Countercyclical Taxation and Price Dispersion*

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Abstract

In this paper, we explore the benefits from a supply-side oriented fiscal tax policy within the framework of a New Keynesian DSGE model. We show that countercyclical tax rules, which are contingent on the observed welfare gap or alternatively on the markup shock and levied on value added, reduce remarkably the impact of nominal frictions. We state that the simple tax rule establishes an efficient path for the evolution of marginal cost at the firm level and largely prevents built up of price dispersion. We highlight that this tax policy is also effective under a balanced-budget regime. Hence, fiscal policy can disencumber monetary policy in the light of stabilizing cost push shocks.

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1 Introduction

Can fiscal policy eliminate the welfare costs of sticky prices by means of simple tax rules? We address this question within the setting of a New Keynesian DSGE model. Using the New Keynesian framework, a rich strand of literature has stressed the role of monetary policy to enhance welfare in an environment of nominal rigidities (Woodford, 2003). However this strand of literature has paid so far little attention to fiscal policy. In particular, only few studies analyze the role of distortionary taxation and debt financed expenditures and their implications for nominal frictions. In this respect, the paper aspires to extend the existing literature in proposing two alternative fiscal policy rules that remarkably reduce price dispersion. The government sector is assumed to maximize lifetime utility of a representative agent. As instrument it relies on a value-added tax, debt or government expenditures. Our framework shares most of the features of recent dynamic optimization sticky price models as e.g. in Woodford (2003), and Gali, Lopez-Salido and Valles (2007).

In the basic New Keynesian framework there is no reason for output to be different across firms, except as a result of price distortions that accrue from staggered price setting (Woodford, 2003). Through this channel infrequent price adjustments create undesired variations in the relative prices of goods across firms. A sufficiently strong feedback of the real interest rate to movements in the inflation rate is argued to be the best response to limit the adverse effects of cost-push shocks on lifetime utility of a representative consumer. Notwithstanding the previous argument the welfare costs of nominal rigidities are estimated to be up to three percent in consumption equivalents (Canzoneri, Cumby and Diba, 2007; and Gertler and Lopez-Salido, 2007).

This highlights that monetary policy does not have a direct leverage on the supply side and thus the price setting behavior of firms. Monetary authorities can only control price dispersion through the aggregate demand channel and thus the reallocation of intertemporal consumption plans, and therefore in the event of a supply shock firms are tempted to increase prices. As best response monetary authorities raise the real interest rate to encourage consumers to reallocate consumption to the future which depresses contemporaneous demand and thus demand-driven production plans. As production plans have to be consistent with the labor supply schedules of workers an equilibrium only occurs if wages and thus marginal cost decline. In contrast demand shocks can be wiped out at zero cost (Clarida, Gali, Gertler, 1999). In essence, it is the lack of an additional instrument on the supply side of the economy which makes markup shocks costly in terms of welfare.
Therefore, we follow the Tinbergen logic and propose that fiscal policy should use its value-added tax as an additional instrument in a state contingent way such that the evolution of marginal cost is stabilized around its deterministic steady state (Tinbergen, 1959). Those firms that are called upon to reset prices will then build on the promise of fiscal authorities to smooth away cost-push shocks and set prices in the neighborhood of those price setters that have to leave prices unchanged.

A key finding of our paper is that fiscal authorities can set up a path for value-added taxes that evolves countercyclically to markup shocks and thus prevents large movements in marginal cost. This seems in particular important as Schmidt-Grohe and Uribe (2006) report evidence from a medium-scale model which comprises a number of real and nominal frictions that price stickiness emerges as the most important distortion. When fiscal policy is allowed to cushion changes in tax rates by debt rather than government expenditures we state that debt prevails a near-random walk behavior in the presence of cost-push shocks. The steady state levels of tax rates and a sufficiently strong feedback from tax rates to changes in the level of debt are determined by long-run solvency considerations such that in steady state the budget is balanced (Canzoneri, Cumbi and Diba, 2003; Linnemann and Schabert, 2003).

Although the general idea of simple fiscal rules has not been new, authors so far have mainly focused on the idea of classical demand management, where government expenditures are conditioned on the output gap such as J.B. Taylor (2000). Only few studies consider stochastic taxation, and its implications for nominal frictions. A notable exception is Leith and Wren-Lewis (2007), who explore the role of countercyclical fiscal policy in a full-fledged DSGE model and analyze commitment solutions. They report evidence that price dispersion can be completely wiped out by commitment solutions when fiscal authorities employ four instruments, namely debt, government expenditures and taxes on labor and value added. We differ from there work in several aspects: (i) instead of modeling commitment solutions we show that optimal fiscal rules under discretion and simple rules are sufficient to substantially improve welfare. (ii) We report analytical evidence that such rules are also effective under a balanced-budget regime by means of MSV-solutions. (iii) We obtain results from a sensitivity analysis with respect to deep parameters. (iv) We simulate the behavior of the economy with the occurrence of markup shocks.

Our findings suggest that countercyclical supply-side taxation rules can remarkably reduce the impact of cost-push shocks on welfare. The paper is structured as follows: In Section 2, the basic model is introduced. Section 3 presents analytical results on fiscal
rules and price dispersion. In Section 4 we compare active fiscal policy, where the fiscal policy maker pursues the countercyclical tax rule, to a passive stance of fiscal policy by using a numerical approach. In Section 5 we conduct robustness analysis. Section 6 summarizes the main findings and concludes.

2 The Model

In this section we present a New Keynesian DSGE model with firms, households, the central bank and fiscal authorities. As standard, firms are partitioned into the final good sector and a continuum of intermediate good producers. Intermediate good producers have some monopoly power over prices that are set in a staggered way following Calvo (1983). Households obtain utility from consumption, public goods, leisure and invest in state contingent securities. Monetary authorities are guided by a simple Taylor rule. The government sector is financed by distortionary taxes levied on value added or debt. Fiscal policy is implemented by tax and spending rules.

The model is built on the framework of Gali, Lopez-Salido and Valles (2007), Leith and Wren-Lewis (2007), and Linnemann and Schabert (2003) by sharing the same kind of features such as debt financed expenditures, state contingent tax rules and staggered price setting as in Calvo (1983). In particular we highlight the role of an active fiscal policy compared to a neutral stance to fight the welfare costs of price dispersion.

2.1 Final Good Producers

The final good is bundled by a representative firm which operates under perfect competition. The technology available to the firm is:

\[ Y_t = \left[ \int_{0}^{1} X_t(i) \frac{x_t - 1}{\epsilon_t} di \right]^{\frac{1}{1-\epsilon_t}}, \]  

(1)

where \( Y_t \) is the final good, \( X_t(i) \) are the quantities of the intermediate goods, indexed by \( i \in (0,1) \) and \( \epsilon_t > 1 \) is the time-varying elasticity of substitution in period \( t \). Profit maximization implies the following demand schedules for all \( i \in (0,1) \):

\[ X_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t. \]  

(2)

The zero-profit theorem implies \( P_t = \left[ \int_{0}^{1} P_t(i)^{1-\epsilon_t} di \right]^{1/(1-\epsilon_t)} \), where \( P_t(i) \) is the price of the intermediate good \( i \in (0,1) \). In a similar way to Smets and Wouters (2003), we assume
that $\varepsilon_t$ is a stochastic parameter. In this context, we define $\Phi_t = \frac{\varepsilon_t}{\varepsilon_{t-1}}$ reflecting the time-varying markup in the goods market and assume that $\Phi_t = \Phi + \hat{\Phi}_t$. Thereby, $\hat{\Phi}_t$ is i.i.d. normal distributed, and $\Phi = \frac{\varepsilon}{\varepsilon - 1}$ is the deterministic markup which holds in the long-run flexible price steady state.

2.2 Intermediate Good Producers

Firms indexed by $i \in (0, 1)$ operate in an environment of monopolistic competition. The typical production technology is given by:

$$Y_t(i) = N_t(i),$$

where $N_t(i)$ denotes labor services. Nominal profits by firm $i$ are given by:

$$\Pi_t(i) = (1 - \tau_t^{\text{VAT}}) P_t(i) Y_t(i) - W_t N_t(i),$$

with $Y_t(i) = X_t(i)$ and $\tau_t^{\text{VAT}}$ denotes a value-added tax with $\tau_t^{\text{VAT}} \in (0, 1)$. As cost minimization implies that marginal costs are equal to wages with $\varphi_t = w_t$ the profit function can be rewritten as follows:

$$\Pi_t(i) = [(1 - \tau_t^{\text{VAT}}) P_t(i) - P_t \varphi_t] Y_t(i).$$

The representative firm is assumed to set prices as in Calvo (1983), which implies that the price level is determined in each period as a weighted average of a fraction of firms $(1 - \theta_p)$ which resets prices and a fraction of firms $\theta_p$ that leaves prices unchanged:

$$P_t = \left[(1 - \theta_p)(\hat{P}_t)^{1-\varepsilon_t} + \theta_p P_{t-1}^{1-\varepsilon_t}\right]^\frac{1}{1-\varepsilon_t},$$

where $\hat{P}_t$ is the optimal reset price in period $t$.

Each firm $i$ that is called upon to reset prices solves the following intertemporal profit maximization problem subject to its demand function for $Y_t(i)$:

$$\max_{\tilde{P}_t(i)} \left\{ E_t \left( \sum_{k=0}^{\infty} (\theta_p \beta)^k \Delta_{t, t+k} \left[ \tilde{P}_t(i)(1 - \tau_{t+k}^{\text{VAT}}) - P_{t+k} \varphi_{t+k} \right] Y_{t+k}(i) \right. \right.$$

$$\left. - \vartheta_{t+k} \left[ Y_{t+k}(i) - \left( \frac{\tilde{P}_{t+k}(i)}{P_{t+k}} \right)^{-\varepsilon_t} Y_{t+k} \right] \right\},$$

where $\vartheta_{t+k}$ denotes the Lagrangian multiplier in period $t + k$, and $\Delta_{t, t+k}$ denotes the stochastic discount factor of shareholders, to whom profits are redeemed. It is defined as $\Delta_{t, t+k} = (U_C(C_{t+k})/U_C(C_t))$. Combining the first-order conditions, we obtain:

$$E_t \left\{ \sum_{k=0}^{\infty} (\theta_p \beta)^k \Delta_{t, t+k} Y_{t+k}(i) \left[ \tilde{P}_t(i)(1 - \tau_{t+k}^{\text{VAT}}) - \Phi_{t+k} P_{t+k} \varphi_{t+k}(i) \right] \right\}. $$
2.3 Households

We assume a continuum of households indexed by \( j \in (0, 1) \). A typical household seeks to maximize lifetime utility:

\[
E_0 \sum_{k=0}^{\infty} \beta^k U_{t+k}(j),
\]

where \( \beta \) denotes a discount factor with \( \beta \in (0, 1) \), and period utility is given by:

\[
U_t(j) = (1 - \chi) \left( \frac{1}{1 - \sigma} C_t(j)^{1-\sigma} \right) + \chi G_t - \frac{1}{1 + \eta} N_t(j)^{1+\eta}.
\]

\( \sigma \) is a coefficient of risk aversion, \( \eta \) is the inverse of the Frisch elasticity of labor supply, and \( \chi \in (0, 1) \) measures the relative weight of public consumption. \( C_t(j) \) are the real consumption expenditures of household \( j \). The sequence of budget constraints reads:

\[
C_t(j) + \frac{B_{t+1}(j)}{P_t} \leq \frac{W_t N_t(j)}{P_t} + \frac{\Pi_t(j)}{P_t} + \frac{B_t(j)}{P_t}.
\]

Each household decides on consumption expenditures \( C_t(j) \) and bond holdings \( B_{t+1}(j) \) and receives labor income \( W_t N_t(j) \), dividends from profits \( \Pi_t(j)/P_t \) and the gross return on bonds purchased \( B_t(j) \).

Maximizing the objective function subject to the intertemporal budget constraint with respect to consumption and bond holdings delivers the following first-order conditions:

\[
(1 - \chi)C_t^{-\sigma} = \lambda_t,
\]

\[
N_t^\eta(j) = \lambda w_t,
\]

\[
\frac{1}{P_t} \lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{1}{P_{t+1}} R_t \right],
\]

where \( \lambda_t \) denotes the Lagrangian from relaxing the budget constraint. Combining the first order conditions yields the consumption Euler equation and the labor supply schedule:

\[
C_t^{-\sigma} = \beta R_t E_t \left[ C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right]
\]

\[
\frac{N_t^\eta(j)}{C_t^{-\sigma}} = \frac{W_t}{P_t} (1 - \chi).
\]

Note that we can drop the index \( j \) for consumption \( C_t \) due to the existence of contingent claims markets.
2.4 Fiscal Authorities

The government issues bonds and collects value-added taxes. It uses its receipts either to finance government expenditures or interest on outstanding debt. The real government budget constraint reads:

\[ R_t - 1 + \frac{B_{t+1}}{P_t} + \tau_t^{VAT} Y_t = \frac{B_t}{P_t} + G_t. \]  (17)

Letting \( b_t = \frac{1}{Y}[(B_t/P_{t-1}) - (\bar{B}/\bar{P})] \) and \( \bar{B} = 0 \), the budget constraint can be rewritten as:

\[ R_t^{-1} b_{t+1} + \tau_t^{VAT} \frac{Y_t}{Y} = b_t \frac{P_{t-1}}{P_t} + \frac{G_t}{Y}. \]  (18)

Government purchases are assumed to move countercyclically to output:

\[ G_t = \alpha Y_t^{-o_Y}, \]  (19)

where \(-1 < o_Y < 0\) denotes the expenditure elasticity with respect to income \( Y_t \) and \( \alpha \equiv \bar{G}Y^{-o_Y} \). The tax rule reads:

\[ \tau_t^{VAT} = \Phi_t^{\chi_1} b_t^{\chi_2}, \]  (20)

which is conditioned on the predetermined state variables \( \Phi_t \) and \( b_t \). In principle a sufficient strong response to the change of the level of outstanding debt \( \chi_2 > 0 \) assures uniqueness and determinacy. A parameter \( \chi_1 < 0 \) denotes a countercyclical fiscal tax policy. Additionally, we consider a simple tax rule. Note that in the literature simple rules are predominantly interpreted as rules where the instrument responds to observable macroeconomic variables, e.g., to the inflation rate or for instance to the welfare gap (e.g., Schmitt-Grohe, Uribe, 2007). Therefore, we opted to consider also an alternative tax rule which is conditioned on the outstanding debt and the welfare gap \( x_t \):

\[ \tau_t^{VAT} = x_t^{\chi_1} b_t^{\chi_2}. \]  (21)

The welfare gap is defined as \( x_t \equiv \hat{Y}_t - \hat{Y}_t' \), i.e., as the gap between the actual output gap and the output gap under flexible prices. We determine the respective parameter \( \chi_1 \) for both types of fiscal tax rules such that the rules are optimal from the perspective of a discretionary fiscal policy. As \( \Phi_t \) and \( b_t \) are the only predetermined state variables equation (20) describes the only optimal feedback rule from a discretionary perspective.

In Section 3, we derive analytical results for the optimal tax rule (20), and we use both types of tax rules in sections 4 and 5, where we consider the welfare implications of both rules and check the robustness by using a numerical approach.
2.5 Market Clearance

In clearing of factor markets and good markets the following conditions are satisfied:

\[
Y_t = C_t + G_t, \\
Y_t(j) = X_t(j), \\
N_t = \int_0^1 N_t(j) dj.
\]

2.6 Linearized Equilibrium Conditions

In this section we summarize the model by taking a log-linear approximation of the key equations around a symmetric equilibrium steady state with zero inflation and zero debt. In the following, a variable \( \hat{X}_t \) denotes the log-linear deviation from the steady state value: \( \hat{X}_t = \log(X_t) - \log(\bar{X}) \), where \( \bar{X} \) represents the deterministic steady state.

**Households** The consumption Euler equation reads:

\[
\hat{C}_t = E_t \hat{C}_{t+1} - \sigma^{-1}(\hat{R}_t - E_t \hat{\pi}_{t+1}) ,
\]

where \( \hat{\pi}_t \) is defined as \( \hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \), and we used that in the steady state \( \hat{R} = \beta^{-1} \) which follows directly from the consumption Euler equation. Under perfectly competitive labor markets the labor supply schedule is equal to:

\[
\hat{w}_t = \eta \hat{N}_t + \sigma \hat{C}_t .
\]

**Firms** Log-linearization of (6) and (8) around a zero inflation steady state yields the dynamics of inflation as a function of the wage \( \hat{w}_t \), a stochastic markup \( \hat{\Phi}_t \) and tax rates \( \hat{\tau}_{VAT} \):

\[
\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \kappa [\hat{w}_t + \hat{\tau}_{VAT} + \hat{\Phi}_t],
\]

with \( \kappa \equiv (1 - \theta_p)(1 - \beta \theta_p)/\theta_p \), and \( \nu \equiv \tau_{VAT}/(1 - \tau_{VAT}) \).

**Fiscal authorities** Log-linearizing the budget constraint around a zero steady state debt yields the following approximation up to first order:

\[
b_{t+1} + \gamma_G (\hat{\tau}_{VAT} \dot{Y}_t) = \beta^{-1} b_t + \gamma_G \dot{G}_t ,
\]
where for the case of a balanced budget (25) simplifies to \( \hat{G}_t = \tilde{\tau}^{VAT} + \hat{Y}_t \). The parameter \( \gamma_G \) denotes the steady state government share which is equal to \( \tau^{VAT} \) implied by a balanced budget in steady state. The fiscal spending rule is the log-linearized version of (19):\(^1\)

\[
\hat{G}_t = o_Y \hat{Y}_t .
\]  

(26)

The simple tax rule is the log-linearized complement to (20):

\[
\tilde{\tau}_t^{VAT} = \chi_1 \hat{\Phi}_t + \chi_2 b_t .
\]  

(27)

Correspondingly, the log-linearization of the alternative tax rule (21) based on the welfare gap \( x_t \) is given by

\[
\tilde{\tau}_t^{VAT} = \chi_1 x_t + \chi_2 b_t .
\]  

(28)

In the following we will refer to a passive fiscal policy if \( \chi_1 = 0 \) such that fiscal policy abstains from following a countercyclical path for taxes.

**Monetary Policy**  
Monetary policy is assumed to follow the Taylor rule:

\[
\hat{R}_t = (1 - \phi_p) \hat{R}_{t-1} + \phi_p [\phi_\pi \hat{\pi}_t + \phi_x x_t] ,
\]  

(29)

where \( \phi_\pi \) and \( \phi_x \) capture the reaction coefficients with respect to the inflation rate and the output gap \( x_t \) as defined below; \((1 - \phi_p)\) with \(0 \leq \phi_p \leq 1\) denotes the degree of interest rate smoothing on part of the central bank. The rule satisfies the Taylor-principle as long as \( \phi_\pi > 1 \), which is a necessary requirement for uniqueness and stability (Woodford, 2003).

**Market Clearing**  
Market clearing requires that the following relation holds:

\[
\hat{Y}_t = \gamma_C \hat{C}_t + \gamma_G \hat{G}_t ,
\]  

(30)

where \( \gamma_C \) denotes the consumption share, which is equal to \( (1 - \tilde{\tau}^{VAT}) \). Using (26) and (30) we can rewrite the consumption Euler-equation as follows:

\[
\hat{Y}_t = E_t \hat{Y}_{t+1} - \gamma_C \frac{\hat{R}_t - E_t \hat{\pi}_{t+1}}{\sigma (1 - \gamma_G o_Y)} .
\]  

(31)

\(^1\)Note that the welfare criterion (see section 4) is derived for the linear case: \( o_Y = 0 \) and \( o_Y = -1 \).
**Flex-price equilibrium** The flex-price equilibrium is obtained by equating $\hat{w}_t = \eta \hat{N}_t + \sigma \hat{C}_t$ and $\hat{\varphi}_t = \hat{w}_t$ which combines the real marginal product of labor to the marginal rate of substitution between consumption and leisure:

$$\varphi^f_t = \Gamma \varphi \hat{Y}^f_t, \quad \text{with} \quad \Gamma \varphi \equiv [\eta + \sigma \gamma_{G}(1 - \gamma_{G}o_{Y})] ,$$

where we additionally used the fiscal spending rule (26) and market clearance condition (30). The superscript $f$ denotes flexible prices. From the optimal price-setting behavior of firms operating in the intermediate good sector under flexible-prices we know that:

$$\varphi^f_t = \Phi^{-1}_t (1 - \gamma_{VAT}^f) ,$$

where we assumed that fiscal policy sets $\chi_{1} = 0$ if prices are flexible as no price dispersion prevails in the flex-price equilibrium such that $\hat{\tau}_{VAT}^f = \chi_{2} b^f_t$. Accordingly the log-deviation of real marginal cost from its deterministic counterpart $(\varepsilon - 1)/\varepsilon$ can then be written in log-linearized terms as: $\hat{\varphi}^f_t = -(\hat{\Phi}_t + \hat{\tau}_{VAT}^f)$. Using the output gap $x_t$ the log-deviation of marginal cost can be written as:

$$\hat{\varphi}_t = \Gamma \varphi (x_t + \hat{Y}^f_t), \quad \text{with} \quad \hat{Y}^f_t = \hat{\Phi}_t + \hat{\tau}_{VAT}^f .$$

We can rewrite the Phillips curve in terms of $x_t$ as:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa [\Gamma \varphi x_t + \nu (\hat{\tau}_{VAT}^f - \hat{\tau}_{VAT}^f)] ,$$

From the Euler-equation we know that the natural rate of interest under flexible prices is equal to:

$$r^n_t - \rho = \sigma E_t (\Delta \hat{Y}^f_{t+1} - \Delta \hat{\tau}^f_{t+1}) ,$$

where $\rho \equiv - \log \beta$. Inserting $\Delta \hat{Y}^f_{t+1}$ and $\Delta \hat{\tau}^f_{t+1}$ the natural rate can be expressed in terms of the exogenous shock $\hat{\Phi}_{t+1}$ and the tax rule $\hat{\tau}^f_{t+1}$ under flexible prices:

$$\hat{r}^n_t = -\sigma (1 - o_Y) \Gamma^{-1} E_t [\Delta \hat{\Phi}_{t+1} + \nu \Delta \hat{\tau}^f_{t+1}] .$$

Using the definitions of the welfare gap $x_t$ it holds that:

$$x_t = E_t x_{t+1} - \gamma_{C}(\sigma (1 - \gamma_{G}o_{Y}))^{-1} [\hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{r}^n_t] ,$$

and

$$b_{t+1} - \gamma_{G} \Gamma^{-1} (1 - o_{Y}) \hat{\tau}^f_{t+1} = \beta^{-1} b_t + \gamma_{G} (o_Y - 1) x_t + \gamma_{G} \Gamma^{-1} (1 - o_{Y}) \hat{\Phi}_t - \gamma_{G} \hat{\tau}^f_{t+1} .$$
Discussion  Notwithstanding that most of the features in the model are standard in particular the value-added tax augmented Phillips curve is worth stressing. First, notice that the inflation rate is simply a weighted average of the expected path of wage costs, the markup shock and the evolution of the value-added taxes. As we will show below this enables the government to design a path for value-added taxes which almost completely offsets any movement in cost pressure such that price dispersion across firms can be reduced. Secondly, as we formulate state contingent tax and spending rules government debt necessarily works as a buffer to accommodate movements in the spending rule and movements of the tax rate. For the case of a balanced budget regime movements in the tax rate call for adjustments in fiscal spending.

2.7 Graphical Illustration of the Model

Throughout Section 2, we have shown the optimization problems of households, intermediate good producers and final good firms, and we have introduced the rules of monetary and fiscal policies. To help the reader to capture how all agents interact with each other, figure 1 illustrates the sequence of the actions for a certain period \( t \) and adumbrates the intertemporal links.

Figure 1: Structure and Sequence of the Model
3 Simple Rules and Price Dispersion

In this Section we analytically examine the role of simple tax rules on the equilibrium allocation of inflation, output, consumption, interest rates and government expenditures. To keep the calculations analytically tractable, we assume that the budget is balanced such that (27) reduces to\( \hat{\tau}_{t}^{VAT} = \chi_{1}\hat{\Phi}_{t} \) and government expenditures are adjusted passively so that the budget equation (25) holds. Additionally, we reduce the system by inserting the natural rate of interest \( \bar{r}^{n} \) and the tax rule (27) into the Phillips curve (35) and the Euler-equation (38). Then the model can be written as the following set of expectational difference equations:

\[
x_{t} = E_{t}x_{t+1} - \sigma^{-1}(\hat{R}_{t} - E_{t}\hat{\pi}_{t+1}) + (\gamma_{G}\gamma_{C}^{-1}\chi_{1} + (\sigma + \eta)^{-1})\hat{\Phi}_{t}, \tag{40}
\]
\[
\hat{\pi}_{t} = \beta E_{t}\hat{\pi}_{t+1} + \kappa[(\sigma + \eta)x_{t} + (\tau - \sigma)\gamma_{C}^{-1}\chi_{1}\Phi_{t}], \tag{41}
\]
\[
\hat{R}_{t} = \phi_{x}\hat{\pi}_{t}, \tag{42}
\]

where the coefficient \( \chi_{1} \) serves as a parameter which can be freely chosen by fiscal authorities. The following propositions summarize the main results.\(^{2}\)

**Proposition 3.1** Suppose that a social planner is only concerned about price dispersion and, hence, inflation variability. Then choosing a coefficient \( \chi_{1} = -\gamma_{G}\gamma_{C}^{-1}(\tau + \eta)^{-1} \) completely eliminates any price dispersion across firms at any date \( t \).

**Proof** Since the simplified model with \( b_{t+1} = b_{t} = 0 \) exhibits no endogenous state variables the fundamental solution takes the form: \( \hat{\pi}_{t} = \delta_{x}\hat{\Phi}_{t} \). Applying the methods of undetermined coefficients leads to the following solution: \( \delta_{x} = [1 + \kappa(\sigma + \eta)\sigma^{-1}\phi_{x}]^{-1}\kappa[1 + \gamma_{C}^{-1}\chi_{1}(\tau + \eta)] \). Inflation is completely stabilized if \( \delta_{x} = 0 \) which holds for \( \chi_{1} = -\gamma_{C}^{-1}(\tau + \eta)^{-1} \).

Thus according to Proposition 3.1 fiscal authorities can completely stabilize the inflation rate by choosing \( \chi_{1} \) appropriately. For the applied calibration, \( \chi_{1} \) would take a numerical value of \( \chi_{1} = -3.20 \) (\( \gamma_{G} = 0.2; \eta = 1; \tau = 0.25 \)). Interestingly the coefficient \( \chi_{1} \) only depends on two deep parameters, namely \( \hat{\tau}_{t}^{VAT} \) and \( \eta \). In line with intuition an increasing steady state government share \( \gamma_{G} \) increases the leverage of fiscal authorities on real

\(^{2}\)For the MSV-solutions, see appendix B.
marginal costs and on prices such that the same effects on equilibrium allocations can be achieved by smaller movements of the instrument \( \tau^{VAT} \). The same holds true for \( \iota \) which is defined as \( \iota = \bar{\tau}^{VAT} - \bar{\tau}^{VAT} \) and is increasing in \( \bar{\tau}^{VAT} \). Additionally, the responsiveness of the coefficient \( \chi_1 \) decreases in the Frisch elasticity \( \eta \) of labor supply. Thus, if labor supply is less responsive to the cycle smaller tax incentives are sufficient to yield the same effects on the evolution of marginal cost.

**Proposition 3.2** Suppose that we compare two economies which are identical except that in one economy fiscal policy implements the simple rule \( \hat{\tau}^{VAT}_t = \chi_1 \hat{\Phi}_t \) whereas in the other economy fiscal policy remains passive with \( \bar{\tau}^{VAT}_t = \tau^{VAT}_t \) and \( \bar{G}_t = G_t \) \( \forall t \). Then, for any policy choice with \( \chi_1 < 0 \) the evolution of the inflation rate \( \hat{\pi}_t \), the welfare gap \( x_t \) and nominal interest rates \( \hat{R}_t \) evolve smoother than in an economy where \( \chi_1 = 0 \).

**Proof** Since in both economies the simplified model exhibits no endogenous state variable the fundamental solution takes in both cases the form \( \hat{X}_t = \delta_X \hat{\Phi}_t \), with \( \hat{X}_t = [\hat{\pi}_t \ x_t \ \hat{R}_t] \) and \( \delta_X = [\delta_\pi \ \delta_x \ \delta_R] \). Thus a necessary and sufficient condition for a smoother evolution of the economy is \( |\delta^{A}_X|_{i,1} < |\delta^{P}_X|_{i,1} \) for \( i = 1, 2, 3 \), where the superscripts \( A \) denote active and \( P \) passive. As shown in appendix B a necessary and sufficient condition for this inequality to hold is that \( \chi_1 < 0 \).

Thus according to Proposition 3.2 it holds that any policy choice with \( \chi_1 < 0 \) accommodates a smoother evolution of the economy.

Without any statement on welfare, we can already conjecture that an active fiscal stance is welfare improving if government expenditure is pure waste as the welfare function for this case would only built on the inflation rate \( \hat{\pi}_t \) and the welfare gap \( x_t \). Note that we know by the Taylor Rule that the nominal interest rate will be smoothed as it is just a linear transformation of the inflation rate itself. This in turn, implies a smoother evolution of the real interest rate which fosters a more stable consumption path \( (C_t/C_{t+1}) \) as can be seen from the Euler-equation:

\[
\frac{1}{\beta} = E_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^\sigma \left( \frac{R_t}{\pi_{t+1}} \right) \right],
\]

which states that the product of the logdeviation of the real interest rate and the ratio of the transformed logdeviations of consumption will always be equal to the inverse of the discount factor.
For the case of a balanced budget, changes in the tax rate have to be cushioned by fiscal spending $\hat{G}_t$. Therefore, fiscal spending is by definition more volatile than under a passive fiscal stance. Notwithstanding the argument the output gap $\hat{Y}_t$, defined as the weighted sum $\gamma_C \hat{C}_t + \gamma_G \hat{G}_t$, evolves less volatile. This reflects that the additional volatility in government expenditure is overcompensated by the stable evolution of consumption, which implies that such a policy is welfare improving as long as consumers attach a higher weight to consumption than to government expenditures in their welfare metric.

4 Welfare

Next we characterize the model if we allow for debt financed expenditures by means of numerical analysis. As shown in the appendix C the welfare criterion is derived by a second-order approximation of the average utility of a household around the deterministic long-run steady state. The welfare function can be written as follows (see Erceg, Henderson, and Levine, 2000, Gali and Monacelli, 2007, and Woodford 2003):

$$W_0 = \sum_{t=0}^{\infty} \beta^t E_0(L_t),$$

where

$$L_t = \frac{\varepsilon}{\kappa} \hat{\pi}_t^2 + (1 + \eta) \hat{Y}_t^2 + \iota (\hat{G}_t - \hat{Y}_t)^2.$$  \hspace{1cm} (45)

In the following we discuss the implementation of the proposed tax rules. We start with the optimal rule under discretion given by (27), which is based directly on the shock $\hat{\Phi}_t$, and check afterwards whether similar results will hold for the simple tax rule (28).

4.1 Optimal Tax Rule under Discretion

Since we do not have a distinctive imagination for an appropriate numerical parameter except that $\chi_1 < 0$, we opt to choose the parameter such that the welfare function (44) is minimized.$^3$

$^3$We also optimized over the parameter $\chi_2$ which governs the feedback from changes in debt and taxes. The algorithm preferred small values which are close to those proposed by Linnemann and Schabert (2003). As the algorithm often fall prey to indeterminacy for too small values of $\chi_2$, we chose a calibration of $\chi_2 = 0.06$. 
Figure 2 portrays the dynamic responses of selected variables to a markup shock. For the baseline case fiscal policy remains passive with $\chi_1 = 0$ whereas for the active stance with $\chi_1 < 0$ fiscal policy aspires to improve welfare by controlling the evolution of marginal cost. The following remark summarizes the main findings:

**Remark:** The implementation of rule (27) largely disconnects the evolution of the inflation rate from exogenous markup shocks. If free to choose fiscal authorities prefer long debt cycles to cushion the exogenous shock.

The impulse responses portray that a sharp cut in taxes $\hat{\tau}_{VT}^t$ levied on the value-added prevents any built up in cost pressure. The tax cut occurs in particular in the first quarter, when the geometrically decaying markup shock hits strongest. As a fraction of firms $\theta_P$ is called upon to reset prices they foresee that any price pressure is undone by fiscal authorities by the targeted tax path that keeps the sum of wage path, markup shock and tax path flat. Due to the moderate evolution of the inflation rate monetary authorities are prevented from sharply raising nominal interest rates. This in turn detains Ricardian households to reallocate planned consumption expenditures by large into the future. As consumption accounts for 80% of output we observe a moderate drop in production. If fiscal authorities are free to choose, they absorb the tax cut by a near-random walk behavior in debt. Note as markup shocks are symmetrically distributed a near-random walk behavior in debt implies that the persistent swings cancel out each other. On the contrary, contemporaneous government-expenditure changes are welfare reducing as they increase the expected variability in consumption of public goods. The point estimate for the parameter $\chi_1$ and the associated standard errors are reported in Table 1. The point estimate for $\chi_1$ is equal to -1.60 with a standard error of 0.11. For the baseline scenario this implies that the implementation of the simple policy rule reduces the value of the loss function by 69 percent. Under the assumption that $\{\sum_{t=0}^{\infty} \beta^t L_t\}^{\text{Passive}} - \{\sum_{t=0}^{\infty} \beta^t L_t\}^{\text{Active}}$ is chi-square distributed with one degree of freedom the loss reduction is significant at the one percent level. Under the header “range” we report evidence that the proposed policy rule is robust with respect to deviations from the optimal reaction coefficient $\chi_1$. To illustrate this we deviate from the optimal coefficient such that the implementation of the policy rule still significantly reduces the business cycle at the one

---

For the choice of the chi-square distribution see e.g. Meier, Müller, 2005, and Wooldridge, 2002.
Notes: Responses of selected variables to a markup shock. Solid lines indicate a state independent passive fiscal policy with $\chi_1 = 0$. The dotted line shows the impulses of the model when fiscal policy is active with $\chi_1 < 0$ and $\chi_2 > 0$. For the applied baseline calibration see appendix A.

percent significance level. Therefore, as a robustness exercise we report how far we can deviate in both directions from the optimal coefficient such that the computed distance $\{\sum_{t=0}^{\infty} \beta^t L_t\}^{Passive} - \{\sum_{t=0}^{\infty} \beta^t L_t\}^{Active,upper,lower}$ is still significant at the one percent level. Generally the results indicate no large asymmetries when fiscal authorities tend to choose too high or too low coefficients $\chi_1$, which indicates that the loss ratio largely behaves linearly when deviating from the baseline by altering $\chi_1$. For the case of large asymmetries we would have expected the reported values for $\chi_1^{lower}$ and $\chi_1^{upper}$ to have a substantially different distance to $-1.60$. The range from -3.15 to -0.06 impressively demonstrates that for a large set of parameters $\chi_1$ the policy rule stabilizes the economy significantly. Therefore we conclude that the proposed rule is robust with respect to variations in $\chi_1$. 


Table 1: The Estimated Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>St.Dev.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction Coefficient</td>
<td>$\chi_1$</td>
<td>-1.60</td>
<td>0.11</td>
<td>[-3.15, -0.06]</td>
</tr>
</tbody>
</table>

4.2 Alternative Simple Tax Rule

Analogously to the procedure in the previous subsection, we simulated the impulse response functions for the simple tax rule based on $x_t$. Figure 3 exhibits that compared to the previous section, the impulse response functions for the selected variables take a very similar course. Hence, we can state that also the implementation of the simple tax rule is highly suitable for stabilizing the economy after the materialization of markup shocks. For the baseline calibration the loss reduction is 48 percent. This is somewhat
worse than for the discretionary optimum which reduced the loss by 69 percent. It might be explained by the following trade-off. In terms of the output gap inflation is driven by \( \Gamma \hat{Y}_t + \iota \hat{\tau}^{VAT} + \hat{\Phi}_t \), such that fiscal authorities target a tax path which sets the linear combination of \( \hat{Y}_t \), \( \hat{\tau}^{VAT} \) and \( \hat{\Phi}_t \) equal to null. If fiscal authorities attach a high weight towards inflation stability as indicated by the loss function there is obviously no strong motive for output gap smoothing, as a decline in the output gap also stabilizes the inflation rate. This in particular prevails for the case of a simple rule where the tax path is not fine tuned towards the discretionary optimum.

The corresponding point estimates for \( \chi_1 \) are given in Table 3: We obtain \( \chi_1 = -7.34 \) with a standard error of 0.38, which implies that the implementation of the simple policy rule reduces the value of the loss function by 48 percent. The loss reduction is significant at the one percent level. The results are still robust over a large range for \( \chi_1 \) from -11.94 to -0.56.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>St.Dev.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction Coefficient</td>
<td>( \chi_1 )</td>
<td>-7.34</td>
<td>0.29</td>
<td>[-11.94, -0.56]</td>
</tr>
</tbody>
</table>

5 Relevance of the Tax-Rule

Markup shocks are costly in terms of welfare as monetary authorities lack an instrument on the supply side of the economy to cushion the adverse effects of cost pressure. Following the Tinbergen (1959) logic we have shown that a state contingent tax can improve welfare remarkably.

In the following we discuss the implications of these issues by computing welfare gains using different parameter constellations. This exercise has two main purposes. On the one hand we want to analyze whether the proposed rule is robust to perturbations of the baseline parametrization. On the other hand we present further insights why the rule works from a micro-founded perspective.

5.1 Robustness of the Optimal Tax Rule

Precisely speaking we compute the expected value of the loss \( E_0\{\sum_{t=0}^{\infty} \beta^t L_t \} \) for the active and the passive fiscal policy stance and then take the ratio of the two. If the ratio takes
the value one, then the loss would be equal under the two regimes. If the value of the ratio is below (above) one, then the loss under an active fiscal policy is smaller (larger) than the loss under the passive fiscal stance. By means of computing these ratios we succeed to uncover those parameter constellations which improve or worsen the relative performance of the proposed policy rule compared to the fallback position of a passive fiscal policy. The solid line indicates how the computed ratio changes when the parameter displayed at the top of the figure is altered, while the rest remains fixed at the baseline calibration. For each altered coefficient, e.g. for $\eta$, the coefficients in the fiscal policy rule $\chi_1$ and $\chi_2$ are reoptimized such that the welfare function (44) is minimized. The inverse of the 

**Figure 4: Recalibrating the Baseline Model – Loss Ratio**

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\phi$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Evolution of the expected loss ratio defined as the ratio of the expected loss if fiscal policy is active with $\chi_1 < 0$ compared to a passive stance $\chi_1 = 0$, $E_0 \left( \{ \sum_{t=0}^{\infty} \beta^t L_t \}^{\text{Active}} / \{ \sum_{t=0}^{\infty} \beta^t L_t \}^{\text{Passive}} \right)$. Appendix A summarizes the ranges of deep parameters typically found in the literature.

Frisch elasticity of labor supply $\eta$ was varied from one to four. The robustness analysis indicates that with an increase of $\eta$ the relative advantage of the policy rule increases from a loss reduction of 69% to a reduction of around 75%. This reflects that the welfare gains attached to rule (27) are larger if households dislike for variations in labor supply triggered by business cycle fluctuations increases, which are successfully stabilized by the rule itself.
With respect to the Taylor rule coefficient $\phi_\pi$ the relative benefit from the fiscal policy rule increases if monetary policy gets somewhat more aggressive on inflation. This reflects to a certain extend that a larger Taylor-rule coefficient $\phi_\pi$ implies that the real interest rate volatility and thus the variations in the consumption aggregate over time increase. One outstanding effect of the fiscal policy rule (27) is that it disencumbers monetary policy such that there is no need, even if monetary policy takes an aggressive stand towards inflation to move the real interest rate a lot.

The robustness analysis indicates that the relative advantage of the proposed policy rule decreases if monetary policy reacts stronger on the welfare gap. Nevertheless, the the loss reduction is still round about twenty percent, even for a coefficient of $\phi_x = 1$, which is higher than the values typically found in literature (e.g., Smets and Wouters, 2003). This might be explained by the fact that the proposed tax rule is successful in reducing inflation, but not so much in reducing output gap variability. This implies that a monetary authority that takes the output gap into account reintroduces real interest rate variability.

The performance of the rule worsens if interest rates are set in a highly inertial fashion. Nevertheless the ratio only deteriorates by 13% when $\phi_\rho$ increases from 0 to 0.75.

The effectiveness of the rule decreases with the degree of correlation in the markup shock $\zeta$. The higher the degree of correlation the larger will be the price dispersion inflicted upon the economy. Those firms that are called upon to reset prices will anticipate further shocks in the same direction which triggers a larger adjustment of prices. Therefore, the rule is welfare enhancing in an environment of correlated shocks as it promises to firms a stable evolution of prices and thus a limited degree of price dispersion for the economy. If, however, the degree of correlation in the markup shock becomes too large, fiscal authorities will not be able to change the tax rate sufficiently as equally solvency considerations have to be fulfilled, which prevents debt explosions.

With respect to the value of the Calvo parameter $\theta_P$ there exists a considerable disagreement in the literature. Del Negro et. al. (2005) for instance estimate an average price duration of three quarters for the euro-area using full information Bayesian techniques; Smets and Wouters (2003) estimate a price duration of 10 quarters. Gali, Gertler, and Lopez-Salido (2001) report a value round about four quarters using single equation GMM approach. Empirical work on price setting in the euro area using micro evidence report relatively low price durations with a median round about 3.5 quarters (see Alvarez et. al., 2006, for a summary of recent micro evidence). Comparable studies for the U.S. like Altig et. al. (2005) report much lower average price durations of just 1.6 quarters,
which they claim to be more consistent with recent evidence drawn from US micro-data. Based on this review of the literature it seems fair to conduct the robustness analysis in a range between 1.8 to 10 quarters, which corresponds to $\theta_p$ ranging between 0.45 to 0.90. The figure illustrates that the performance of the rule is almost constant when the degree of nominal stickiness increases from 0.45 to 0.9. The implementation of the policy rule prevents that a wedge can be driven between the production schedules and thus enhances welfare as the variability of inflation decreases. However, for a very high degree of price rigidity ($\theta_p > 0.80$) the ratio increases, as the fiscal rule loses its effectiveness.

5.2 Robustness of the Simple Rule

Figure 5 portrays that the evolution of the ratios in particular the baseline ratios are quantitatively almost identical under the modified rule (28), but the figure shows that the loss ratios are shifted upward for the baseline calibration around 20%. Note that there

Figure 5: Recalibration for the Simple Rule – Loss Ratio

![Graphs showing the recalibration for the Simple Rule - Loss Ratio](image)

**Notes:** Evolution of the expected loss ratio defined as the ratio of the expected loss if fiscal policy is active with $\chi_1 < 0$ compared to a passive stance $\chi_1 = 0$, $E_0\left(\left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Active}} / \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Passive}} \right)$. Appendix A summarizes the ranges of deep parameters typically found in the literature.

is one notable difference standing out: Variations in $\eta$ almost have no influence on the
reported ratios. This result can be traced back to the almost identical evolution for the output gap \( x_t \) under the simple rule and the passive policy stance. Put differently, as the simple tax rule is not so effective in stabilizing the output gap, the attached welfare gain does not increase in agents’ dislike business cycle fluctuations and thus variations in the labor supply more strongly.

6 Conclusions

In this paper we addressed the question whether fiscal policy can wipe out price dispersion by implementing a countercyclical tax rule. Our motivation stems from the fact that there is a large strand of literature which stresses the role of monetary policy to enhance welfare in an environment of nominal rigidities (Woodford, 2003). However this strand of literature has paid so far little attention to the question whether fiscal policy can improve welfare with respect to nominal frictions. In the event of cost push shocks Woodford (2003) shows that monetary policy faces a trade off between stabilizing the inflation rate and stabilizing the output gap. A sufficiently strong feedback from movements in the inflation rate is argued to be the best response to limit the adverse effects of cost-push shocks on lifetime utility of a representative consumer to generate a unique and determinate equilibrium. Notwithstanding these arguments, the costs of nominal rigidities are estimated to be still up to three percent in consumption equivalents (Canzoneri, Cumby and Diba 2007).

This highlights that monetary policy does not have a direct leverage on the supply side of the economy. Therefore, we followed the Tinbergen logic and proposed that fiscal policy should use its value-added tax, as an additional instrument in a state contingent way such that the evolution of marginal cost is stabilized around its deterministic steady state. Our findings suggest that a countercyclical taxation approach can remarkably reduce the impact of cost push shocks on welfare. The reduction in expected losses, when fiscal authorities switch from a passive towards an active fiscal stance are quantified around 69% for the optimal tax rule and 48% for the simple tax rule, and depend on the particular parameter settings. Key to the functioning of the tax-rule is that the fraction of firms that adjusts prices anticipates the promise of fiscal authorities to target a value-added tax path that eliminates any cost pressure at the firm level. Accordingly, those firms that are called upon to reset prices will set them in the neighborhood of those firms that leave prices unchanged. This prevents any inefficient built-up in prices across firms at any date \( t \).
The Keynesian tradition considers fiscal policy as operating over the aggregate demand effect. We showed that fiscal policy can use its distortionary instruments to unfold stabilizing effects on the economy upon an aggregate supply channel.
Appendices

A Calibrated Parameters

In Section 5 of the main text we conduct some sensitivity analysis to demonstrate the robustness of the proposed policy rule. While conducting this exercise we rely on ranges of the deep parameters chosen in a way to best represent the uncertainty found in the literature as reported in Table 3.

Table 3: Values and Ranges for the Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Household</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\sigma$</td>
<td>1.00</td>
</tr>
<tr>
<td>Inverse of the Labor Supply Elasticity</td>
<td>$\eta$</td>
<td>1.00 – 4.00</td>
</tr>
<tr>
<td><strong>B. Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price Elasticity of Demand for an Intermediate Good</td>
<td>$\varepsilon$</td>
<td>11.00 – 25.00</td>
</tr>
<tr>
<td>Price Stickiness</td>
<td>$\theta_p$</td>
<td>0.75 – 0.95</td>
</tr>
<tr>
<td><strong>C. Monetary Policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor Rule: Smoothing</td>
<td>$\phi_\rho$</td>
<td>0.50 – 0.75</td>
</tr>
<tr>
<td>Taylor Rule: Inflation</td>
<td>$\phi_\pi$</td>
<td>1.10 – 2.00</td>
</tr>
<tr>
<td>Taylor Rule: Welfare Gap</td>
<td>$\phi_x$</td>
<td>0.25 – 1.00</td>
</tr>
<tr>
<td><strong>D. Fiscal Authorities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiscal Rule (optimal): Mark-up shock</td>
<td>$\chi_1$</td>
<td>-1.60</td>
</tr>
<tr>
<td>Fiscal Rule (simple): Welfare Gap</td>
<td>$\chi_1$</td>
<td>-7.34</td>
</tr>
<tr>
<td>Fiscal Rule (both): Debt</td>
<td>$\chi_2$</td>
<td>0.06</td>
</tr>
<tr>
<td>Steady State VAT Level</td>
<td>$\bar{\tau}^{VAT}$</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>E. Exogenous Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mark-up Shock: Persistence</td>
<td>$\zeta$</td>
<td>0.75 – 0.90</td>
</tr>
</tbody>
</table>

Remarks: The table displays the calibrated values. The respective upper and lower bounds are taken from related studies in literature. The reviewed literature is Smets and Wouters, 2003; Leith and Maley, 2005; Rabanal, 2003; Coenen, McAdam and Straub, 2006; Del Negro, Schorfheide, Smets and Wouters, 2004; Welz, 2005; Linnemann and Schabert, 2003.
\[ x_t = E_t x_{t+1} - \sigma^{-1}(\phi_x \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \left(\frac{\gamma_G}{\gamma_C} \chi_1 + (\sigma + \eta)^{-1}\right) \hat{\Phi}_t, \]  
\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \left( (\sigma + \eta) x_t + (\nu - \sigma) \frac{\gamma_G}{\gamma_C} \hat{\Phi}_t \right), \]  
\[ \hat{R}_t = \phi_x \hat{\pi}_t. \]

The rest of the system is recursive and can be solved afterwards. Let us posit a fundamental (minimum state variable) solution of the following generic form (McCallum, 1983):

\[ \hat{\pi}_t = \delta_{\pi} \hat{\Phi}_t \]  
\[ x_t = \delta_x \hat{\Phi}_t, \]

where the coefficients \( \delta_{\pi} \) and \( \delta_x \) remain to be determined. With 
\[ E_t x_{t+1} = E_t \delta_x \hat{\Phi}_{t+1} = 0 \]  
and 
\[ E_t \hat{\pi}_{t+1} = E_t \delta_{\pi} \hat{\Phi}_{t+1} = 0, \]

this leads to the following conditions for the undetermined coefficients:

\[ \delta_{\pi} = \kappa (\sigma + \eta) \delta_x + \kappa (\nu - \sigma) \frac{\gamma_G}{\gamma_C} \chi_1, \]  
\[ \delta_x = -\sigma^{-1} \phi_x \delta_{\pi} + (\sigma + \eta)^{-1} + \frac{\gamma_G}{\gamma_C} \chi_1. \]

Inserting (B.5) into (B.4) yields

\[ \delta_{\pi} = [1 + \kappa (\sigma + \eta) \sigma^{-1} \phi_x]^{-1} \cdot \kappa [1 + \gamma_G \gamma_C^{-1} \chi_1 (1 + \eta)], \]

and

\[ \delta_x = \frac{\sigma \gamma_C + (\sigma + \kappa (\sigma - \nu) \phi_x) (\sigma + \eta) \gamma_G \chi_1}{(\sigma + \eta) (\sigma + \kappa \phi_x (\sigma + \eta)) \gamma_C}. \]

**Balanced budget and passive policy**

Let us define the neutral benchmark system as \( \hat{\pi}_t = \hat{\pi}^{VAT}_t = 0 \). Then the model can be stated as:

\[ x_t = E_t x_{t+1} - \sigma^{-1}(\phi_x \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + (\sigma + \eta)^{-1} \hat{\Phi}_t, \]  
\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\sigma + \eta) x_t, \]

where the MSV solution reads:

\[ \delta_x = [1 + \sigma^{-1} \phi_x \kappa (\sigma + \eta)]^{-1} (\sigma + \eta)^{-1}, \]

and

\[ \delta_{\pi} = \kappa [1 + \sigma^{-1} \phi_x \kappa (\sigma + \eta)]^{-1}. \]
Comparison of active versus passive fiscal policy  In the following, we compare the MSV solutions for an economy where fiscal policy implements policy rule (26) versus an economy where fiscal policy remains passive with $\dot{G}_t = \dot{\tau}^{VAT} = 0$. The superscript $P$ denotes passive whereas the superscript $A$ denotes active.

Inflation:

$$\delta^P_\pi > \delta^A_\pi$$

$$\Rightarrow \kappa[1 + \kappa \sigma^{-1}(\sigma + \eta)\phi_\pi]^{-1} > \kappa[1 + \kappa \sigma^{-1}(\sigma + \eta)\phi_\pi]^{-1}[1 + \gamma_G \gamma_C^{-1} \chi_1(1 + \eta)]$$

$$\Rightarrow 1 > 1 + \gamma_G \gamma_C^{-1} \chi_1(1 + \eta)$$

$$\Rightarrow 0 > \gamma_G \gamma_C^{-1}(1 + \eta) \chi_1 \Rightarrow \chi_1 < 0, \eta, \gamma_G, \gamma_C > 0$$

Welfare gap:

$$\delta^P_x > \delta^A_x$$

$$\Rightarrow [1 + \kappa \sigma^{-1}(\sigma + \eta)\phi_\pi]^{-1}(\sigma + \eta)^{-1} \sigma \gamma_C + (\sigma + \kappa(\sigma - \iota)\phi_\pi)(\sigma + \eta)\gamma_G \chi_1$$

$$\Rightarrow \sigma \gamma_C > \sigma \gamma_C + (\sigma + \kappa(\sigma - \iota)\phi_\pi)(\sigma + \eta)\gamma_G \chi_1$$

$$\Rightarrow 0 > (\sigma + \kappa(\sigma - \iota)\phi_\pi)(\sigma + \eta)\gamma_G \chi_1$$

$$\Rightarrow \chi_1 < 0, \gamma_G, \kappa, \sigma, \iota, \eta > 0$$

\[\Box\]

C Utility-Based Welfare Function

The utility function is given by:

$$U(C, N, G) = (1 - \tau) \log C + \tau \log G - \frac{N^{1+\eta}}{1 + \eta}. \quad (C.1)$$

Note that the weight $\tau$ in the utility function is equal to the steady state share of government spending $\tau = G/Y$. Taking a second-order approximation around the consumption part of the utility function yields:

$$\log(C_t) = \log(Y_t - G_t) = \frac{1}{1 - \tau}(\hat{y}_t - \tau \hat{y}) - \frac{1}{2} \frac{\tau}{(1 - \tau)^2}(\hat{y}_t - \hat{y})^2 + tip + o(||a^3||). \quad (C.2)$$
Where it holds that: $\hat{x}_t = \tilde{x}_t + (\bar{x}_t - x)$. We denote the gap $\hat{y}_t = y_t - \tilde{y}_t$ and the fiscal gap $\hat{g}_t = g_t - \tilde{g}_t$. Note that $\hat{y}_t$ comprises the sum of the deviation of output from the distorted (short term) steady state and the deviation of the distorted steady-state output from the efficient long-term steady state. Taking a second-order approximation around the disutility of labor term yields:

$$\frac{N^{1+\eta}}{1+\eta} = \hat{n}_t + \frac{1}{2} \tilde{n}_t^2 + tip + o(||a^3||).$$  \hspace{1cm} (C.3)

We find the relationship $N_t = Y_t Q_t$, which is derived in the following:

$$N_t = \int_0^1 N_t(i)\,di = \int_0^1 Y_t(i)\,di = Y_t \int_0^1 \frac{Y_t(i)}{Y_t} \,di.$$

$$\Rightarrow N_t = Y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} \,di = Y_t Q_t.$$  \hspace{1cm} (C.4)

After log linearization, we obtain:

$$\hat{n}_t = \hat{y}_t + q_t.$$  \hspace{1cm} (C.6)

Where $q_t = (\varepsilon/2)\sigma_t^2$ and $q_t$ is defined as:

$$q_t \equiv \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} \,di.$$  \hspace{1cm} (C.7)

The intertemporal welfare function is given by the discounted sum of the approximated utility functions:

$$W_t = \sum_{t=0}^{\infty} \beta^t U_t(C_t, G_t, N_t) = \sum_{t=0}^{\infty} \beta^t \left[ (1 + \eta)\hat{y}_t^2 + \iota(\hat{g}_t - \tilde{g}_t)^2 + \varepsilon\sigma_t^2 \right].$$  \hspace{1cm} (C.8)

Now, we aim at expressing $\sigma_t^2$ in terms of $\pi_t^2$ while following the proof given by Woodford 2003:

$$\sum_{t=0}^{\infty} \beta^t \sigma_t^2 = \sum_{t=0}^{\infty} \beta^t \left[ tip + \sum_{s=0}^{t} \frac{\theta_P^{t-s}}{1-\theta_P} \hat{\pi}_s^2 + o(||a^3||) \right]$$

$$= \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2 + tip + o(||a||^3).$$  \hspace{1cm} (C.9)

Using this result (C.8) can be rewritten as follows:

$$W_t = \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon}{\kappa} \hat{\pi}_t^2 + (1 + \eta)\hat{y}_t^2 + \iota(\hat{g}_t - \tilde{g}_t)^2 \right].$$  \hspace{1cm} (C.10)
D Matrix Notation of the Model

The linearized equilibrium dynamics can be represented as follows (Söderlind, 1999):

\[
A_0 \begin{bmatrix} X_{1,t+1} \\ E_tX_{1,t+2} \end{bmatrix} = A_1 \begin{bmatrix} X_{1,t} \\ E_tX_{1,t+1} \end{bmatrix} + B \hat{R}_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2,t+1} \end{bmatrix}, \quad \text{and} \quad \hat{R}_t = F \begin{bmatrix} X_{1,t} \\ E_tX_{1,t+1} \end{bmatrix},
\]

with \( X_{1,t+1} = [\hat{\Phi}_{t+1} \ \hat{R}_t \ b_t \ \hat{\pi}_t^{VAT} \ \hat{\pi}_t^n b_t'] \), and \( E_tX_{1,t+1} = [E_tx_{t+1} \ E_t\hat{\pi}_{t+1}] \). The matrices \( A_0, A_1 \) and \( B \) are given by:

\[
A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_G & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \gamma_G & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \gamma_G & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
A_1 = \begin{bmatrix}
\zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\gamma_G \Gamma_{\varphi}^{-1}(\gamma_{\varphi} - 1) & \beta^{-1} & 0 & \gamma_G \Gamma_{\varphi}^{-1}(1 - \gamma_{\varphi}) \chi_2 & \gamma_G(\delta_{\varphi} - 1) & 0 \\
0 & \chi_1 & 0 & \chi_2 & 0 & 0 & 0 & 0 & 0 \\
\sigma(1 - \gamma_{\varphi}) \Gamma_{\varphi}^{-1} & 0 & 0 & 0 & \sigma \Gamma_{\varphi}^{-1}(1 - \gamma_{\varphi}) \chi_2 & 0 & 0 & 0 & 0 \\
-\gamma_G(\gamma_{\varphi} - 1) \Gamma_{\varphi}^{-1} & 0 & 0 & 0 & \beta^{-1} - (\gamma_G(\delta_{\varphi} - 1)) \Gamma_{\varphi}^{-1} + \gamma_G \chi_2 & 0 & 0 & 0 & 0 \\
0 & \chi_1 & 0 & \chi_2 & 0 & 0 & 0 & \sigma \chi_2 & -\sigma \chi_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma \chi_2 & -\sigma \chi_2 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 - \phi_{\varphi} \phi_x & (1 - \phi_{\varphi}) \phi_x \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
0 & \phi_{\varphi} & 0 & 0 & 0 & 0 & (1 - \phi_{\varphi}) \phi_x & (1 - \phi_{\varphi}) \phi_x
\end{bmatrix}
\]
References


