L)^d A(L) y_t = C(L) \epsilon_t \tag{27}
$$


\(^{36}\)Frankel and Froot (1987, 1990b) provide a closer discussion of expectation forming in the foreign exchange market.
Since the white noise error term $\epsilon_t$ is stationary, $d$ is the number with which the left-hand side must be differenced to obtain stationarity. The time series $y_t$ is integrated of order $d$, i.e. $y_t \sim I(d)$. At this point, the parameter $d$ shall only be a non-negative integer. For $d = 1$ the time series is non-stationary meaning that a disturbance rests indefinitely in the series. In the case of $d = 0$ the time series is stationary and an innovation dissipates over time, so that the series wander around a specific mean value. $^38$ The autocorrelation function of the process decays exponentially within a very short period. Stationarity must be correctly denoted by covariance-stationarity. This means that not only the mean is independent of time but the covariances and the variance are, too. Hence, for $d = 0$ the time series is stationary and mean reverting.

The parameter $d$ does not need to be a non-negative integer. As Granger (1980), Granger and Joyeux (1980) and Hosking (1981) demonstrate, $d$ can also be a fraction, i.e. any real number. Again by using (27), the term $(1 - L)^d$ is defined more generally. In this context, one speaks about fractional integration. $^39$ Hence, if the parameter $d$ is a non-integer, the time series under observation is fractionally integrated. $^40$ For this reason, the integrated autoregressive moving average (ARIMA) model in (27) becomes a fractionally integrated autoregressive moving average (ARFIMA) model (if $d$ is a fraction the series is still differenced by order $d$ to achieve a pure $I(0)$ process). In that case, the autocorrelation function decays hyperbolically. A disturbance has a long-lasting effect and only dissipates over a long-run horizon whereas it dies out quickly for pure $I(0)$ processes and never for non-stationary processes. For this reason, the case of fractional integration is also referred to as long-memory. Following Hosking (1981), the series is stationary if $0 \leq d < \frac{1}{2}$ (pp. 169-171). If the integration parameter lies within the interval $\frac{1}{2} \leq d < 1$ the time series is not stationary any longer but mean-reverting. A disturbance still dies out in the long run; only the covariances depends on time (Baillie, 1996, pp.21/22). $^41$

Usual tests that exclude the possibility of fractional integration can thus only distinguish between $I(1)$ (non-stationarity) and $I(0)$ (stationarity). The classical tests for testing stationarity depart from the Dickey-Fuller (DF-) test and start with a simple regression of changes in $y_t$ on its one period lagged levels $y_{t-1}$ (Fuller, 1976; Dickey and Fuller, 1979, 1981). $^42$ In order to account for serial correlations, the original DF test is extended by allowing for lagged independent variables of a higher order (augmented Dickey-Fuller (ADF-) test). After algebraic manipulations, the equation (28) is used to test for unit roots

$$\Delta y_t = \mu + \gamma y_{t-1} + \sum_{i}^{k} \beta_i \Delta y_{t-i} + \epsilon_t$$  \hspace{1cm} (28)

where $\epsilon_t$ is normally i.i.d. distributed. A unit root can be rejected if $\gamma$ is significantly different from zero by using the corresponding test statistics. A problem with ADF tests is that the choice of the test statistic for the unit root problem is not straightforward.

$^37$ $\epsilon_t$ is a white noise error process. $L$ represents the lag operator and $d$ is the parameter of integration. $A(L)$ and $C(L)$ are the polynomials in the lag operators of $y_t$ and $\epsilon_t$ respectively with roots outside the unit circle.

$^38$ In eq. (27) the mean is not different from zero.

$^39$ As given in Granger and Joyeux (1980) and Hosking (1981), the function $(1 - L)^d$ can be expressed in terms of a gamma function $\Gamma$: $(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k - d) L^k}{\Gamma(-d) \Gamma(k + 1)}$.

$^40$ $d$ can also have negative values but this case shall be excluded here.

$^41$ The definitions for long memory models vary depending on the field in which they are used. A broad literature review is provided by Baillie (1996) including a deeper statistical discussion.

$^42$ To be more precise, these are no tests for stationarity, rather for a unit root, i.e. the null hypothesis is non-stationarity.
lag length must be done very carefully because serial correlations can distort the test statistics as well as heteroskedasticity can do. Phillips (1987), Perron (1988), and Phillips and Perron (1988) develop a semi-parametric test that can deal with serial correlations and heteroskedasticity. The critical values of this test are equal to those of the ADF test. In the PP-test, the lagged variables in eq. (28) on the right-hand side are omitted because the serial dependencies are caught by using the robust Newey-West estimator. The ADF and the PP test have low power when a moving average is important. For this reason Elliot et al. (1996) revised the ADF test insofar as they use a GLS instead of an OLS estimator. These tests yield more efficient results than the ADF and the PP-test. A unit root test with an alternative formulation of the null hypothesis is provided by Kwiatkowski et al. (1992). Here, it is tested for stationarity against non-stationarity under the alternative. All aforementioned unit root tests assume the stability of the data-generating process. In the real world, it is possible that structural breaks cause the series under observation to change its behaviour. At this, two unit root tests are provided that can deal with one structural break occurred at unknown time. Zivot and Andrews (1992) and Perron (1997) develop unit root tests that take account of a structural break in the level of the series. Both contributions depart from eq. (28). Zivot and Andrews (1992) omit the lagged variables under the null hypothesis and assume that the time series behaves like a random walk with a non-zero mean (p. 254). Perron (1997) builds up on Perron (1989) and explicitly takes the structural break into the null hypothesis (Perron, 1989, p. 1363/1364; Perron, 1997, p. 357). The models under the alternative hypothesis look very similar in both approaches. Only the case of a break in the intercept and the linear trend is reported below. 

\[ y_t = \hat{\mu} + \hat{\theta} D U_t(\lambda) + \hat{\beta} t + \hat{\gamma} D T^*_t(\lambda) + \hat{\alpha} y_{t-1} + \hat{\epsilon}_t \]  

(29)

with \( D U_t(\lambda) = 1 \), if \( t > T \lambda \) and 0 otherwise, and \( D T^*_t(\lambda) = t - T \lambda \), if \( t > T \lambda \) and zero otherwise. \( D U \) and \( D T \) are impulse dummy variables. \( \lambda \) denotes the value that corresponds to the minimum value of the \( t \)-statistic regarding \( \alpha \). Both Zivot and Andrews (1992) and Perron (1989, 1997) allow for models in which either the intercept or the linear trend is faced only with a structural break. \( \hat{\gamma} \) in the first case and \( \hat{\theta} \) in the second case are restricted to zero. To test for a unit root with the alternative of stationarity and one structural break occurring at unknown time, the test statistic is estimated for each potential break point determined by the dummy variable that is successively varied over a specific range. Then, the minimum value of the test statistic is compared with the critical values. A rejection of the null hypothesis would be evidence of stationarity and one structural break.

An approach for the estimation of fractional integrated processes is provided by Geweke and Porter-Hudak (1983) (GPH hereafter). This approach can estimate the parameter \( d \) sufficiently for values \( d < 1/2 \) and is the most popular technique to detect long-memory behaviour of time series. GPH depart from the periodogram (finite sample estimate of the spectral density) that is closely linked to the autocorrelation function of the process. The idea of the GPH estimator is to separate the low-frequency behaviour of a time series from higher frequency components and to investigate the series at harmonic frequencies. For this reason, only frequencies near 0 are evaluated. In that case, the integration parameter can be estimated by OLS with the regression of the log periodogram at

\[ \text{Exact critical values and p-values for the ADF test are provided by MacKinnon (1996) who uses a response surface regression for their computation.} \]

\[ \text{See Granger and Joyeux (1980) and Hosking (1981) for a more detailed discussion.} \]
a specific ordinate on a constant and on the harmonic ordinates. The parameter \( d \) is then the slope coefficient of the harmonic ordinates (Geweke and Hudak-Porter, 1983, p. 225). When \( d \) is zero, none of the periodogram can be explained by the frequency at the specific level of the periodogram.\(^{45}\) A major drawback of the log periodogram regression in the fashion of GPH is that the estimate \( \hat{d} \) is inconsistent when the true value of \( d \) is greater than one, i.e. in the unit root case (Kim and Phillips, 2006).\(^{46}\) Phillips (2007) revises the GPH estimator by using an exact log periodogram regression and additionally develops an asymptotic theory for unit root cases.\(^{47}\)

### 3.2 Tests for Cointegration

As discussed in the previous section, the cointegration methodology is applied for the investigation of exchange rates as same assets. We have already pointed out that two non-stationary time series are cointegrated if a linear combination of them exists that is stationary. A further explanation can be made by using eq. (27) again. In addition to the first time series \( y_t \), a second time series \( x_t \) shall be taken into account. With the help of both time series a third \( (z_t) \) shall be generated by differencing

\[
z_t = y_t - a \cdot x_t, \tag{30}\]

If the time series \( y_t \) and \( x_t \) are integrated of the same order \( d \) and the time series \( z_t \) is integrated of order zero, \( y_t \) and \( x_t \) are called cointegrated and the parameter \( a \) is the cointegration parameter.\(^{48}\) Furthermore, some time series are contained in the vector \( X_t \). The elements of \( X_t \) are cointegrated of order \( d, b \) \((X_t \sim CI(d,b))\), if they are all integrated of the same order and a vector exists such that

\[
Z_t = \beta' X_t \sim I(d - b) \tag{31}\]

with \( b > 0.\)\(^{49}\) In this multivariate case, the vector \( \beta_i \) is the cointegration vector and contains the cointegration parameters (Engle and Granger, 1987, pp. 252-255).

The original way to test for cointegration bases upon eq. (30). Engle and Granger (1987) propose to run a linear regression and to test the resulting error series for stationarity. A second technique developed by Johansen (1988, 1991) makes use of eq. (31) as well as Granger’s representation theorem that states that a cointegration relationship is equivalent with an error correction representation. Johansen departs from a vector autoregressive model and formulates it as a first difference VAR with an error correction term

\[
\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-1} + \epsilon_t, \tag{32}\]

whereas \( \Gamma_i = -I + \Pi_1 + \ldots + \Pi_i \) with \( i = 1, \ldots, k - 1 \) and \( \Pi = -(I - \Pi_1) \) (Johansen, 1988, 1991; Johansen and Juselius, 1990). The matrices \( \Gamma_i \) include information about short run adjustment

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\(^{45}\)The least squares regression equation is \( \ln(I(\omega_j)) = c - d \cdot \ln(4 \sin^2(\omega_j/2)) + \text{error}_j \) with \( j = 1, \ldots, n \) and \( \omega_j = 2\pi j/T \) where \( j = 1, \ldots, T - 1 \). \( I(\omega) \) is the periodogram of the series under observation at frequency \( \omega \). The spectrum of the time series \( Y_t \) \((I(\omega)) \) is defined as \( [1 - e^{-1T}]^{-1/2} I_n(\omega) \) where \( I_n(\omega) \) is the spectrum of \( u_t = (1 - L)^d Y_t \). See Granger and Joyeux (1980), Hosking (1981), Geweke and Porter-Hudak (1983), Agiakloglou et al. (1993), and Baillie (1996).

\(^{46}\)When the true process is non-stationary, Hurvich and Ray (1995) quantify the bias of the GPH estimator.

\(^{47}\)See Phillips (2007) for a more detailed discussion, derivations and proofs. One substantial change affects the number of frequency ordinates. GPH propose \( m = O(T^{1/2}) \). Phillips (2007) provides a more rigorous \( m = O(T^{1/2-1/p}) \).

\(^{48}\)For a closer explanation see Granger (1981) and Granger and Weiss (1983).

\(^{49}\)\( d \) denotes the number of independent cointegration vectors.
coefficients. The expression $\Pi X_{t-1}$ denotes the error correction term and contains the cointegration relationship. A test for cointegration is directed to test for the rank of $\Pi X_{t-1}$. The rank of a matrix is the number of independent columns and is here automatically equivalent with the number of cointegration vectors. Thus, the rank of $\Pi X_{t-1}$ shows the cointegration rank.

Both techniques, that of Engle and Granger (1987) and that of Johansen (1988, 1991), implicitly assume that the data-generating process is stable over the whole period of observation. Gregory and Hansen (1996) develop a test for the case in which a structural break occurs. Their technique bases upon the Engle and Granger approach and tests the residuals of a linear regression for stationarity under one endogenous break. Gregory and Hansen (1996) allows for a mean in eq. (30) that changes at unknown time. In a second setting, they also allow the cointegration parameter to change additionally to the constant term.

$$y_t = \mu_1 + \mu_2 \phi_{t\tau} + \beta_1 x_t + \beta_2 x_t \phi_{t\tau} + e_t$$ \hspace{1cm} (33)$$

with

$$\phi_{t\tau} = \begin{cases} 0 & \text{if } t \leq [n\tau] \\ 1 & \text{if } t > [n\tau] \end{cases}$$ \hspace{1cm} (34)$$

where $\tau$ is unknown and expresses the occurrence of the structural break (Gregory/Hansen, 1996, pp. 102/103).\textsuperscript{50} If only the constant term changes, $\beta_2$ is restricted to zero. To test for cointegration, eq. (33) is estimated for each possible break point first, i.e. the dummy variable varies over a specific range whereas for each break point the error series is tested for a unit root. Finally, the minimum value of the test statistic is compared with the critical values based upon usual unit root tests. A rejection of the null hypothesis will be evidence in favour of cointegration and one structural break. Granger (1986) introduces the concept of fractional cointegration which is implicitly contained in the generalisation of Engle and Granger (1987).\textsuperscript{51} Cheung and Lai (1993) provide a practical application of the concept of fractional cointegration on exchange rates and the validity of purchasing power parity. Time series are fractionally cointegrated if the equilibrium error’s integration parameter is less than 1. Then, a shock to the system dies out over the long-run, so that the equilibrium relationship between the time series is retained (p. 106). Furthermore, Cheung and Lai (1993) argue that the least squares estimate is consistent in the case of fractional cointegration and that only the rate of convergence differs.\textsuperscript{52} As an estimator they applied the GPH method to the residual series.

For this reason, a test for fractional cointegration is a test for $I(1)$ versus $I(d)$ with $d < 1/2$. From this point of view, the Phillips (2007) estimator can also be used to test for (fractional) cointegration of exchange rates within a triangular framework. If the estimate of $d$ is significantly different from zero and smaller than 1, two exchange rates are fractionally cointegrated inasmuch they are integrated of the same order.

Granger and Ding (1996) and Granger and Hyang (2002) depart partly from the notion of long-memory behaviour of financial time series. In both contributions, it is pointed out that fractional

\textsuperscript{50}The parameter $\tau \in (0, 1)$ can be seen as the timing of the relative change point, as Gregory/Hansen (1996) write.

\textsuperscript{51}Granger (1986) refers to Granger and Joyeux (1980) and argues that fractional differencing can be applied and the general representation still holds (p. 222). Engle and Granger (1987) formulate the general definitions but concentrate only on the $I(0)$ and $I(1)$ cases (pp. 252/253).

\textsuperscript{52}Cheung and Lai (1993) show the consistence of the least squares estimate analytically (p.106).
integration can arise from occasional structural breaks. Instead of using a fractional integration framework more emphasis shall be laid on the modelling of structural changes. We take account of this critique by using rolling regressions. The PP and the KPSS tests as well as the modified log periodogram regression are performed with a fixed sample size and are rolled over the whole period of observation.

4 Empirical results

For the purpose of detecting periods in which exchange rates are the same assets, the weekly end of day data for the four most traded currencies are taken into account. The used currencies are the Euro, the US dollar, the Japanese yen and the British pound sterling. Between these currencies each exchange rate is examined. Before the introduction of the Euro in January 1999, the Deutsche Mark (DEM) rates are used and converted with the official DEM/Euro conversion rate into Euro. While doing so, the DEM is not seen as the economic predecessor of the Euro. The reason why the DEM is used traces back to the fact that the DEM was the second most traded currency before the introduction of the Euro. From this point of view, the Euro replaced the DEM in the quantitative importance. For sake of simplicity, it is referred to the Euro rates and the Euro area when the DEM and Germany are investigated in periods up to 1998. All exchange rates are expressed in natural logs and taken from Datastream. The period of observation starts on 5 January 1994 after the turbulences caused by the EMS crises diminished and ends on 29 June 2007.

Since the aim of this paper is to detect periods in which exchange rate pairs are cointegrated, i.e. currencies are the same asset, it must be checked for the premises of cointegration. For this reason, all exchange rates in logs are tested for stationarity. This is done for the whole period of observation but the sample is also subdivided into the period before and after the introduction of the Euro. The results for the stationarity tests are reported in tables 1, 2 and 3. According to the above outlined unit root tests, the PP, KPSS and the DF-GLS tests are applied to the levels and first differences. If the PP and DF-GLS tests can reject the null hypothesis for first differences but not for levels, it is concluded that the exchange rates under investigation are first difference stationary. For the KPSS test the reverse argumentation applies. A non-rejection of the null hypothesis for first differences is evidence for first difference stationarity when it can be rejected for levels. As can be seen in table 1, the stylized facts of Meese and Singleton (1982) and Baillie and Bollerslev (1989b), that exchange rates are first difference stationary, can be confirmed. For the exchange rates EUR/USD and GBP/USD the DF-GLS test cannot reject the null hypothesis of non-stationarity in first differences. Since the KPSS test cannot reject the null hypothesis of stationarity based upon robust grounds, these exchange rates are treated as first difference stationary. In the second sub-period the KPSS test cannot reject the null hypothesis for levels while the DF-GLS test can at 5% significance level for the JPY/USD rate. Here, evidence that the JPY/USD is stationary seems weak.

It is known that the KPSS test and the DF-GLS test are not robust when the true data-generating process is fractionally integrated. For this reason, more attention shall be directed to the integration

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53 The exchange rates are averages of the bid and ask rates.
54 Together these exchange rates make up a share of approximately 160% of the average daily turnover. The share is greater than 100% because it is measured the turnover in which the currency is involved.
55 In this period significant interventions are only expected when the JPY is concerned.
parameter. At this, we apply the estimator developed by Phillips (2007) that is a variant of the GPH estimator. Jin et al. (2006) is the most recent contribution that investigates exchange rates for long memory and they use the so called OLS wavelet estimator. They compare the estimates of their estimator with those of the GPH estimator and conclude that the results are completely disparate (p.132). While the wavelet estimator gives evidence in favour of long-memory, the GPH estimates contrast these results. For this point of view, they assert the theoretical investigation by Jensen (1999) who developed the OLS wavelet estimator. Jensen (1999) shows that the mean squared error of his estimator is significantly smaller than that of the GPH estimator. In addition, the GPH based tests are biased positively away from zero (Agiakloglou et al., 1993) whereas the quantity of the bias is negligible (Liebermann, 2001). In order to test for fractional integration, the modified log periodogram regression by Phillips (2007) is applied because it is able to determine the order of integration over a broad interval. Although Phillips (2007) also mentions the lower consistency of his estimator we decide to apply his test anyway because of the attractiveness of estimating $d$ in unit root cases. We are aware of the critiques and thus use the modified log periodogram regression in addition to the basic unit root tests.

In Cheung (1993), Barkoulas et al. (2004), and Jin et al. (2006) the (log) changes of the exchange rates are inspected for long-memory behaviour. Their studies are focussed on cases in which $-0.5 < d < 0.5$. Since our aim is to identify periods in which the level series are stationary and/or mean-reverting, we analyse the (log) levels of exchange rates. The modified GPH test by Phillips (2007) is able to test for the case of $I(1)$. The application of the exact log periodogram regression (ELP) is given in table 4. The hypothesis that $d$ is equal to 1 can broadly be confirmed for all periods. Only in the case of GBP/USD, is there evidence for fractional integration. An increase of the power of the test from the usual 0.5 to 0.6 changes the result and provides evidence in favour of $I(1)$ in all cases. Thus, it can be concluded that all exchange rates can be used to perform a cointegration analysis. They are all integrated of the same order. The above outlined economic argumentation departs from the assumption that the no-arbitrage condition holds. Hence, it must first be carefully tested whether this is true in reality. The no-arbitrage condition in eq. (6) can be slightly rearranged in bringing the right-hand side to the left-hand side.

$$s_{t}^{12} - s_{t}^{13} - s_{t}^{32} = \mu \quad (35)$$

Equation (35) states that no excess profit can be earned after cross converting. When transaction costs as defined above are relevant ($\mu$), the left hand-side of eq. (35) is equal to a constant term. As Baffes (1994) argues, the no-arbitrage condition is fulfilled when the three non-stationary exchange rates are cointegrated. This is tested by applying the Johansen (1988, 1991) approach to each triplet. In all cases, the validity of the no-arbitrage condition can be shown (see tables 11, 12, 13, 56 Agiakloglou et al. (1993) show for moderate sample sizes (100 observations) that the estimator of $d$ is substantially biased when the autoregressive parameters have large positive values. In general, the hypothesis of pure stationarity is too frequently rejected. Furthermore, Agiakloglou et al. (1993) argue that the bias is only a serious problem when the autoregressive parameters are large and positive (pp. 236/237). The same is true for large positive values of moving-average parameters (pp. 240-242). Lieberman (2001) quantifies the bias and concludes that the GPH estimates are sufficiently reliable. 57 In this way, exchange rates returns are investigated. Granger and Hyung (2004) scrutinize also the return series. 58 In Barkoulas et al. (2004) only the case of $0 \leq d < 0.5$ is treated. 59 This result is not printed here but available from the author.
The validity of the no-arbitrage condition makes possible to conclude that two exchange rates denominated in the same currency are cointegrated when they are non-stationary while the cross rate is stationary. A test for cointegration is equivalent to a test for stationarity of the cross rate. By reviewing the results of the unit roots tests, it can be concluded that no cointegration of exchange rates is expected over the whole period of observation because all exchange rates are integrated of the same order. Based upon the observations of Sephton and Larsen (1991) and Kühl (2007) that cointegration is sensitive to the selected period of observation, we apply tests that account for structural breaks. For this reason, the unit root tests of Zivot and Andrews (1992) and Phillips (1997) and the cointegration approach by Gregory and Hansen (1996) are carried out. Both unit roots tests taking a structural break into account can reject non-stationarity against stationarity and one break in the intercept for the GBP/EUR exchange rate approximately at the same time in the period since the introduction of the euro (tables 7, 8, 9, and 10). The GH approach confirms that finding because it can reject the hypothesis of no cointegration against the alternative of cointegration and one structural change for all pairs in which the Euro and the pound are expressed in a different currency, namely the USD and the JPY (see tables 5 and 6). In both cases the break occurs at the same time, around the end of 2002 or the beginning of 2003, i.e. in the second subsample after the introduction of the Euro. In addition, the estimated break dates coincide for both the unit root tests and the cointegration test. The two unit root tests also display that the GBP/JPY and the EUR/JPY exchange rates are stationary when a structural break is taken into account. This might be the reason why the GH test can reject the null hypothesis in cases in which one or both of these exchange rates are included. Besides these results, there is evidence that the USD/EUR and the GBP/EUR are cointegrated in the first sub-period with a break point in November 1996 based upon the GH approach. Both unit root tests cannot confirm this finding because the GBP/USD exchange rate is non-stationary.

Since there is evidence that structural changes seem to play a significant role in answering the question whether currencies can be the same asset in reality, we return to the basic unit root tests and the ELP. The previous analysis has shown that cointegration seems to be present in shorter periods. In the following, we try to detect those periods in which exchange rates are cointegrated. For this reason, the KPSS and the PP test as well as the ELP estimator are applied within a rolling regression framework. The estimation period is held fixed at a size equivalent to two years (96 observations). This period is rolled over the whole period of observation and at each time the test statistics are reported. The graphical results are presented in figures 1, 2 and 3. For the PP-test the p-values based upon the exact critical values of MacKinnon (1996) are used to evaluate the significance of the test statistic. If the calculated p-value is less than the corresponding significance level, the null hypothesis of non-stationarity is rejected. A similar argumentation can also be applied to the ELP estimator. Here, there is evidence in favour of stationarity if the p-value exceeds the

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60. There is one caveat: the used exchange rates are averages of bid and ask rates. Economically, bid rates should be used because they are relevant by converting one currency in another. Unfortunately, in all cases in which the DEM/JPY rate is involved only average rates are available. Due to the lack of data, the available average data are used. For all other exchange rates the same tests are applied by using bid rates and the results do not differ. Thus, it is expected that the same is true for the DEM/JPY, although the doubts cannot be ruled out completely.

61. Needless to say, this approach cannot be applied in a multivariate framework with more than two exchange rates denominated in the same currency.

62. Similar result have also been shown by Kühl (2007) for daily data.
corresponding significance level. The p-value corresponds with the null hypothesis that \( d \) is zero. The higher the p-value is, the more robust the result is in favour of stationarity. By using the KPSS test, the critical values are reported and compared with the critical values of the corresponding significance level.

Based upon the tests, three different periods can be identified. The first starts in the mid 1990s. Here, the GBP/USD exchange rate is stationary while for example the USD/EUR and GBP/EUR are non-stationary.\(^6\) After the introduction of the Euro, the PP-test again shows stationarity of EUR/USD, GBP/USD and GBP/EUR rates, which is also confirmed by the KPSS test. From this point of view, the currencies involved are expected to be the same assets, i.e. the exchange rates denominated in a fourth currency and not belonging to the triplet should be cointegrated. The third period starts in the mid of 2004. Here, the GBP/EUR exchange rate seems to be stationary again while the other exchange rates are non-stationary. The results of the GBP/USD during the 1990s and the GBP/EUR after the introduction of the Euro have already been indicated by the GH test.\(^4\)

The outlined results are remarkably robust because they do not change by applying a sensitivity analysis. The fixed sample within the rolling regression framework is extended to three years (144 observations). Although the length of the period in which stationarity can be shown becomes smaller, the same quantitative results arise. An increase of the power of the ELP estimator to 0.6 yields the same quantitative results whereas the period of stationarity of exchange rates decreases noticeably.

Finally, it can be concluded that exchange rate pairs comove during subsequent periods and the involved currencies can be seen as the same asset.

5 Economic interpretation

We have shown that currencies can be classified as the same asset during short periods. Referring to the above outlined economic argumentation, we shall look at the reasons why the exchange rates exhibit such a strong comovement, i.e. whether fundamental or behavioural aspects are responsible for that finding. Since the periods are very short and an econometric analysis cannot be applied properly an interpretation with the help of a visual investigation will suffice. From this point of view, this section has case study character.

Since the most interesting behaviour occurs in the triplet EUR/USD, GBP/USD and GBP/EUR, these weekly exchange rates are presented in figure 4 (first graphic in the left column).\(^5\) The periods in which the above mentioned rolling regressions can find stationarity for at least one exchange rate are very similar across the techniques. All three tests indicate approximately the same point in time when the period of stationarity starts; only the duration differs. For the further purpose, the KPSS test results are predominantly used for the categorisation of periods with slight adjustments due to the evaluation of the ELP regression. In addition to the exchange rates, the periods of stationarity are shaded in figure 4. As already pointed out, three different regimes are classified. The first one

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\(^6\) The PP test exhibits stationarity for these exchange rates, too. But the two other tests do not. This allows us to conclude to treat only one exchange rate as stationary.

\(^4\) In the case of the GBP/EUR, Kühl (2007) investigates the period after the introduction in more detail and finds by using the Johansen approach cointegration between EUR/USD and GBP/USD in sub-periods. His investigation is directed to market efficiency considerations.

\(^5\) The exchange rates are normalised to the first observation, i.e. first week in January 1994.
starts approximately at the beginning of October 1996 and ends around mid May of 1999. In this period the GBP/USD exchange rate is stationary and hence the EUR/USD and GBP/EUR rates are cointegrated. Although the visual inspection suggests evidence that a comovement between the EUR/USD and EUR/GBP rates is already present before the classified period, none of the tests can confirm that result statistically. Hence, this comovement is not that strong as indicated by cointegration.66 This period is referred to as period 1. In the second period (mid October 1999 till mid April 2002), inferring from our tests all exchange rates seem to be stationary. Although the GBP/EUR exchange rate becomes stationary at an earlier stage, this conclusion can be drawn because the periods overlap due to the fixed sample size. The third period starts at the end of June 2004 and runs till the end of the period of observation. In this period the GBP/EUR exchange rate is stationary and because the EUR/USD and GBP/USD rates are non-stationary, they are cointegrated.

In the first period the GBP/USD exchange rate is stationary which means that the concerned fundamentals should either comove or be stationary. For a deeper analysis we refer again to figure 4. As one can see, the interest differentials (short term interest rates) in the undermost left column show a very similar behaviour but predominantly from 1994 onwards. The interest rate differentials’ comovement between the differentials of the USA and the UK to the Euro area is less elaborated in the first period than in the third period. In contrast, the short-term real interest rates differentials exhibit a much more pronounced parallel movement. In addition, the UK has a positive interest rate differential (nominal and real) vis-à-vis the USA and the Euro area. The nominal interest rate differential between the UK and the USA starts to comove with the UK-Euro differential at the beginning of 1997 whereas the comovement between the UK-Euro and the USA-Euro differentials diminishes. However, the short-term real interest rates comove to a higher extent than the nominal rates during this sub-period. During the first part of the period the Euro depreciates both vis-à-vis the US dollar and the Pound sterling. This movement is completely covered by the sign of interest rates differentials. When the Bank of England changed its monetary policy and lowered central bank interest rates, both the Pound sterling and the Euro appreciate vis-à-vis the US dollar. The effect on the GBP/USD exchange rate is less pronounced. The period of a narrower comovement between the EUR/USD and EUR/GBP stops when the US-Euro interest rate differential reverts its sign which happens at the same time as the UK-US interest rate differential exceeds the UK-Euro differential. In addition, at this point in time the real interest rate differential between the UK and the USA begins its upward tendency. The reason of the US dollar’s and the Pound Sterling’s appreciation vis-à-vis the Euro seems to be plausible while only looking at interest rate differentials. The comovement of the EUR/USD and EUR/GBP exchange rates takes place at the same time as the real interest rate differentials between USA and the Euro area and the UK respectively comove. Against the background of the above outlined theoretical considerations only looking at nominal interest rate differentials seems to be counterintuitive. When looking at the real interest rates, it becomes much clearer that these might be the dominating factors on the foreign exchange market. From this point of view, the comovement can be explained broadly without behavioural

66The PP test and the ELP test indicate cointegration earlier than the KPSS test does. Qualitatively, the broader classification has no consequences.
considerations.\textsuperscript{67}

During the second period the comovement of all three short-term real interest rates seems to be plausible for explaining the tendencies of stationarity with respect to all three exchange rates. Again, short-term real interest rate seems to dominate the exchange rates. For this reason, this period shall not be investigated more intensively.

Looking at the interest rate differentials (short-term money market rates with a maturity of three months), the most elaborated period of fundamentals' comovements is between the US-Euro area and the US-UK interest rate differentials in the third period. Both interest rate differentials behave largely parallel which implicitly shows that the interest rates in the Euro area and the UK perform similarly. It is remarkable that the comovement starts at that point when the period of stationarity of the GBP/EUR, as indicated by the empirical tests, begins. Both the Euro and the Pound sterling depreciate vis-à-vis the US dollar when short term nominal interest rate differentials diminish and appreciate when the both differentials widen. In general the Euro area has a negative interest rate differential while the UK has predominantly a positive one. During the same period the inflation rate differentials of the Euro area and the UK vis-à-vis the USA, as given in figure 4 on the first position in the right column, are also quite synchronous and consistent with the observed movement, even though the UK has a higher rate of inflation than the USA at the end of 2006. The long term interest rate differentials (yields of government bonds with a maturity over 10 years) between the USA and the Euro area and the UK respectively are positive and also move quite similarly. Also the GDP growth rates are very similar across the period of observation (not reported here). We see an indication that cointegration arises because of synchrony between important fundamentals but that common sentiments drive both exchange rates. The change in the monetary policy of the USA reflects increased inflation expectations. But this is not a pure sentiment factor it is more a sign for changed expectations and is consistent with the rational expectation approach. The market participants expect growing interest rates in the USA. By only looking at the changes of the interest rate differential, the Euro and the UK series coincide. Regarding the reasons for the common movement of the exchange rate without a more intensive investigation, which is beyond the scope of this paper. For the last period it can be concluded that behavioural aspects, i.e. sentiments, drive both exchange rates but that fundamentalists are still present. In the face of the US current account deficit and increasing fears about a slowdown of the US economy, we speculate that a common sentiment term is into play instead of spill over effects.

\section{6 Conclusion}

We can show that stronger comovements between exchange rates exist in reality. Our results give an explanation for Sephton and Larsen's (1991) results who conclude that cointegration of exchange rates depends on the period of observation. At the beginning, we theoretically derived under which circumstances exchange rates can be bivariately cointegrated. This is first done by applying a rational expectation approach in the vein of Mussa (1976, 1977) and Frenkel and Mussa (1980, 1985). Based upon this approach we extend the rational expectations approach and introduce a behav-

\textsuperscript{67}Although the result counteracts the real interest differential approach by Frenkel (1979) in which the currency with a positive real interest rate differential is expected to depreciate.
ournal finance framework. The results of those approaches can be used to explain why cointegration is only present in short sub-samples. It requires that the sensitivity coefficients with respect to fundamentals’ impact on the exchange rates must coincide and that the fundamentals of the two countries whose currency is not the denomination currency must comove (if non-stationary). This must also be true if the market is driven by noise traders and thus is influenced by sentiments or fads. Only if fundamentals do not change in a specific period, do they have no significant impact on the exchange rates’ comovement. This requires that no attention is drawn to fundamental factors and that the whole market is driven by sentiments and fads. Hence, both currencies denominated in the same currency are truly the same asset, as introduced by Copeland (1991) or Baffes (1994). The results yield that periods of comovements between the USD and GBP based upon the Euro prevail during the 1990s and periods of comovements between EUR and GBP prevail since the introduction of the Euro. Furthermore, no long-run relationships can be discovered across the observed exchange rates. In the first period there is graphical evidence that real interest rates are the dominating factors. In contrast, we can observe a stronger comovement between nominal short-term interest rates. From the practitioner’s point of view (or arguing in line with the Mundell-Fleming model of an open economy), there are positive nominal interest rate differentials vis-à-vis the USA during most of the period, whereas the US-dollar depreciates. We see evidence in favour of a significant impact of sentiments and fads probably linked to the current account deficit of the USA.

This paper gives four major innovations to the literature: firstly, it is theoretically shown under which conditions exchange rates can be bivariately cointegrated, secondly, it is tested for cointegration by using the cross-rate identity, i.e. deducing recursively, thirdly, from this point of view, the cointegration methodology is applied within a triangular framework, i.e. detecting cointegration between exchange rates that are not only denominated in U.S. dollars, and fourthly, it is shown that comovements between two exchange rates in a narrower sense exist but only in short periods.
References


rejection would be evidence in favour of stationarity. In the case of the KPSS test the null hypothesis is stationarity. A non-rejection would be evidence in favour of stationarity. Critical values for the KPSS test are 0.463 for 5% and 0.347 for 1% significance level, for the DF-GLS test -1.95 for 5% and -2.58 for 1% significance level. * (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The p-values of the Phillips-Perron test are calculated on the basis of Kwiatkowski et al. (1992). The KPSS test bases upon MacKinnon (1996). The KPSS test bases upon Kwiatkowski et al. (1992). The autovariances of the KPSS test are weighted by Bartlett kernel. The DF-GLS test bases upon Elliot et al. (1996) and the lag selection is carried out by using the modified Akaike information criterion (MAIC) as developed by Ng and Perron (2001). For the Phillips-Perron test and the DF-GLS test the null hypothesis it is assumed that the time series are non-stationary. A rejection would be evidence in favour of stationarity. Critical values for the KPSS test are 0.463 for 5% and 0.347 for 1% significance level, for the DF-GLS test -1.95 for 5% and -2.58 for 1% significance level.

### Table 1: Unit Root Tests for the exchange rates for the whole period of observation

<table>
<thead>
<tr>
<th>Exchange rates</th>
<th>Z(t)</th>
<th>p-value</th>
<th>lags</th>
<th>Test statistic</th>
<th>lags</th>
<th>DF-GLS $\mu$</th>
<th>Z(t)</th>
<th>p-value</th>
<th>lags</th>
<th>Test statistic</th>
<th>lags</th>
<th>DF-GLS $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td>-0.494</td>
<td>0.625</td>
<td>18</td>
<td>0.676 **</td>
<td>1</td>
<td>-0.918</td>
<td>-26.892</td>
<td>0.000</td>
<td>22</td>
<td>0.221</td>
<td>19</td>
<td>-5.130 **</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>-0.872</td>
<td>0.797</td>
<td>18</td>
<td>1.650 **</td>
<td>12</td>
<td>0.791</td>
<td>-25.198</td>
<td>0.000</td>
<td>26</td>
<td>0.157</td>
<td>19</td>
<td>-1.197</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>-2.176</td>
<td>0.215</td>
<td>18</td>
<td>0.639 *</td>
<td>1</td>
<td>-1.323</td>
<td>-25.395</td>
<td>0.000</td>
<td>17</td>
<td>0.048</td>
<td>15</td>
<td>-4.656 **</td>
</tr>
<tr>
<td>GBP/EUR</td>
<td>-1.319</td>
<td>0.620</td>
<td>18</td>
<td>1.680 **</td>
<td>1</td>
<td>-0.765</td>
<td>-28.417</td>
<td>0.000</td>
<td>5</td>
<td>0.138</td>
<td>19</td>
<td>-1.679</td>
</tr>
<tr>
<td>JPY/EUR</td>
<td>-1.025</td>
<td>0.744</td>
<td>18</td>
<td>0.594 *</td>
<td>1</td>
<td>-0.943</td>
<td>-26.701</td>
<td>0.000</td>
<td>14</td>
<td>0.181</td>
<td>10</td>
<td>-6.645 **</td>
</tr>
<tr>
<td>JPY/GBP</td>
<td>-0.972</td>
<td>0.763</td>
<td>18</td>
<td>1.620 *</td>
<td>1</td>
<td>-0.315</td>
<td>-25.803</td>
<td>0.000</td>
<td>16</td>
<td>0.103</td>
<td>19</td>
<td>-3.130 **</td>
</tr>
</tbody>
</table>

### Table 2: Unit Root Tests for the exchange rates before the introduction of the Euro

<table>
<thead>
<tr>
<th>Exchange rates</th>
<th>Z(t)</th>
<th>p-value</th>
<th>lags</th>
<th>Test statistic</th>
<th>lags</th>
<th>DF-GLS $\mu$</th>
<th>Z(t)</th>
<th>p-value</th>
<th>lags</th>
<th>Test statistic</th>
<th>lags</th>
<th>DF-GLS $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td>-1.354</td>
<td>0.604</td>
<td>11</td>
<td>1.090 **</td>
<td>1</td>
<td>-0.856</td>
<td>-16.481</td>
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<td>14</td>
<td>0.284</td>
<td>7</td>
<td>-4.831 **</td>
</tr>
<tr>
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<td>0.196</td>
<td>11</td>
<td>1.560 **</td>
<td>1</td>
<td>-0.588</td>
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<td>12</td>
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<td>JPY/USD</td>
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<td>11</td>
<td>1.700 **</td>
<td>1</td>
<td>-1.165</td>
<td>-15.245</td>
<td>0.000</td>
<td>10</td>
<td>0.144</td>
<td>13</td>
<td>-3.059 **</td>
</tr>
<tr>
<td>GBP/EUR</td>
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<td>0.874</td>
<td>11</td>
<td>1.490 **</td>
<td>3</td>
<td>-0.533</td>
<td>-16.975</td>
<td>0.000</td>
<td>17</td>
<td>0.300</td>
<td>15</td>
<td>-1.395</td>
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<td>JPY/EUR</td>
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<td>0.372</td>
<td>11</td>
<td>1.360 **</td>
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<td>-1.442</td>
<td>-14.021</td>
<td>0.000</td>
<td>10</td>
<td>0.062</td>
<td>8</td>
<td>-3.800 **</td>
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<td>0.781</td>
<td>11</td>
<td>1.900 **</td>
<td>3</td>
<td>-0.910</td>
<td>-14.788</td>
<td>0.000</td>
<td>11</td>
<td>0.135</td>
<td>13</td>
<td>-2.347 *</td>
</tr>
</tbody>
</table>

### Table 3: Unit Root Tests for the exchange rates after the introduction of the euro

<table>
<thead>
<tr>
<th>Exchange rates</th>
<th>Z(t)</th>
<th>p-value</th>
<th>lags</th>
<th>Test statistic</th>
<th>lags</th>
<th>DF-GLS $\mu$</th>
<th>Z(t)</th>
<th>p-value</th>
<th>lags</th>
<th>Test statistic</th>
<th>lags</th>
<th>DF-GLS $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td>-0.446</td>
<td>0.883</td>
<td>13</td>
<td>2.330 **</td>
<td>1</td>
<td>-0.710</td>
<td>-21.229</td>
<td>0.000</td>
<td>24</td>
<td>0.407</td>
<td>17</td>
<td>-2.985 **</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>-0.378</td>
<td>0.914</td>
<td>13</td>
<td>2.350 **</td>
<td>3</td>
<td>-0.445</td>
<td>-19.304</td>
<td>0.000</td>
<td>42</td>
<td>0.310</td>
<td>1</td>
<td>-14.097 **</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>-2.130</td>
<td>0.233</td>
<td>13</td>
<td>0.251 **</td>
<td>1</td>
<td>-2.078</td>
<td>-20.443</td>
<td>0.000</td>
<td>23</td>
<td>0.075</td>
<td>15</td>
<td>-4.275 **</td>
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<td>GBP/EUR</td>
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<td>0.268</td>
<td>13</td>
<td>1.680 **</td>
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<td>-1.327</td>
<td>-22.988</td>
<td>0.000</td>
<td>12</td>
<td>0.179</td>
<td>17</td>
<td>-1.695</td>
</tr>
<tr>
<td>JPY/EUR</td>
<td>-0.279</td>
<td>0.928</td>
<td>13</td>
<td>2.440 **</td>
<td>1</td>
<td>-0.564</td>
<td>-22.895</td>
<td>0.000</td>
<td>11</td>
<td>0.173</td>
<td>17</td>
<td>-3.336 **</td>
</tr>
<tr>
<td>JPY/GBP</td>
<td>-0.227</td>
<td>0.935</td>
<td>13</td>
<td>2.420 **</td>
<td>1</td>
<td>-0.493</td>
<td>-21.703</td>
<td>0.000</td>
<td>16</td>
<td>0.161</td>
<td>15</td>
<td>-3.968 **</td>
</tr>
</tbody>
</table>

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The p-values of the Phillips-Perron test are calculated on the basis of MacKinnon (1996). The KPSS test bases upon Kwiatkowski et al. (1992). The KPSS test bases upon Kwiatkowski et al. (1992). The autovariances of the KPSS test are weighted by Bartlett kernel. The DF-GLS test bases upon Elliot et al. (1996) and the lag selection is carried out by using the modified Akaike information criterion (MAIC) as developed by Ng and Perron (2001). For the Phillips-Perron test and the DF-GLS test the null hypothesis it is assumed that the time series are non-stationary. A rejection would be evidence in favour of stationarity. Critical values for the KPSS test are 0.463 for 5% and 0.347 for 1% significance level, for the DF-GLS test -1.95 for 5% and -2.58 for 1% significance level.
Table 4: Log Periodogram Regression by Phillips (2007) on levels

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d</td>
<td>p(H0: d=0)</td>
<td>p(H0: d=1)</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>1.070</td>
<td>0.000</td>
<td>0.578</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>1.208</td>
<td>0.000</td>
<td>0.097</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>0.802</td>
<td>0.000</td>
<td>0.116</td>
</tr>
<tr>
<td>GBP/EUR</td>
<td>1.132</td>
<td>0.000</td>
<td>0.294</td>
</tr>
<tr>
<td>JPY/EUR</td>
<td>0.978</td>
<td>0.000</td>
<td>0.863</td>
</tr>
<tr>
<td>JPY/GBP</td>
<td>1.034</td>
<td>0.000</td>
<td>0.787</td>
</tr>
</tbody>
</table>

\( \hat{d} \) is the integration parameter and shows the number a time series must be differenced to obtain stationarity. The parameter \( \hat{d} \) is the estimate of the least squares regression \( \log(\lambda_s) = \hat{c} - \hat{d} \log(1 - e^{-\lambda_s^2}) + \text{resid} \).

Table 5: Gregory/Hansen Cointegration test with a break in the constant term

|Gregory/Hansen Cointegration test - Break in constant term in the cointegration equation|
|----------------|----------------|----------------|
| Lags | Test Statistic | Breakpoint | Lags | Test Statistic | Breakpoint | Lags | Test Statistic | Breakpoint |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| GBP/USD, EUR/USD | 1 | -4.985 * | 22-Jan-97 | 3 | -3.503 | 1 | -4.885 * | 29-Jan-03 |
| GBP/USD, GBP/USD | 4 | -4.537 | 0 | -4.970 * | 06-Nov-96 | 4 | -3.757 |
| JPY/USD, EUR/USD | 0 | -3.884 | 4 | -3.812 | 1 | -3.348 |
| EUR/USD, JPY/USD | 0 | -3.884 | 2 | -3.795 | 0 | 3.348 |
| JPY/USD, GBP/USD | 4 | -3.795 | 0 | -4.743 * | 23-Oct-96 | 1 | -3.348 |
| GBP/USD, JPY/USD | 2 | -3.795 | 2 | -3.319 | 0 | -3.348 |
| GBP/EUR, EUR/USD | 4 | -3.693 | 0 | -4.752 * | 06-Nov-96 | 1 | -3.934 |
| USD/EUR, GBP/USD | 4 | -4.537 | 0 | -4.970 * | 06-Nov-96 | 4 | -3.447 |
| JPY/EUR, EUR/USD | 2 | -3.883 | 2 | -3.795 | 0 | 3.349 |
| USD/EUR, JPY/USD | 0 | -3.883 | 2 | -3.795 | 0 | 3.349 |
| JPY/EUR, GBP/EUR | 0 | -5.229 ** | 04-Dec-96 | 4 | -3.795 | 0 | -4.536 |
| GBP/EUR, JPY/EUR | 0 | -3.795 | 1 | -3.421 | 0 | -3.977 |
| USD/GBP, EUR/GBP | 1 | -4.985 * | 22-Jan-97 | 3 | -3.503 | 1 | -4.885 * | 29-Jan-03 |
| EUR/GBP, USD/GBP | 4 | -4.500 | 0 | -4.752 * | 06-Nov-96 | 1 | -3.936 |
| JPY/GBP, EUR/GBP | 1 | -4.726 * | 04-Dec-96 | 0 | -2.810 | 1 | -4.979 * | 29-Jan-03 |
| EUR/GBP, JPY/GBP | 0 | -4.259 | 1 | -3.421 | 0 | -3.978 |
| JPY/GBP, USD/GBP | 3 | -4.500 | 2 | -4.262 | 1 | -3.349 |
| USD/GBP, JPY/GBP | 2 | -4.500 | 2 | -3.120 | 0 | -3.349 |
| GBP/JPY, EUR/JPY | 1 | -4.726 * | 04-Dec-96 | 0 | -2.810 | 1 | -4.978 * | 29-Jan-03 |
| EUR/JPY, GBP/JPY | 0 | -5.230 ** | 04-Dec-96 | 4 | -3.450 | 0 | -4.536 |
| USD/JPY, EUR/JPY | 0 | -4.802 * | 29-Jun-05 | 4 | -2.869 | 1 | -3.349 |
| EUR/JPY, USD/JPY | 2 | -4.802 * | 29-Jun-05 | 2 | -3.155 | 0 | -3.349 |
| USD/JPY, GBP/JPY | 4 | -3.349 | 0 | -4.744 * | 23-Oct-96 | 1 | -3.349 |
| GBP/JPY, USD/JPY | 3 | -3.354 | 2 | -4.263 | 1 | -3.349 |

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The test bases upon Gregory and Hansen (1996). It is tested the null hypothesis of no cointegration against the alternative of cointegration with one endogenously estimated breakpoint. Critical values are -4.61 for 5% and -5.13 for 1% significance level.
Table 6: Gregory/Hansen Cointegration test with a break in the constant term and the cointegration parameter

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lags</td>
<td>Test Statistic</td>
<td>Breakpoint</td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>GBP/USD, EUR/USD</td>
<td>1</td>
<td>-5.072 *</td>
<td>04-Dec-96</td>
</tr>
<tr>
<td>EUR/USD, GBP/USD</td>
<td>4</td>
<td>-4.968 *</td>
<td>05-Apr-00</td>
</tr>
<tr>
<td>JPY/USD, EUR/USD</td>
<td>0</td>
<td>-3.795</td>
<td></td>
</tr>
<tr>
<td>EUR/USD, JPY/USD</td>
<td>0</td>
<td>-3.795</td>
<td></td>
</tr>
<tr>
<td>JPY/USD, GBP/USD</td>
<td>4</td>
<td>-3.348</td>
<td></td>
</tr>
<tr>
<td>GBP/USD, JPY/USD</td>
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<td>-4.047</td>
<td></td>
</tr>
<tr>
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<td>-3.798</td>
<td></td>
</tr>
<tr>
<td>USD/EUR, GBP/EUR</td>
<td>4</td>
<td>-4.968 *</td>
<td>05-Apr-00</td>
</tr>
<tr>
<td>USD/EUR, JPY/EUR</td>
<td>0</td>
<td>-3.795</td>
<td></td>
</tr>
<tr>
<td>JPY/EUR, GBP/EUR</td>
<td>0</td>
<td>-5.149 *</td>
<td>04-Dec-96</td>
</tr>
<tr>
<td>GBP/EUR, JPY/EUR</td>
<td>0</td>
<td>-3.977</td>
<td></td>
</tr>
<tr>
<td>USD/GBP, EUR/GBP</td>
<td>1</td>
<td>-5.072 *</td>
<td>04-Dec-96</td>
</tr>
<tr>
<td>EUR/GBP, USD/GBP</td>
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<td>-3.956</td>
<td></td>
</tr>
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<td>-4.253</td>
<td></td>
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<tr>
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<td>-3.978</td>
<td></td>
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<tr>
<td>JPY/GBP, USD/GBP</td>
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<td>-3.700</td>
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<td>USD/GBP, JPY/GBP</td>
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<td>GBP/JPY, EUR/JPY</td>
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<td>-4.322</td>
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</tr>
<tr>
<td>EUR/JPY, GBP/JPY</td>
<td>0</td>
<td>-5.149 *</td>
<td>04-Dec-96</td>
</tr>
<tr>
<td>USD/JPY, EUR/JPY</td>
<td>0</td>
<td>-3.349</td>
<td></td>
</tr>
<tr>
<td>EUR/JPY, USD/JPY</td>
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<td>-3.349</td>
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</tr>
<tr>
<td>USD/JPY, GBP/JPY</td>
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<td>-3.548</td>
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</tr>
<tr>
<td>GBP/JPY, USD/JPY</td>
<td>3</td>
<td>-3.704</td>
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</tbody>
</table>

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The test bases upon Gregory and Hansen (1996). It is tested the null hypothesis of no cointegration against the alternative of cointegration with one endogenously estimated breakpoint. Critical values are -4.61 for 5% and -5.13 for 1% significance level.

Table 7: Zivot/Andrews Unit Root test with a break in the intercept

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lags</td>
<td>Test Statistic</td>
<td>Breakpoint</td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>0</td>
<td>-3.799</td>
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<tr>
<td>GBP/USD</td>
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</tr>
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<td></td>
</tr>
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<td>-4.769</td>
<td></td>
</tr>
<tr>
<td>JPY/EUR</td>
<td>0</td>
<td>-4.918 *</td>
<td>October-98</td>
</tr>
<tr>
<td>JPY/GBP</td>
<td>0</td>
<td>-3.913</td>
<td></td>
</tr>
</tbody>
</table>

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The test bases upon Zivot and Andrews (1992). It is tested the null hypothesis of non-stationarity the alternative of stationarity with one endogenously estimated breakpoint. Critical values are -4.80 for 5% and -5.34 for 1% significance level.
Table 8: Zivot/Andrews Unit Root test with a break in the intercept and the trend term

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lags</td>
<td>Test Statistic</td>
<td>Breakpoint</td>
</tr>
<tr>
<td>EUR/USD</td>
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<td>0</td>
</tr>
<tr>
<td>GBP/USD</td>
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<td>-4.321</td>
<td>0</td>
</tr>
<tr>
<td>JPY/USD</td>
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<td>-4.449</td>
<td>2</td>
</tr>
<tr>
<td>GBP/EUR</td>
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</tr>
<tr>
<td>JPY/EUR</td>
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</tr>
<tr>
<td>JPY/GBP</td>
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<td>-5.360</td>
<td>October-08</td>
</tr>
</tbody>
</table>

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The test bases upon Zivot and Andrews (1992). It is tested the null hypothesis of non-stationarity the alternative of stationarity with one endogenously estimated breakpoint. Critical values are -4.80 for 5% and -5.34 for 1% significance level.

Table 9: Perron (1997) Unit Root test with a break in the intercept and the slope coefficient

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Statistic</td>
<td>Breakpoint</td>
<td>Test Statistic</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>-4.234</td>
<td>-2.654</td>
<td>-3.669</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>-3.995</td>
<td>-3.740</td>
<td>-3.214</td>
</tr>
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<td>JPY/USD</td>
<td>-3.799</td>
<td>-3.821</td>
<td>-3.471</td>
</tr>
<tr>
<td>GBP/EUR</td>
<td>-4.388</td>
<td>-3.417</td>
<td>-4.9089</td>
</tr>
<tr>
<td>JPY/EUR</td>
<td>-4.886</td>
<td>-5.074</td>
<td>-4.652</td>
</tr>
</tbody>
</table>

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The test bases upon Perron (1997). It is tested the null hypothesis of non-stationarity the alternative of stationarity with one endogenously estimated breakpoint. Critical values are -5.08 for the 5% significance level and -5.57 for the 1% level. a - here a rejection at the 10 % level can be considered; the corresponding critical value is -4.82.

Table 10: Perron (1997) Unit Root test with a break in the intercept

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Statistic</td>
<td>Breakpoint</td>
<td>Test Statistic</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>-2.973</td>
<td>-3.752</td>
<td>-3.915</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>-3.015</td>
<td>-3.694</td>
<td>-3.235</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>-3.987</td>
<td>-3.464</td>
<td>-3.873</td>
</tr>
<tr>
<td>JPY/EUR</td>
<td>-4.791</td>
<td>-4.951</td>
<td>-4.436</td>
</tr>
<tr>
<td>JPY/GBP</td>
<td>-3.939</td>
<td>-3.544</td>
<td>-3.495</td>
</tr>
</tbody>
</table>

* (**) denotes rejection of the null hypothesis at the 5% (1%) significance level. The test bases upon Perron (1997). It is tested the null hypothesis of non-stationarity the alternative of stationarity with one endogenously estimated breakpoint. Critical values are -4.80 for the 5% significance level and -5.44 for the 1% level.
Table 11: Johansen Cointegration Analysis for the validity of the no arbitrage condition - EUR/USD, GBP/USD and GBP/EUR exchange rates

<table>
<thead>
<tr>
<th>Rank</th>
<th>Trace Test p-value</th>
<th>Maximum Eigenvalue Test p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>144.46</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>4.55</td>
<td>0.958</td>
</tr>
<tr>
<td>2</td>
<td>1.69</td>
<td>0.83</td>
</tr>
</tbody>
</table>

lags = 3

Cointegration analysis bases upon Johansen (1988, 1991). The constant term is restricted and lies within the cointegration space. The AR 1-7 test tests for serial correlations from lag 1 to 7. Here, the corresponding p-values are in brackets. Regarding the rank tests, the p-values base upon Doornik (1998) and are approximated by a Gamma distribution.

Table 12: Johansen Cointegration Analysis for the validity of the no arbitrage condition - EUR/USD, JPY/USD and JPY/EUR exchange rates

<table>
<thead>
<tr>
<th>Rank</th>
<th>Trace Test p-value</th>
<th>Maximum Eigenvalue Test p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>111.17</td>
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</tr>
<tr>
<td>1</td>
<td>6.34</td>
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</tr>
<tr>
<td>2</td>
<td>1.14</td>
<td>0.918</td>
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</tbody>
</table>

lags = 1

Cointegration analysis bases upon Johansen (1988, 1991). The constant term is restricted and lies within the cointegration space. The AR 1-7 test tests for serial correlations from lag 1 to 7. Here, the corresponding p-values are in brackets. Regarding the rank tests, the p-values base upon Doornik (1998) and are approximated by a Gamma distribution.

Table 13: Johansen Cointegration Analysis for the validity of the no arbitrage condition - GBP/USD, JPY/USD and GBP/JPY exchange rates

<table>
<thead>
<tr>
<th>Rank</th>
<th>Trace Test p-value</th>
<th>Maximum Eigenvalue Test p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>364.66</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>6.34</td>
<td>0.927</td>
</tr>
<tr>
<td>2</td>
<td>1.85</td>
<td>0.801</td>
</tr>
</tbody>
</table>

lags = 1

Cointegration analysis bases upon Johansen (1988, 1991). The constant term is restricted and lies within the cointegration space. The AR 1-7 test tests for serial correlations from lag 1 to 7. Here, the corresponding p-values are in brackets. Regarding the rank tests, the p-values base upon Doornik (1998) and are approximated by a Gamma distribution.

Table 14: Johansen Cointegration Analysis for the validity of the no arbitrage condition - GBP/EUR, JPY/EUR and GBP/JPY exchange rates

<table>
<thead>
<tr>
<th>Rank</th>
<th>Trace Test p-value</th>
<th>Maximum Eigenvalue Test p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>211.6</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>4.67</td>
<td>0.991</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>0.811</td>
</tr>
</tbody>
</table>

lags = 2

Cointegration analysis bases upon Johansen (1988, 1991). The constant term is restricted and lies within the cointegration space. The AR 1-7 test tests for serial correlations from lag 1 to 7. Here, the corresponding p-values are in brackets. Regarding the rank tests, the p-values base upon Doornik (1998) and are approximated by a Gamma distribution.
Figure 1: KPSS test - rolling regression with a fixed sample size of two years

Figure 2: Phillips/Perron test - rolling regression with a fixed sample size of two years
Figure 3: Modified log periodogram regression - rolling regression with a fixed sample size of two years
Figure 4: Key fundamental variables for the Euro area, the UK and the USA