Monetary Policy, Rule-of-Thumb Consumers and External Habits: An International Comparison*

Giovanni Di Bartolomeo  
*University of Teramo and University of Crete  
gdibartolomeo@unite.it

Lorenza Rossi  
University Cattolica del Sacro Cuore, Milan  
lorenza.rossi@unicatt.it

Massimiliano Tancioni  
University of Rome La Sapienza  
massimiliano.tancioni@uniroma1.it

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Abstract. This paper extends the standard New Keynesian dynamic stochastic general equilibrium (DSGE) model to agents who cannot smooth consumption (i.e. spenders) and are affected by external consumption habits. Although these assumptions are not new, their joint consideration strongly affects some theoretical and empirical results addressed by the recent literature. By deriving closed-form solutions, we identify different demand regimes and show that they are characterized by specific features regarding dynamic stability and monetary policy effectiveness. We also evaluate our model by stochastic simulations obtained from the Bayesian parameters estimates for the G7 economies. From posterior impulse response we address the empirical relevance of the different regimes and provide comparative evidence on the asymmetric effects of monetary policy, resulting from the heterogeneity of the estimated model structures.

Keywords: Rule-of-thumb, habits, monetary policy transmission, determinacy, New Keynesian DSGE model, monetary policy, Monte Carlo Bayesian estimators.

JEL codes: E61, E63.

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1. Introduction

This paper extends the standard New Keynesian DSGE model by considering that some agents may not be able to smooth consumption and may have consumption habits. Although both assumptions are not new in the literature, the joint consideration of these two features of consumption behavior strongly affects the results addressed by the recent literature, both theoretically and empirically.

Evidence on the existence of heterogeneous consumers has been first provided, nearly fifteen years ago, by Campbell and Mankiw (1989, 1990, 1991). According to them, only a fraction of households (savers) is able to plan consumption along with the standard Hall’s consumption function, while a relevant fraction of households (spenders) equates current consumption to current income period by period (violating the permanent income hypothesis).1

The policy implications of the introduction of spenders in the model are important. Considering fiscal policy, if consumers not able to smooth consumption, the Barro-Ricardo equivalence does not hold. For this reason, savers are often referred to as Ricardian consumers and spenders as non-Ricardian consumers.2

Recently, economists have instead focused on the effects of spenders on monetary policy by considering agents that consume according to a rule-of-thumb behavior within the New Keynesian theoretical apparatus. They find that the presence of rule-of-thumb consumers may overturn some of the conventional policy prescriptions addressed by the literature.

Gali et al. (2004), e.g., explore the Taylor rule properties when considering rule-of-thumb households and show that the Taylor principle 1) may be not a sufficient criterion for stability when there are many rule-of-thumb consumers; 2) becomes a sufficient but non necessary condition for stability when monetary policy is set according to a standard (feedback) Taylor rule. Instead, in the

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1 Spenders’ behavior can be interpreted in various ways. One can view their behavior as resulting from consumers who face binding borrowing constraints. Alternatively, myopic deviations from the assumption of fully rational expectations should be assumed (rule-of-thumb), i.e. consumers naively extrapolate their current income into the future, or weigh their current income too heavily when looking ahead to their future income because current income is the most salient piece of information available. See Mankiw (2000) and references therein. Note that whatever the reason why some agents do not smooth consumption, their analytical modeling is however similar and for this reason we will generically refer to rule-of-thumb consumers to include both categories of non-smoothing consumers.

2 See Mankiw (2000) and Muscatelli et al. (2006).
case of forward-looking interest rate rules, conditions for a unique equilibrium are somewhat different from the usual ones.

Amato and Laubach (2003) explore the optimal monetary rule when considering rule-of-thumb households and firms. By modeling consumers’ rule-of-thumb behavior as a consumption habit, households’ current decisions mimic past behavior of all agents (including optimizing agents). They show that, while the monetary policy implications of rule-of-thumb firms are minimal, the presence of rule-of-thumb consumers alters the determination of the optimal interest rate. As their fraction increases, higher inertial monetary policy is required.

Similar results are found by Di Bartolomeo and Rossi (2007), who however focus on the effectiveness of monetary policy. They find that, although an increase in consumers who cannot access to the financial markets (and thus cannot smooth consumption) reduces effects of interest rate policies via the consumption inter-temporal allocation (according to the permanent income effect), monetary policy becomes more effective as the degree of financial markets participation falls. In fact, after a change in the interest rate, spenders and savers revise their consumption plans in the same direction, because the fall of the interest rate supports the increase of current output also by affecting spenders’ consumption through higher real wages (staggered prices in fact imply a decline in the mark-up after an initial increase in economic activity; this allows real wages to increase, leads to a boom in rule-of-thumb consumption, generates inflation and improves the effectiveness of monetary policy).

By using a simplified version of Gali et al. (2004), Bilbiie (2005) and Di Bartolomeo and Rossi (2005) find that, for high fractions of rule-of-thumb (ROT) consumers, the interest rate increase becomes expansionary, thus showing that two-demand regimes can emerge (according to the “slope” of IS curve).³

On the empirical side, rule-of-thumb consumption has been considered consistent with the puzzling

³ More specifically, Bilbiie (2006) addresses the implications of limited asset market participation for optimal monetary policy, from both a theoretical and empirical³ point of view. His main finding is that when limited asset market participation is considered a passive interest rate rule is consistent with a welfare-maximizing monetary policy. In this context, a passive policy does not lead to indeterminacy.
result of a weak or positive relationship between expected consumption growth and real interest rates (Ahmad, 2005, Bilbiie 2006, Canzoneri et al., 2006). The empirical relevance of the New Keynesian DSGE theoretical predictions is however still ambiguous, since its evaluation has been generally obtained by estimating reduced-form forward-looking IS curves, whose coefficients are only a convolution of the deep parameters.4

The contribution of this paper is to extend the aforementioned literature both theoretically and empirically. We firstly derive an analytical closed-form solution of the model and study its stability regions, and then we evaluate the model by stochastic simulations, obtained from Bayesian parameters estimates for the G7 economies.

In order to derive a closed-form solution of the model, as in the standard New-Keynesian models, we do not consider capital accumulation. This also allows us to simply discuss the dynamic properties of the model. According to the fraction of spenders, we analytically discriminate between two demand regimes (i.e. two IS-curves), defined according to the response of the aggregate demand to nominal interest rate movements. We show that the possibility of a demand regime shift has a dramatic importance for the analysis of monetary policy effectiveness (discussed by Amato and Laubach, 2003; Di Bartolomeo and Rossi, 2005, 2007). We then show that the consideration of external habits reduces the probability of obtaining a regime shift in the demand schedule, as it increases the threshold fraction of spenders above which an inversion of the slope of the IS-curve is obtained.5

The possibility of a demand regime shift has remarkable implications for the analysis of equilibrium determinacy, as discussed in Bilbiie (2005, 2006) and Di Bartolomeo and Rossi (2005). On this respect we show that, the unconventional results stressed by Gali et al. (2004) hold only if the relationship between the nominal interest rate and the aggregate demand is positive, i.e. when the

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4 Fuhrer and Olivei (2004) provide empirical evidence for the parameters of a reduced form IS equation, defined in a standard New Keynesian model augmented with habits. Sensitivity of income to changes in the real interest rate is weakly negative or insignificant. Bilbiie (2005) explicitly deals with the issue of the monetary policy implications of the presence of relevant liquidity constraints in consumption behavior. Even in this case, the use of a reduced-form IS curve does not allow a direct estimate of the fraction of rule-of-thumb consumers.

5 In this case the numerical solution of the model is needed, since the joint consideration of external habits and of ROT consumers increases the complexity and nonlinearity of the model.
The IS-curve is positively sloped.

The second stage of the analysis focuses on the empirical evaluation of the theoretical predictions of the model. Our investigation aims to evaluate the empirical relevance of the regime inversion from a direct estimate of the structural parameters of the model. Moreover, our analysis aims to providing an assessment of the heterogeneous effects of monetary policy. The values of the structural parameters are not calibrated or fixed on the basis of previous evidence, as in the standard practice. Because of our strong empirical bearing, we estimate the structural coefficients employing quarterly data for the seven most industrialized economies (G7) for the 1963-2003 period. Differently from the common practice emerging in recent studies (see Smets and Wouters, 2003; Coenen and Straub, 2005), we consider country-level data separately in order to stress the cross-country heterogeneity. The complexity and nonlinearity of the resulting structure of the model suggests the implementation of a Bayesian Monte-Carlo Markov Chain estimation procedure (MCMC).\(^6\)

The remainder of the paper is organized as follows. Section 2 outlines the basic theoretical framework and describes the two demand regimes implied by the presence of rule-of-thumb consumers and external habits; it further discusses the properties of the model by closely analyzing the rational expectation equilibrium determinacy and the transmission mechanism of monetary policy. Section 3 provides the details of the empirical evaluation of the model and the interpretation of the main results. Section 4 concludes.

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\(^6\) Our analysis is close in spirit to the strategy proposed by Smets and Wouters (2003) for the estimation of their New Keynesian model. The main differences with respect to their analysis are that we do not consider capital accumulation and that we introduce non-Ricardian consumers.
spenders’ behavior we do not consider the capital accumulation process. The economy is populated by a continuum of infinitely-lived heterogeneous agents normalized to one. A fraction \(1 - \lambda\) of them consumes and accumulates wealth as in the standard setup (savers). The remaining fraction \(\lambda\) is composed by agents who do not own any asset, cannot smooth consumption, and therefore consume all their current disposable income (spenders). We also assume that savers consumption at time \(t + i\) depends on habits inherited from past consumption, i.e. on a fraction \(\gamma\) of lagged aggregate consumption. Type-specific representative consumers are indexed by \(R\) (Ricardian or savers) and \(N\) (non-Ricardian or spenders). At the date zero, they plan to maximize the following utility:

\[
E_i \sum_{j=0}^{\infty} \beta^j u\left(C_{t+i}^j, N_{t+i}^j, \phi^j\right) \quad j \in \{R, N\}
\]

where \(\beta \in (0,1)\) is the discount factor, \(C_i\) is household consumption at time \(t\), while \(N_i\) is labor supply. \(\phi^j\) is a binary variable such that when \(j = R\), \(\phi^R = 1\) and when \(j = N\), \(\phi^N = 0\).

Regarding the functional form, we assume logarithmic utility to permit the analytical derivation of the closed-form solution of the model. Although simplistic, the logarithmic utility hypothesis allows the derivation of the log-linear model representation without having to impose the evenly restrictive assumption of equal income between savers and spenders in the steady state. The instantaneous utility is thus:

\[
u(\cdot) = \ln\left(C_{t+i}^j - \gamma \phi^j C_{t+i-1}^j\right) + \kappa \ln\left(1 - N_i^j\right)
\]

where \(0 < \gamma \leq 1\) and \(\kappa > 0\) are parameters. The former measures the impact of the consumption habits, the latter measures labor disutility with respect to consumption.

In addition, the following budget constraint holds:

\[
C_i^j = \frac{W_i}{P_i} N_i^j + \phi^j \left[\Pi_i^j - \frac{B_i^j - (1 + i_{t+1}) B_{t+1}^j}{P_i}\right]
\]

where \(W_i\) is the nominal wage and \(\Pi_i\) is profit sharing, \(B_i\) represents the quantity of one-period nominally risk-less discount bonds purchased in period \(t\), and maturing in period \(t+1\) and paying a net interest rate equal to \(i\).
Real wages are the unique source of spenders’ disposable income; therefore, they are subject to a static budget constraint, while savers face a standard dynamic constraint. Since spenders do not save, they consume all their current income.

By solving the inter-temporal optimization problems of savers and spenders, aggregating, and then linearizing around the steady-state, we obtain the following description of the demand side of the economy:

\[
\begin{align*}
(4) \quad c_t &= -\frac{1-\sigma - \lambda \zeta^N}{1+\sigma} (i_t - E_t \pi_{t+1}) + \frac{1}{1+\sigma} E_t c_{t+1} + \frac{\sigma}{1+\sigma} c_{t-1} - \frac{\lambda \zeta^N}{1+\sigma} E_t \Delta (w_{t+1} - p_{t+1}) \\
(5) \quad w_t - p_t &= \nu m_t + (1-\sigma) c_t - \sigma (1-\sigma) c_{t-1}
\end{align*}
\]

where \(c_t\) is consumption, \(i_t\) is the nominal interest rate, \(\pi_t\) is the inflation rate and \(w_t - p_t\) is the real wage. Concerning parameters, \(\sigma = \gamma (1-\lambda)\) is the “aggregate” habit parameter, since consumption habits are only relevant for savers; \(\nu = N (1-N)^{-1} = \theta \kappa^{-1} (1-\sigma)^{-1}\) is the inverse Frish elasticity; \(\theta = (\eta - 1) \eta^{-1} \in (0,1)\) is inverse mark-up, which in turn depends on the elasticity of substitution among intermediate goods \(\eta\); \(\kappa\) indicates labor disutility, and \(\zeta^N = \kappa (1+\kappa)^{-1} (1+\nu) (1-\sigma)\) is the share of spenders’ consumption at the steady state.

Equation (4) is a modified version of the standard consumption Euler equation while equation (5) describes the aggregate labor supply. Our Euler equation (4) differs from the standard version considering habits formation only, since consumption also depends on expected changes in the real wage. The economic rationale is that the presence of savers establishes a relationship between the demand for goods and the real wage.

Considering the economy production function \(y_t = a_t + n_t\) (\(a_t\) is a technology level variable), the resource constraint, \(y_t = c_t\) and equation (5), the consumption Euler equation (4) can be expressed as a modified IS-curve:

\[
(6) \quad y_t = \frac{1 - \lambda \zeta^N [(1+\nu) + \sigma]}{1 + \sigma - \lambda \zeta^N (1+\nu - \sigma^2)} E_t y_{t+1} + \frac{\sigma [1-(1+\sigma) \lambda \zeta^N]}{1 + \sigma - \lambda \zeta^N (1+\nu - \sigma^2)} y_{t-1} +
\]
The next subsection will discuss equation (6) in more detail.

As in the standard New-Keynesian framework, the supply side of the economy is described by a continuum of firms producing differentiated intermediate goods for a perfectly competitive final goods market. Intermediate sector firms cannot adjust their prices period by period; conversely, each firm in each period faces a certain probability of being able to do it (the Calvo’s lottery). Thus, in setting their price firms consider the future marginal costs (by considering inflation expectations) in addition to the current marginal cost. As a result, the price-adjustment mechanism is described by the following forward-looking relationship:

\( \pi_t = \beta E_{t+1} \pi_{t+1} + \tau mc_t \)

with \( \tau = (1-\varphi)(1-\beta\varphi)^{-1} \). The parameter \( \varphi \) defines the degree of price staggering, i.e. the fraction of firms maintaining their price fixed each period. By considering labor as the sole input of the intermediate sector and a standard linear production function, the sticky-price equilibrium real marginal cost is given by:

\( mc_t = \frac{1+\nu(1-\sigma)}{1-\sigma} y_t - \frac{\sigma}{1-\sigma} y_{t-1} - (1+\nu) a_t. \)

Since we assume that markup is constant at the steady-state, under flexible-price equilibrium the linearized real marginal costs are zero. Substituting (8) in (6) and solving for \( y_t \) we obtain the natural rate of output, i.e. output under flexible-price equilibrium \( y_t^f \),

\( y_t^f = \frac{(1+\nu)(1-\sigma)}{1+\nu(1-\sigma)} a_t + \frac{\sigma}{1+\nu(1-\sigma)} y_{t-1}^f. \)

The flexible-price output is a weighted average of technology and of its past value. The inertial component of output is increasing in the aggregate habit parameter and decreasing in the inverse Frisch labor elasticity. Thus, the introduction of rule-of-thumb consumers reduces the role played by the inertial component in the natural rate of output adjustment process since it reduces the
aggregate habit parameter. If habit persistence is not present, equation (9) collapses to the standard natural output equation.

Considering equations (8) and (9), from the price adjustment equation (7) we derive the New Keynesian Phillips curve:

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{\tau (\kappa + \theta)}{\kappa (1 - \sigma)} (y_t - y_t^f).
\]

Notice that if we assume nonzero habit persistence in consumption, the fraction of spenders affects the coefficient for the inflation response to the output gap, otherwise it has no role.

Model dynamics is fully described by three equations: the demand side, i.e. the modified IS curve (6), the supply-side, i.e. the New Keynesian Phillips curve (10), and equation (9), defining the flexible-price natural rate of output.

The assumption of rule-of-thumb consumers crucially affects the first of the above relationship that, in turn, is relevant for the dynamic and stability properties of the model, as we are going to show in the next two subsections.

2.2. Demand regimes and rule-of-thumb consumers

The existence of spenders has serious implications for the determination of the size and sign of the relationship between the demand and the nominal interest rate. Ceteris paribus by increasing the fraction of rule-of-thumb consumers, we can generate an inversion in the sign of the interest rate impact on aggregate demand, i.e., on aggregate income.

According to the sign of the interest rate elasticity, defined as \( \Omega = \partial y_t / \partial i_t \), equation (6) identifies two different demand regimes:

- A standard demand regime – which implies a standard negatively sloped IS curve – holds if the interest rate elasticity is positive (since it enters with a negative sign). Such a regime is consistent with the life-cycle permanent income hypothesis and thus with consumption smoothing;
• An inverse demand regime – which implies a positively sloped IS curve – holds if the interest rate elasticity is negative. In other words, the demand regime is dominated by rule-of-thumb behavior; an increase in real interest rates is expansionary and interest rate cuts imply demand contractions.

As in Di Bartolomeo and Rossi (2005), if external habits are not present, i.e. $\gamma = 0$ or $\sigma = 0$, $\zeta^N$ and $\nu$ do not depend on the fraction of rule-of-thumb consumers and $y_i = a_i$ and it is easy to show that the emergence of a particular demand regime only depends on a threshold value of $\lambda$.

It can be shown that the standard regime holds for:

$$\lambda < \lambda^* = \frac{1}{\zeta^N (1 + \nu)} = \frac{\kappa (1 + \kappa)}{(\kappa + \theta)^2}. \quad (11)$$

If inequality (11) is not satisfied, the inverse demand regime emerges. For relatively low values of $\theta$ (thus for low values of the elasticity of substitution among intermediate goods) and high values of labor disutility $\kappa$, the threshold value can be greater than one ($\lambda^* > 1$). In such a case, only the standard regime occurs since $\lambda \in [0,1]$. For relatively high values of $\theta$ and low values of $\kappa$, the liquidity-constrained regime can emerge. Notice that if $\theta$ is greater than 0.5 $\lambda^*$ is always smaller than one. Thus, in such a likely case, the inverse regime can emerge for sufficiently high values of $\lambda$.

The intuition of the regimes can be explained by comparing the main macro frameworks based on general equilibrium model and considering that the interest rate has a direct effect (by the agent decision of smoothing) and an indirect effect (by the labor market and real wage) on consumption of output. More in detail, in a standard real business cycle model an increase of the interest rate generates a reduction in current consumption (direct effect) and an increase in the labor supply of the agents who aim to increase their savings; as result the real wage falls (i.e. the labor market indirect effect of the change of the interest rate on consumption has the same direction of the direct

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7 The $\theta$ parameter is bigger than 0.5 if the intermediate goods elasticity of substitution $\eta$ is bigger than 2. The inverse regime is thus possible for mark-up values below 100%.
effect). By introducing nominal rigidities, as in New Keynesian DSGE models, the increase of the interest rate still generates a reduction in current consumption and an increase in the labor supply, but because of the price stickiness it also implies a markup fall and deflation. Labor demand also shifts and real wage increases instead of falling. However, in a standard New Keynesian model the real wage effect is small and as result consumption fall even if less than in the real business cycle case. By introducing the rule-of-thumb consumer the effect of the real wage increases is amplified, a small increase of the real wage has strong effect on current consumption since spenders do not save. Thus, when the spender fraction is high, the indirect effect via labor market can dominate the direct effect of an increase of the interest rate, which now support an higher consumption (output) level instead of a lower one.

The effects of the spenders’ fraction on regime inversion are non linear. On the one hand, a reduction in the savers’ fraction supports a more strong impact of the real wage on consumption (because spenders are more), but, on the other hand, it also reduces real wage increase caused by a positive change in the interest rate. When external habits are taken into account the study of the sign of the interest rate elasticity is even more complex, since the parameters $\zeta^N$, $\nu$ and $\sigma$ also depend on $\lambda$. Thus, the analytical derivation of the conditions for regime shifts becomes problematic. An implicit condition for observing the standard regime can be derived:

(12) $\lambda < \frac{(1-\sigma^2)\kappa(1+\kappa)}{[\kappa(1-\sigma)(1-\sigma^2)+\theta][\kappa(1-\sigma)+\theta]}$

From the expression above it is clear that if $\theta$ increases the inverse regime is more likely to emerge for given values of rule-of-thumb consumption. The effects of labor disutility $\kappa$ and of the “aggregate” habit parameter $\sigma$ are more ambiguous since they appear both at the numerator and denominator of the ratio therefore the sign of derivatives of (12) with respect of $\kappa$ and $\sigma$ is hard to be derived analytically. However, by numerical simulations we obtain that for high values of labor disutility the inverse regime is never observed; most importantly, we also obtain that, ceteris paribus, the threshold value of $\lambda$ needed to obtain the regime inversion increases with the value of
the habit parameter. Thus, the consideration of external habits in the analysis reduces the probability of observing a shift in the demand regime. The intuition of the result is that habits reduce the savers’ smoothing behavior (in this case, they also look at the past values of aggregate consumption) after a change in the interest rate and thus reduce also the effects of the interest rate on the real wage, which will increase less after a positive change in the interest rate.

### 2.3 Demand regimes and equilibrium determinacy

The recent literature on central banking has shown that one of the fundamental tasks of the monetary authority is to support rational expectation equilibrium determinacy. In order to close the model and study determinacy, we consider the following simple feedback Taylor rule:

\[
\begin{align*}
\pi_t &= \alpha_0 + \alpha_1 y_t + k_t,
\end{align*}
\]

where \( \alpha_0 \) and \( \alpha_1 \) are positive parameters and \( k_t \) is a constant or a stochastic term (representing a stationary disturbance process), which does not affect the conditions for determinacy.

Determinacy depends on two factors: the particular demand regime and monetary policy effectiveness. These factors correspond to, respectively, the sign and the size of the elasticity of income with respect to the interest rate. As previously stated, a positive (negative) sign occurs in the standard (inverse) regime; policy effectiveness increases in the elasticity modulus, i.e. \( |\Omega| \).

Under a contemporaneous Taylor rule, in the standard regime, determinacy requires an active policy rule satisfying:

\[
\begin{align*}
\alpha_1 > 1 - \frac{1 - \beta}{k} \alpha_2,
\end{align*}
\]

where \( k = \frac{\gamma_{\pi}(1 - \theta)}{\sigma_{\pi}(1 - \alpha)} \) is the elasticity of the price adjustment with respect to the real output (see equation (10)). The condition has the usual interpretation: a rule satisfies the Taylor principle if, in the event of an increase of the inflation rate by one percentage point, the nominal interest rate is

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8 John Taylor has proposed that the Fed monetary policy can be described by a rule as that considered here (see Taylor, 1993). Note also that the Taylor rule can be used to study the determinacy properties of an endogenous policy derived from the so-called flexible inflation targeting approach (Svensson, 1999; Evans and Honkapohja, 2006) or from utility-based welfare maximization (Woodford, 2003: Ch. 6).
raised by more than one percentage point. Each percentage point of permanent increase in the inflation rate implies an increase in the long-run average output gap of \((1 - \beta)k^{-1}\) percent. An exogenous Taylor rule thus satisfies the Taylor principle if and only if \(a_i + (1 - \beta)k^{-1}a_z > 1\) (see Woodford, 2004).

In the inverse demand regime, determinacy requires (see Appendix A):

\[
(15) \quad a_i > \max \left\{ 1 - \frac{1 - \beta}{k} a_z, \left( \frac{2}{\Omega} - a_z \right) \frac{1 + \beta}{k} - 1 \right\}
\]
or

\[
(16) \quad a_i < \min \left\{ 1 - \frac{1 - \beta}{k} a_z, \left( \frac{2}{\Omega} - a_z \right) \frac{1 + \beta}{k} - 1 \right\}
\]

From equation (15) we obtain that a rule satisfying the Taylor Principle can result insufficient for model determinacy. Thus a more aggressive rule, i.e. a rule that strongly react to current inflation, may be requested. Equation (16) implies that even a passive policy can lead to determinacy. However, if both values between brackets on the r.h.s. of the last term are negative, the equilibrium is always indeterminate. This occurs if the central bank places a high weight to output stabilization.

In the standard regime, the Taylor principle is thus the necessary and sufficient condition for determinacy. By contrast, in the alternative regime, we have to consider three different cases. More specifically, determinacy may be related to the monetary policy effectiveness as follows.

1. For a relative high effectiveness of monetary policy, i.e. \(\Omega > \frac{1 + 3\beta}{k + \beta a_z}\), the Taylor principle is a necessary and sufficient condition for determinacy.

2. For relative medium effectiveness, i.e. \(\Omega \in \left( \frac{1 + \beta}{k + \beta a_z}, \frac{1 + 3\beta}{k + \beta a_z} \right)\), the Taylor principle is a sufficient condition for determinacy, even if not necessary, since a loose policy implies determinacy as well.

3. Finally, if monetary policy has a relatively low effectiveness (i.e. \(\Omega < \frac{1 + \beta}{k + \beta a_z}\)), the satisfaction of the Taylor principle is neither a necessary nor a sufficient condition for determinacy. In fact,
even a more aggressive policy condition leading to determinacy would be only sufficient, since under low effectiveness all passive policies lead to determinacy in a sort of “inverse Taylor principle”.

The rationale of the inverse Taylor principle is that a positive non-fundamental shock in expectations reduces the real interest rate; in the liquidity-constrained regime this implies that, if the interest rate does not change at all or is set according to a passive rule, i.e. it does not increase a lot, output falls (by the aggregate demand), inflation decreases (by the aggregate supply), and expectations are thus not self fulfilled. By contrast, if the interest rate increases a lot (e.g. it satisfies the Taylor principle), the real interest rate will increase, then output and inflation will also increase and non-fundamental shock in expectations will be self-fulfilled.

2.4. Monetary policy effectiveness and transmission mechanisms

As long as different demand regimes can emerge, different policy regimes (i.e. transmission mechanisms of the monetary policy) may be required for model (price) stability. Policy regimes are related to both the sign of the elasticity of demand with respect to the real interest rate (demand regime) and to its size (monetary policy effectiveness). If monetary policy is set according to a Taylor rule of the kind of equation (13) augmented with a white-noise shock (i.e. a monetary policy innovation), three different policy regimes can be identified (see Appendix B).

1. In the standard demand regime ($\Omega < 0$) a positive policy shock has deflationary effects, as it leads to a reduction of inflation and of real output. The real interest rate increases.

2. In the inverse regime ($\Omega > 0$) two different policy regimes can emerge:

   (a) If $\Omega > (a, k + a_z)^{-1}$, an unexpected positive policy shock has the same effects as in the standard regime. Even if the semi-elasticity of demand is positive, the real interest rate is moving in the opposite direction of the nominal rate.

   (b) If $\Omega < (a, k + a_z)^{-1}$, a positive policy shock leads to increased output and inflation,
and the real interest rate falls.\textsuperscript{9}

The rationale of the two non-standard policy regimes (a) and (b) can be interpreted as follows. In the inverse regime a positive monetary policy shock initially shifts the aggregate demand schedule backwards, leading to a reduction of real output and inflation. Lower real output and inflation stimulate an expansionary central bank’s reaction, eventually leading to either an increase or a reduction of the real interest rate, depending on monetary policy effectiveness. Neglecting habits formation for simplicity, monetary policy effectiveness is increasing in the fraction of spenders in the standard demand regime and decreasing in the inverse demand regime. Considering habits, the study of transmission mechanisms becomes analytically intractable and numerical solutions are needed. In line with the results of the analysis of the model properties, from numerical simulations we have obtained that, other things equal, the probability of observing a standard regime and thus the typical policy regime increases with the size of the habit parameter.

3. Bayesian MCMC estimation of the structural parameters

3.1 A brief description of the estimation approach

In this section we provide the details of the empirical evaluation of the theoretical model described in the previous section. The computational task is cumbersome, since the convergence performances of numerical methods for Full Information Maximum Likelihood (FIML) estimation may be affected by the presence of nonlinearities in model parameters.\textsuperscript{10} In such cases, a viable solution is to restrict the parameters estimates within a range that we deem as \textit{reasonable}, i.e. to employ a restricted FIML estimator. However, since outcomes would by definition depend on the assumptions on the range of admissible values (priors), we adopt a more structured Bayesian Monte

\textsuperscript{9} Note that this regime is potentially compatible with the prize puzzle, i.e. the (apparent) positive empirical relationship between the federal funds rate and inflation (Sims, 1992).

\textsuperscript{10} For some reference applications of the methodology, see Ireland (2004).
Carlo estimation approach.\textsuperscript{11}

Bayesian estimation for DSGE models is close in spirit to restricted FIML estimation, since the subjective element is specified in both cases. The peculiarity of the Bayesian MCMC approach is that, instead of employing interval restrictions on parameters, it requires to nest formalized \textit{distributional} priors on parameters with the conditional distribution (i.e. the likelihood) in order to obtain the posterior distribution.

We will consider the posterior density as the benchmark distribution for Monte Carlo integration. The final estimates will be obtained employing the Metropolis-Hastings procedure implemented in Dynare for Matlab (Juillard, 2004).

The posterior distribution is the result of a weighted average of the prior non sample information and the conditional distribution (i.e. the empirical information); weights are inversely related to, respectively, the variance of the prior distributions and the variance of the sample information (“precisions”). The bigger the informative power of the likelihood (i.e. the lesser the variances of the likelihood-based estimates), the closer the posterior will be to the conditional distribution. In the limiting case in which data allow a perfect knowledge of parameters, the posterior distribution collapses to the conditional distribution. Contrary, if empirical information is weakly informative, the priors will correspondingly have more weight in estimation. Formalizing a tight prior will result in highly constrained estimation, while a diffuse prior will result in weakly constrained estimation.

Formally, our procedure requires nesting the prior distribution $P(\theta)$ for the vector of parameters $\theta \in \Theta$ and the conditional distribution\textsuperscript{12} $P(Y_T | \theta)$, $Y_T = \{y_t\}_{t=1}^T$ to get the posterior distribution $P(\theta | Y_T)$. This is basically obtained employing the Bayes rule:

\begin{equation}
(17) \quad P(\theta | Y_T) = \frac{P(Y_T | \theta)P(\theta)}{P(Y_T)},
\end{equation}

where $P(Y_T)$ is the marginal distribution.


\textsuperscript{12} The conditional distribution is obtained employing the Kalman filter (Sargent, 1989).
Once the posterior distribution is obtained, it is employed as the “proposal density” to initialize the Metropolis-Hastings MCMC sampling method\textsuperscript{13}, which substantially generates a large number of random draws from the posterior density in order to obtain a Monte-Carlo estimate of the parameters’ distributions.

The model is estimated employing four observable variables: log real private output, first differences of the log GDP deflator (i.e. the inflation rate), the quarterly nominal interest rate and a measure of log real output gap. Sample information is quarterly and spans from 1963:1 to 2003:2 for each of the G7 countries being considered. In the benchmark formulations, we employ short-term nominal interest rate definitions such as the Federal Funds Rate for the United States, the Overnight Rate for Canada and the United Kingdom and the Money Call Rate for the remaining countries. In order to check for robustness, we also re-run the estimations by substituting the reference short-term rates with the three months Treasury Bill Rate and the 10-years Government Bonds Rate. Data are all drawn from the IMF International Financial Statistics (IFS) database. Log real output gap is obtained as the difference between log real output and its trend, the latter approximated by the Hodrick-Prescott filter. Following Smets and Wouters (2003), real output is de-trended assuming a linear trend while both inflation and the nominal interest rate – because of their co-trending behavior – are de-trended on the basis of the estimated linear component in inflation. Results are qualitatively robust to the specific output gap measure and de-trending procedure being considered.\textsuperscript{14}

### 3.2 Operational structure of the model and prior distributions

The empirical version of the theoretical model described in section two is obtained by adding five structural \textit{i.i.d.} shocks, a definition equation for the output gap and the policy reaction function. To improve model fit, we assume a contemporaneous policy reaction rule with stochastic inflation

\textsuperscript{13} More precisely, the algorithm employs the mode and the Hessian evaluated at the mode for the initialization of the Metropolis-Hastings procedure.

\textsuperscript{14} We have employed alternative approximations of the output gap and different de-trending procedures for output, inflation and the nominal interest rate. Alternative approximations for the output gap are based on the Baxter-King and Christiano-Fitzgerald filters, while a second order polynomial has been employed as alternative de-trending procedure. Results can be obtained upon request from the authors.
target and interest rate smoothing. The resulting stochastic structural model is fully described by eleven equations:

\begin{align}
(18.1) \quad y_t &= \frac{1 - \lambda \zeta^N}{1 + \sigma - \lambda \zeta^N (1 + \nu - \sigma^2)} E_t y_{t+1} + \frac{\sigma [1 - (1 + \sigma) \lambda \zeta^N]}{1 + \sigma - \lambda \zeta^N (1 + \nu - \sigma^2)} y_{t-1} \nonumber \\
&\quad - \frac{1 - \sigma - \lambda \zeta^N}{1 + \sigma - \lambda \zeta^N (1 + \nu - \sigma^2)} \left( i_t - E_t \pi_{t+1} - \mu_t + E_t \mu_{t+1} \right) + \frac{\lambda \zeta^N \nu}{1 + \sigma - \lambda \zeta^N (1 + \nu - \sigma^2)} \Delta E_t a_{t+1} \\
(18.2) \quad \pi_t &= \beta E_t \pi_{t+1} + \frac{\tau (k + \theta)}{k (1 - \sigma)} m c_t \\
(18.3) \quad m c_t &= \frac{1 + \nu (1 - \sigma)}{1 - \sigma} y_t - \frac{\sigma}{1 - \sigma} y_{t-1} - (1 + \nu) a_t + u^{ep}_t \\
(18.4) \quad i_t &= \rho_i i_{t-1} + (1 - \rho_i) \left[ \pi^*_t + \psi_s \left( \pi_t - \pi^*_t \right) + \psi_s \lambda_{x_t} \right] + u^i_t \\
(18.5) \quad x_t &= y_t - y^f_t \\
(18.6) \quad y^f_t &= \frac{(1 + \nu) (1 - \sigma)}{1 + \nu (1 - \sigma)} \frac{a_t}{1 + \nu (1 - \sigma)} y^f_{t-1} \\
(18.7) \quad \pi^*_t &= \rho_{\pi} \pi^*_{t-1} + \varepsilon^\pi_t \\
(18.9) \quad a_t &= \rho_a a_{t-1} + \varepsilon^a_t \\
(18.9) \quad \mu_t &= \rho_{\mu} \mu_{t-1} + \varepsilon^{\mu_{tref}}_t \\
(18.10) \quad u^{ep}_t &= \varepsilon^{ep}_t \\
(18.11) \quad u^i_t &= \varepsilon^i_t
\end{align}

Equations (18.1) and (18.3) are equivalent, respectively, to the IS relation (6) and to the marginal cost equation (8), augmented with preference and cost push shocks. The fourth equation (18.4) is a Taylor-like rule in the spirit of that employed by Smets and Wouters (2003), which has proven good performances in terms of fit; \( \pi^*_t \) is the stochastic policy target and \( u^i_t \) is a serially uncorrelated policy shock. The fifth equation (18.5) is the standard output gap definition; The last five equations (18.7)-(18.11) specify the stochastic processes driving the dynamics of the model.

For empirical identification, five structural (independent) shocks are considered: \( i \) a preference...
shock $\varepsilon_{\text{pref}}^i$; ii) a technology shock $\varepsilon_t^a$; iii) a cost-push shock $\varepsilon_{\text{cp}}^i$; iv) a monetary policy shock $\varepsilon_{\text{it}}^i$; v) a shock to the monetary policy target, i.e. to targeted inflation, $\varepsilon_{\text{it}}^{\bar{\pi}}$. We assume three persistent (albeit stationary) components and two serially uncorrelated components. Preference, technology and monetary policy target shocks are somewhat persistent, giving rise to autoregressive stationary processes. The other stochastic components are serially uncorrelated $i.i.d.$ innovations.

This characterization of the shocks is needed to reproduce the persistence and hump-shaped responses found in the data. It represents a quite weak assumption from a theoretical point of view, since it is commonly accepted that technology shocks, as well as preference shocks, have long-lasting effects, while the persistence of the monetary policy target can be justified on the grounds that, once committed on a given target, authorities change their mind slowly.

The shape of the prior distributions is chosen according to the following standard assumptions: the reference distribution for the structural shocks is the inverted gamma distribution with two degrees of freedom, which is consistent with a diffuse prior on perturbations and positive variances; for parameters theoretically defined in a 0-1 range, we assume a beta distribution; for the other parameters we assume a normal distribution. Prior means and standard deviations are defined on the basis of the empirical reliability of the information obtainable from other studies and from our preliminary GMM and ML estimates conducted on reduced-form equations for the seven countries.\footnote{Results from this preliminary evaluation can be obtained upon request from the authors.}

Differently from Smets and Wouters (2003), we do not employ fixed parameters values, with the exception of the discount factor $\beta$ which is fixed at 0.995 (consistent with a steady state real rate of 2%). Anyway, we adopt relatively tight priors for the elasticity of substitution across intermediate goods $\eta$ and for labor disutility $\kappa$.

Given the model assumptions described above, we estimate 17 parameters, of which 5 define the distribution of the structural innovations and 3 their persistence.

Concerning prior mean values, in line with Gali et al. (2004), the expected elasticity of substitution
across intermediate goods $\eta$ is set to 6, which is consistent with a steady-state mark-up of 20%. The mean of the labor disutility parameter $\kappa$, set to 3, is chosen on the basis of the ratio between hours spent at work and total available time. For both parameters, we assume a relatively small prior variability of, respectively, 0.3 and 0.15, and a normal prior shape. Concerning the Taylor rule parameters, we assume that the mean values for the parameter on expected inflation and for the parameter on output gap are, respectively, 1.5 and 0.125. Prior standard deviations are, respectively 0.15 and 0.05 and the prior shape of the distribution is again the normal. The chosen variability implies a moderately diffuse prior for the first parameter and a very diffuse prior for the second parameter. These values are also consistent with the average ML estimates of the Taylor rule parameters conducted for the seven countries included in the analysis. The prior mean of the interest rate smoothness parameter, consistently with the average ML estimates, is set to 0.8, while for its variability we assume a prior of 0.10, which can be considered relatively large with respect to the empirical standard deviations obtained with the ML estimates. The chosen prior shape for the distribution of the interest rate smoothness parameter is the beta distribution.

For the fraction of firms maintaining the price fixed $\phi$ we assume a prior mean of 0.75, which is consistent with the results of Galì et al. (2001). These authors obtained an average duration of the price contracts of approximately one year and a rather small prior variability, leading to a range of duration between 3 and 6 quarters.

For the parameters defining the persistence of shocks, following Smets and Wouters (2003), we adopt a common mean value of 0.85 and a prior variability of 0.10. The choice of a relatively concentrated prior for the persistence parameters is justified by the need of having a tight separation between persistent and transitory shocks, enhancing the identification of the two shocks entering the interest rate equation. The prior shape is the beta distribution.

For the habits persistence parameter we assume a prior mean value of 0.7 and a moderately diffuse prior variability of 0.1. The shape of the prior distribution is again the beta distribution. Prior mean and variability are chosen on the basis of the evidence emerged in a number of previous studies and
on the basis of our Euler equation GMM estimates, modified in order to account for habit persistence.

For the rule-of-thumb parameter we set a prior mean of 0.5 and a prior standard deviation of 0.10, while the reference distributional shape is again the beta. These prior values are consistent with the findings of Campbell and Mankiw (1989) and with our modified consumption equation GMM estimates for the seven major economies\textsuperscript{16}.

For the structural shocks we adopt a parameterization which is similar to that employed by Smets and Wouters (2003). Apart from the large interval implied by the assumption of 2 degrees of freedom for the inverted gamma distribution, the prior mean values are obtained from previous estimations conducted with very diffuse priors.

The table below summarizes the structural parameters’ prior distributions considered in the analysis.

Table 1 about here

3.3 Parameter estimates and country-specific simulations

Table 2 summarizes the MCMC estimates of the structural parameters and their posterior distributions, obtained with the Metropolis-Hastings sampling algorithm.

Concerning regime inversion, results are in line with the theoretical expectations. Given the estimated values for the structural parameters of the model, the regime shift is never observed. The size of the estimated habit persistence and rule-of-thumb parameters rule out the emergence an inverted IS relation.

We find relevant heterogeneity across countries, in particular for the parameters indicating the fraction of rule-of-thumb households and habit persistence. Since the other parameters show a lower cross-country variability, the heterogeneity found with respect to the rule-of-thumb and the habit

parameters emerges as the main cause of the differences that we get when the model is simulated employing the country-specific parameterization.

Italy shows the highest rate of habits (0.8), while Germany the lowest (0.6). The average habit persistence parameter is 0.7, a value that is strictly in line with the results obtained in previous empirical investigations.

The average fraction of spenders for the G7 economies is 26%, a value that is well below the prior mean employed in the estimations. This value is broadly consistent with the outcomes of the analysis of Campbell and Mankiw (1991), who obtained a fraction of spenders of approximately 35% for the United States and 20% for the United Kingdom. It is also marginally consistent with the results obtained by Banerjee and Batini (2003) who, employing the AIM solution procedure of Anderson and Moore, obtained a fraction of spenders of nearly 26% for the United States and of nearly 15% for the United Kingdom.

Interestingly, the fraction of rule-of-thumb households in Italy, Germany and Japan is relatively low (nearly 7% on average), while it is high in France (0.44), in the United Kingdom (0.42), in the United States (0.37) and in Canada (0.30). This result is surprising, since it requires explanations that are not in line with the standard view on the meaning of rule-of-thumb consumption. In many studies the existence of spenders is considered a proxy of the development and efficiency of the financial sector. As long as our estimates are reliable, since the higher fraction of spenders is found for countries in which the financial markets are considered developed and efficient, the standard interpretation of rule-of-thumb consumption appears misleading. Under this perspective, differences are more likely to be related to psychological and cultural factors rather than to financial factors.17

Table 2 about here

The estimates also show a considerable degree of Calvo price stickiness, whose average estimate is 0.84, consistent with an average duration of the price contracts of approximately 6 quarters.

17 Despite different in many respects, Japan, Germany and Italy have some relevant similarities, as for the importance of the generational and family transfers and for the role and the features of the banking sector. Moreover, they show the highest saving rates among industrialized countries.
We find a significant positive central bank’s short-term reaction to the current change in inflation and the output gap. Our estimation delivers plausible parameters for the long and short-run reaction function of the monetary authorities, and results are broadly in line with those discussed in Taylor (1993). The parameter for the policy reaction to inflation is rather stable across countries and in line with the prior assumptions. Some heterogeneity is found with respect to the policy elasticity to the output gap. The highest values are obtained for the United States and for Italy (nearly 0.2), while the lowest for Germany (0.11), Japan, France and the United Kingdom. In agreement with the large literature on estimated interest rate rules, we also find evidence of a substantial degree of interest rate smoothing, which in addition is also rather stable across countries.

The simulation of the DSGE model conducted employing the estimated structural parameters provides an appreciation of the degree of heterogeneity of the dynamic properties of the stylized economies. In particular, the simulations allow us to recognize the country specific effectiveness of monetary policy and the degree of asymmetry of its effects. Figure 1 contains the impulse responses to a monetary policy shock, while Figure 2 the impulse responses to a technology shock.

In spite of the spenders, a positive monetary shock disturbance has the standard effects discussed above by reduction both inflation and real activity and increasing the real interest rate. Hump-shaped reactions are the usual result of habit persistence.

Concerning the reaction to monetary policy in different countries, the biggest impulse response of inflation to a positive interest rate shock is found for Japan, for which the half-life deviation from price stability is approximately 4 quarters, while the smaller if found to the United States which half life is nearly 2 quarters. The responses of output are even more differentiated among countries. A common feature is that the maximum effect on output of the monetary policy shock is reached after 2 quarters. The maximum responsiveness and duration of effects is found for the United Kingdom, the minimum for the United States.

Figure 1 about here
The half life of the response is approximately 4 quarters for the United States, 6 quarters for Italy, Germany and Japan, 7 for Canada and 8 quarters for United Kingdom and France. In line with the theoretical predictions, with the exception of the United States, the output sensitivity to monetary policy is thus stronger in those countries that show the highest fraction of rule-of-thumb consumers. A technology shock \( \epsilon_t \) also has standard effects on the variables of the model. Inflation decreases at the impact following marginal costs (Fig 8a-b). According to the monetary policy reaction rule, the nominal interest rate is decreased (Fig 8c), i.e. the policy accommodates the shock. The hump-shaped response of output, i.e. its deviation from the flexible price standard response, depends on the degree of inertia in policy.

Figure 2 about here.

As long as the nominal interest rate adjustment is smoothed by the monetary policy authorities, the real interest rate response may become positive, with counter-intuitive contractionary effects on output (via the IS equation). The estimated and simulated high degree of heterogeneity in policy response explains the heterogeneous impact and medium-term effects on output: they are in fact virtually zero at the impact for the majority of the countries considered in the analysis and negative for France and the United Kingdom.

Even if emerging from a different perspective, this result is in line with the evidence produced by Gali (1999) on the possibility of “contractionary” supply shocks. If monetary policy does not fully accommodate the positive supply shock, the demand response is unable to match the potential output response, inducing the counter-cyclical employment (hours) conditional dynamics which has been generally addressed as “productivity-employment puzzle.” The main difference here is that we do not consider this puzzle explicitly and, most importantly, that it implicitly emerges even considering a Taylor-like monetary rule instead of a money supply rule as in Gali (1999).

4. Conclusions

We consider strong violations of the Hall’s benchmark consumption function in a simple New
Keynesian DSGE model. In particular, we analyze the implications of the joint presence of spenders and external habits in consumption for the local stability of the model and the effectiveness of the conduct of monetary policy.

We find that the presence of spenders can potentially alter conventional policy prescriptions as a stream of the recent literature has also shown. However, we find that the important and unconventional results only hold under some particular circumstances related to the response of aggregate demand to nominal interest rate movements, i.e. demand regimes.

More specifically, we first show that models with rule-of-thumb consumers are consistent with two different demand regimes. In the standard regime an increase of the interest rate, other things equal, reduces inflation and output as usual. By contrast, in the inverse regime the reverse mechanism emerges: An increase of the nominal interest rate may increase output since deflation and decreasing markups, push-up the real wage, and induce spenders to consume more. Unconventional results only apply to the inverse regime, thus when a large number of spenders is present.

The analysis has evidenced that two sub-regimes may emerge in the inverse demand regime. A monetary policy shock initially shifts aggregate demand backwards, reducing real output and inflation. The reaction of the central bank to this change can imply either an increase or a reduction of the real interest rate. Which of the two outcomes will emerge depends on the size of the monetary policy effectiveness. The reverse behavior is only observed for low values of the monetary policy effectiveness, rendering the central bank’s reaction insufficient to reverse the effect of the shock. The policy regime that will emerge thus depends on both monetary policy effectiveness and the demand regime.

The probability of an inverse regime increases with the size of the fraction of spenders. However, the inversion is also strongly influenced by the presence of consumption habits in a highly non-linear manner. From the numerical solution and simulation of the model, we have obtained that, \textit{ceteris paribus}, the threshold value of spenders needed to obtain the regime inversion increases
with the size of the habit persistence parameter. Hence, by introducing habits, the probability of observing a demand regime shift decreases.

The empirical relevance of our theoretical hypotheses has been evaluated by estimating the structural parameters of the DSGE model for the seven most industrialized economies. Then, the structural estimates have been employed for obtaining country-specific simulations of the dynamics of the stylized economies.

The analysis has evidenced the effectiveness of the monetary policy in stabilizing the business cycle in all the countries considered. However, it has also highlighted the presence of relevant international asymmetries in the monetary transmission mechanisms. The presence of asymmetries in the monetary transmission channels stimulates a serious reconsideration of the policy prescriptions neglecting that the differences among economies may result decisive in the determination of the effects of the policy, in particular monetary policy.

An additional result of our analysis is that, despite the heterogeneous sensitivity to shocks, the dynamic properties of all the model economies are qualitatively in line with those predicted by the conventional New Keynesian DSGE model. In particular, the estimated values of the structural parameters rule out the possibility of a demand regime inversion due to the presence of rule-of-thumb consumers. Even though the fraction of spenders is relevant in many countries (0.26 on average), in none of them this fraction is high enough to generate the regime inversion. A further interesting result is that, despite the model is theoretically able to generate the so-called “price puzzle” for habits and rule-of-thumb parameters values that are not prohibitively high, the estimation has generated a parameterization that is not consistent with this result.

**Appendix A – Determinacy**

Determinacy is studied by augmenting the log-linearized dynamic system with a simple feedback
rule, we obtain:\textsuperscript{18}
\begin{equation*}
A1 \begin{bmatrix} 1 & -\Omega \\ 0 & \beta \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 - \Omega a_2 & -\Omega a_1 \\ -k & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}
\end{equation*}

Stability depends on the eigen-structure of the following matrix:
\begin{equation*}
A2 M = \begin{bmatrix} 1 & -\Omega \\ 0 & \beta \end{bmatrix} \begin{bmatrix} 1 - \Omega a_2 & -\Omega a_1 \\ -k & 1 \end{bmatrix} = \begin{bmatrix} 1 - \Omega(a_2 + k\beta^{-1}) & \Omega(\beta^{-1} - a_1) \\ -k\beta^{-1} & \beta^{-1} \end{bmatrix}
\end{equation*}

By indicating with \( D(\cdot) \) and \( T(\cdot) \) the determinant and trace operators, we have:
\begin{equation*}
A3 \begin{cases} 
D(M) = \beta^{-1} + \Omega (a_2 + ka_1) \beta^{-1} \\
T(M) = 1 + a_2\Omega + (1 + k\Omega) \beta^{-1}
\end{cases}
\end{equation*}

The eigen-structure of matrix \( M \) is studied as in Woodford (2003: Appendices to Chapter 4). Since the analysis of the standard regime does not differs from Woodford (2003), we only consider the liquidity-constrained regime.

Determinacy requires either:
\begin{itemize}
  \item[i)] \( D(M) > 1 \), i.e. \( a_1 < \left[(1 - \beta)\Omega^{-1} - a_2\right]k^{-1} \), \( D(M) \pm T(M) + 1 > 0 \) or
  \item[ii)] \( D(M) \pm T(M) + 1 < 0 \).
\end{itemize}

Being:
\begin{equation*}
A4 D(M) + T(M) + 1 = \left\{ 2 (1 + \beta) + \Omega \left[ (1 + \beta) a_2 + (1 + a_1) k \right] \right\} \beta^{-1}
\end{equation*}
\begin{equation*}
A5 D(M) - T(M) + 1 = \Omega \left[ (1 - \beta) a_2 + (a_1 - 1) k \right] \beta^{-1}
\end{equation*}

from equations A4 and A5 we derive conditions (11) and (12), respectively.

Regarding the relationship between determinacy and effectiveness of monetary policy under a standard Taylor rule, determinacy requires (11) and (12), but since \( 1 - \frac{1 - \beta}{k} a_2 > \left( \frac{2}{\Omega} - a_2 \right) \frac{1 + \beta}{k} - 1 \) if
and only if \( \Omega > \frac{1 + \beta}{k + \beta a_2} \), the following statements hold.

\textsuperscript{18} In order to investigate the stability properties we do not need to look at the stochastic part that is thus omitted for the sake of brevity. We assume stationary disturbance processes.
1. For $\Omega > \frac{1+\beta}{k + \beta a_z}$ determinacy requires: 1a) $a_i > 1 - \frac{1 - \beta}{k} a_z$ or 1b) $a_i < \frac{2}{\Omega - a_z} \frac{1+\beta}{k} - 1$ if
   $$a_i < \frac{1 - \beta}{\Omega k} - \frac{a_z}{k}.$$

2. For $\Omega < \frac{1+\beta}{k + \beta a_z}$ determinacy requires: 2a) $a_i > \frac{2}{\Omega - a_z} \frac{1+\beta}{k} - 1$ or 2b) $a_i < 1 - \frac{1 - \beta}{k} a_z$ if
   $$a_i < \frac{1 - \beta}{\Omega k} - \frac{a_z}{k}.$$

From conditions 1a) and 2a) follow that a standard Taylor principle holds for a relatively high effectiveness a more aggressive principle should be used for a relatively low degrees of effectiveness. In addition, note that

$$A6 \quad \frac{1 - \beta}{\Omega k} - \frac{a_z}{k} > \frac{2}{\Omega - a_z} \frac{1+\beta}{k} - 1 \text{ for } \Omega < \frac{1+3\beta}{k + \beta a_z}$$

$$A7 \quad \frac{1 - \beta}{\Omega k} - \frac{a_z}{k} > 1 - \frac{1 - \beta}{k} a_z \text{ for } \Omega > \frac{1 - \beta}{k + \beta a_z}.$$

Thus condition 1b is binding if $\Omega < \frac{1+3\beta}{k + \beta a_z}$ and condition 2b is binding if $\Omega > \frac{1 - \beta}{k + \beta a_z}$. By putting all together, condition 1b is binding if $\Omega \in \left( \frac{1 + \beta}{k + \beta a_z}, \frac{1+3\beta}{k + \beta a_z} \right)$ and condition 2b is always binding.

Summarizing the above results we obtain the result reported in section 2.3.

**Appendix B – Monetary policy transmission (policy regimes)**

By simple derivation we obtain
$$\frac{\partial y_i}{\partial \epsilon_i} = \Omega \frac{\partial i}{\partial \epsilon_i} ; \quad \frac{\partial \pi}{\partial \epsilon_i} = \kappa \frac{\partial y_i}{\partial \epsilon_i} ; \quad \frac{\partial i}{\partial \epsilon_i} = a_i \frac{\partial \pi}{\partial \epsilon_i} + a_z \frac{\partial y_i}{\partial \epsilon_i} + 1,$$
where $\epsilon_i$ is a white-noise monetary disturbance. From equation (6) and (10), using the above expressions, it is easy to derive
$$\frac{\partial y_i}{\partial \epsilon_i} = \Omega \left( a_i \frac{\partial y_i}{\partial \epsilon_i} + a_z \frac{\partial y_i}{\partial \epsilon_i} + 1 \right),$$
and thus,
$$\frac{\partial y_i}{\partial \epsilon_i} = \frac{\Omega}{1 - \Omega (a_i \kappa + a_z)}.$$
\[ \frac{\partial \pi}{\partial \varepsilon_t} = \frac{\Omega \kappa}{1 - \Omega (a_1 \kappa + a_2)}, \quad \text{and} \quad \frac{\partial \iota}{\partial \varepsilon_t} = \frac{1}{1 - \Omega (a_1 \kappa + a_2)}, \]

from which the discussion in the main text can be derived.

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Figure 1. Impulse responses to a monetary policy shock, M-H MCMC estimates

a) inflation

b) marginal costs

c) nominal interest rate
d) output

Computations obtained with Dynare for Matlab.
Figure 2. Impulse responses to a technology shock, M-H MCMC estimates

a) inflation

b) marginal cost

c) nominal interest rate

d) output

Computations obtained with Dynare for Matlab.