THE IMPACT OF FOREIGN STOCK MARKETS ON MACROECONOMIC DYNAMICS IN SMALL OPEN ECONOMIES:
A STRUCTURAL ESTIMATION

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ABSTRACT. This paper employs a structural New Keynesian-style model with an international wealth channel to estimate the effect of foreign stock market fluctuations on small open economy countries. The magnitude of the wealth channel in the model depends on the length of economic agents' planning horizon when taking financial decisions.

The model is estimated on data for a sample of small open economies, which can in principle be affected by changes in a largest foreign stock market: the country considered are Australia, Canada, New Zealand, Ireland, Austria, and Netherlands.

With the increased size of international portfolio holdings (documented for example in Lane and Milesi-Ferretti 2006), swings in asset prices in the country in which the largest financial market is located may have significant effects on domestic aggregate consumption. The financial market of reference typically consists of the U.S., although also U.K. (for Ireland), Australia (for New Zealand), and Germany (for Austria) are considered in some cases in the estimation.

[PRELIMINARY AND INCOMPLETE VERSION]

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THE IMPACT OF FOREIGN STOCK MARKETS ON SMALL OPEN ECONOMIES

1. Introduction

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where \( i = H, F \) denotes the country and \( j \) the cohort, \( \beta \) is the intertemporal discount factor, subject to a sequence of budget constraints

\[
C^H_t(j) + \frac{\varepsilon_t}{P^H_t} E^H_t F^{j}_{t,t+1} + B^H_{F,t+1}(j) + \frac{\varepsilon_t}{P^H_t} \int_n^1 Q^{*H}_{F,t}(i) Z^{H}_{t+1}(f,j) df \leq W^H_t \frac{N^H_{j,t}}{P^H_t} + \frac{1}{P^H_t} \int_0^n D^*_{H,t}(h,j) dh + \frac{\varepsilon_t}{P^H_t} \Omega^H_{F,t}(j) \tag{2.2}
\]

if living in the Home country and

\[
C^F_t(j) + \frac{1}{P^F_t} E^F_t F^{j}_{t,t+1} + B^F_{F,t+1}(j) + \frac{1}{P^F_t} \int_n^1 Q^{*F}_{F,t}(f) Z^{F}_{t+1}(f,j) df \leq W^F_t \frac{N^F_{j,t}}{P^F_t} + \frac{1}{P^F_t} \Omega^F_{F,t}(j) \tag{2.3}
\]

if living in the Foreign economy, and subject to a No-Ponzi-game condition

\[
\lim_{k \to \infty} E_t \{ F^{j}_{t,t+k} (1 - \gamma)^k \Omega^{i}_{F,t+k}(j) \} = 0, \tag{2.4}
\]

where \( \varepsilon_t \) denotes the nominal exchange rate in terms of units of domestic currency needed to purchase foreign currency, \( P^i_t \) is the Consumer Price Index in country \( i \), \( F^{j}_{t,t+k} \) is the stochastic discount factor between period \( t \) and \( t + k \), \( B^i_{F,t}(j) \) are holdings of state-contingent assets in country \( i \) expressed in \( F \)-currency, \( Q^{*i}_{F,t}(f) \) denotes the nominal price of equities (in \( F \)-currency), \( Z^i_{t}(f,j) \) denote the equity shares issued by firms located in country \( F \), \( W^i_t \) is the nominal wage rate in country \( i \), \( D^*_i(t',j) \) are dividends paid by firms that produce good \( i' \) in country \( i \) to cohort \( j \)'s households, and where nominal financial wealth, denoted by \( \Omega^i_{F,t}(j) \) and expressed in foreign currency equals

\[
\Omega^i_{F,t}(j) = \frac{1}{1 - \gamma} [ B^i_{F,t}(j) + \int_n^1 (Q^{*i}_{F,t}(f) + D^*_i(t,f)) Z^i_{t}(f,j) df ], \tag{2.5}
\]

since it is assumed, as in Blanchard (1985), that an insurance contract exists so that the wealth that is carried over from the previous period is redistributed within the living cohort.

Each household in each country consumes a bundle of domestic and foreign goods

\[
C^i_t(j) = \left[ n^{\frac{1}{\sigma}} C^i_{H,t}(j)^{\frac{\sigma - 1}{\sigma}} + (1 - n)^{\frac{1}{\sigma}} C^i_{F,t}(j)^{\frac{\sigma - 1}{\sigma}} \right]^{-\frac{\sigma}{\sigma - 1}} \tag{2.6}
\]
with

\begin{align*}
C_{H,t}(j) &= \left[ \left( \frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n C_t(h,j)^{\frac{1}{\epsilon-1}} \, dh \right]^{\frac{1}{\epsilon-1}} \\
C_{F,t}(j) &= \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_1^n C_t(f,j)^{\frac{1}{\epsilon-1}} \, df \right]^{\frac{1}{\epsilon-1}}
\end{align*}

(2.7)

and where $\theta > 0$ denotes the elasticity of substitution among domestic and foreign goods and $\epsilon > 1$ the elasticity of substitution among differentiated goods, which is assumed to be the same across countries.

**UIP**

The terms of trade are defined as $S_t = \frac{P_{F,t}}{P_{H,t}}$, i.e. as the relative price of foreign-produced goods in terms of domestically-produced goods. The Law of One Price and the Purchasing Power Parity hold at each point in time:

\[ P_{H,i,t} = \epsilon_t P_{F,i,t}, \quad P_{H,t} = \epsilon_t P_{F,t} \]

2.2. **Firms.** Monopolistically-competitive firms in each country produce a continuum of differentiated goods. Each firm supplies good $i'$, which is produced according to the production technology $Y_t(i') = A_i N_t(i')$, where $i' = h, f$, $i = H, F$, $N_t(i')$ is labor input, and $A_i$ is a country-specific technology shock. Following Calvo (1983), a fraction $0 < 1 - \alpha^i < 1$ of firms in country $i$ are allowed to change their price in a given period. Firms face a common demand curve $Y_t(i') = Y_t^i \left( \frac{P_t^i(i')}{{P_{F,i,t}}} \right)^{-\frac{1}{\epsilon}}$ for their product, where $Y_t^i$ is aggregate output in country $i$.

Each firm faces the same decision problem and, if allowed to re-optimize, sets the common price $P_t^*(i')$ to maximize the expected present discounted value of future profits.
2.3. **Aggregate Dynamics.** The macroeconomic dynamics in the domestic small open economy can be characterized by the following set of equations

\[
x_t^H = \frac{1}{1 + \psi} \Delta_t x_{t+1}^H + \frac{\psi}{1 + \psi} (q_t^F + (1 - n)\theta s_t)
\]

\[
- \frac{1}{1 + \psi} (i_t^H - \Delta_t \pi_{t+1}^H - r_t^H) - (\theta - 1)(1 - n)\Delta s_{t+1}^H
\]

\[
\pi_t^H = \tilde{\beta} \Delta_t \pi_{t+1}^H + \lambda^H (1 + \varphi)x_t^H - (1 - n)(\theta - 1)\lambda^H s_t + u_t^H
\]

\[
i_t^H = \rho^H i_{t-1}^H + (1 - \rho^H)\Delta_t s_t + (1 + \chi^H \pi_{t+1}^H + \chi^H s_{t-1}^H + \chi^H q_{t-1}^F) + \varepsilon_t^H
\]

which represent a New Keynesian-style model, extended to include the impact of foreign stock prices and the terms of trade on the domestic economy.

Equation (2.8) is the log-linearized Euler equation arising from households’ optimal choice of consumption and re-expressed in terms of the output gap. Output gap in period \( t \) depends on expected output gap in \( t + 1 \), on the real stock price gap \( q_t^F \), on the current and expected terms of trade \( s_t \), and on the ex-ante real interest rate, with \( i_t \), \( \pi_t \), \( r_t^H \) denoting the nominal interest rate, inflation, and the natural interest rate. The wealth effect from international stock market fluctuations to the domestic economy depends on the reduced-form term \( \frac{\psi}{1 + \psi} \), in which \( \psi \) is a composite function of structural coefficients that depend positively on the probability of exiting the market \( \gamma \), i.e. \( \psi \equiv \gamma (1 - n) \frac{1 - \beta (1 - \gamma) \Omega^F p}{(1 - \gamma) p Y} \). A higher \( \gamma \), indicating a shorter planning horizon by agents, therefore, leads to a stronger wealth channel. It reduces, instead, the degree of intertemporal consumption smoothing and the sensitivity of output to real interest rates.

Equation (2.9) is a New Keynesian Phillips curve, in which domestic inflation \( \pi_t \) depends on expected inflation in \( t + 1 \), on the domestic output gap, and on the terms of trade. Here \( \tilde{\beta} \equiv \frac{\beta}{1 + \psi} \) and \( \lambda^i \equiv \frac{(1 - \alpha)(1 - \beta \gamma)}{\alpha (1 + \psi)} \), hence, a higher \( \gamma \) will cause inflation to depend less on expectations and more on current real activity and terms of trade.
Equation (2.10) denotes a Taylor rule, which describes the monetary policy implemented by the central bank in the domestic economy. The central bank reacts to domestic inflation, output gap, the terms of trade, and to the stock price gap; \( \chi^H_{\pi}, \chi^H_x, \chi^H_s \), and \( \chi^H_q \) are the feedback coefficients, while \( \rho \) accounts for the degree of policy inertia.

The real foreign stock price gap and the terms of trade evolve according to the structural equations:

\[
q^F_t = \tilde{\beta} \hat{E}_t q^F_{t+1} + (1 - \tilde{\beta}) \hat{E}_t (x^F_{t+1} + n s_{t+1}) - \left( i^F_t - \hat{E}_t \pi^F_{t+1} - r^F_{n,F} \right)
- n \hat{E}_t \Delta s_{t+1} - \frac{\tilde{\beta} Y/Q^F}{\mu} \hat{E}_t \left[ (1 + \varphi)x^F_{t+1} + n(\theta - 1)s_{t+1} \right] + e(2.11)
\]

\[
s_t = \tilde{E}_t s_{t+1} + \left( i^F_t - \tilde{E}_t \pi^F_{t+1} - r^F_{n,F} \right) - \left( i^H_t - \tilde{E}_t \pi^H_{t+1} - r^H_{n,F} \right)
+ \phi_t \tag{2.12}
\]

where equation (2.11) is derived from loglinearization of the asset-pricing equation that arises from the household optimization problem and states that the foreign stock price gap is affected by expected stock price gap, output gap, terms of trade, and by the ex-ante real interest rate. The parameter \( \varphi \equiv \frac{N}{1-N} \) denotes the inverse of the Frisch elasticity of labor supply in steady-state, \( \mu \equiv \frac{\epsilon}{\epsilon-1} \) denotes the steady-state mark-up on prices; \( N, \epsilon, Y, \) and \( Q^F \) all refer to steady-state values. Equation (2.12) describes the dynamics of the log of the terms of trade, which also depends on expectations as well as on the real interest rate differential.

The foreign economy, in which the stock market is situated, can be represented by the following laws of motion

\[
x^F_t = \frac{1}{1 + \psi} \hat{E}_t x^F_{t+1} + \frac{\psi}{1 + \psi} (q^F_t - n\theta s_t)
- \frac{1}{1 + \psi} (i^F_t - \hat{E}_t \pi^F_{t+1} - r^F_{n,F}) + (\theta - 1)n \hat{E}_t \Delta s_{t+1} \tag{2.13}
\]

\[
\pi^F_t = \tilde{\beta} \hat{E}_t \pi^F_{t+1} + \lambda^F (1 + \varphi)x^F_t + n(\theta - 1) \lambda^F s_t + u^F_t \tag{2.14}
\]

\[
i^F_t = \rho^F i^F_{t-1} + (1 - \rho^F) r^F_{n,F} +
+ (1 + \chi^F_{\pi}) \pi^F_{t-1} + \chi^F_x x^F_{t-1} + \chi^F_q q^F_{t-1} + \varepsilon^F_t, \tag{2.15}
\]
which characterize aggregate demand, aggregate supply, and monetary policy in the foreign economy.

The dynamics of macroeconomic variables in the home country is, therefore, affected by the stock price dynamics in the foreign country, in which, we assume, the largest financial market is situated. The magnitude of the effect is an empirical question, on which the paper will try to shed light in the next section.

Expectations $\hat{E}_t$ can be formed differently in the two countries, that is $\hat{E}^H_t \neq \hat{E}^F_t$. This makes sense, since domestic economic agents will mostly make use of domestic economic variables to form their expectations, while foreign economic agents will rely on economic variables in their economy. The next section describes in more detail the assumed expectations formation and the empirical section will try to examine alternative possible cases.

### 2.4. Expectations.

I relax the assumption of rational expectations, by assuming that economic agents form near-rational expectations and learn about economic relationships over time.

Agents in country $i$ use a linear model as their Perceived Law of Motion (PLM)

$$Z^i_t = a^i_t + b^i_t Z^i_{t-1} + \epsilon^i_t$$

(2.16)

where $Z^i_t \equiv [x^i_t, \pi^i_t, i_t, q^F_t, s_t]'$, $a^i_t$ is a $5 \times 1$ vector and $b^i_t$ is a $5 \times 5$ matrix of coefficients. Agents are assumed not to know the relevant model parameters and they use historical data to learn them over time. Each period, they update their estimates of $a^i_t$ and $b^i_t$ according to the constant-gain learning formula

$$\hat{\phi}_t = \hat{\phi}_{t-1} + \bar{R} (R^i_t)^{-1} X^i_t \left[ Z^i_t - (X^i_t)' \hat{\phi}_{t-1} \right]$$

(2.17)

$$R^i_t = R^i_{t-1} + \bar{R} \left[ X^i_{t-1} (X^i_{t-1})' - R^i_{t-1} \right]$$

(2.18)
where (2.17) describes the updating of the learning rule coefficients collected in $\hat{\phi}_i = ((a_i^t)', \text{vec}(b_i^t))'$, and (2.18) characterizes the updating of the precision matrix $R_i^t$ of the stacked regressors $X_i^t \equiv \{1, x_{t-1}^i, s_{t-1}, \pi_{t-1}^i, \bar{y}_{t-1}, q_{t-1}^F, s_{t-1}\}_0^{t-1}$. $\tilde{g}^i$ denotes the constant gain coefficient.

The PLM assumes that economic agents in each country use only information in domestic output, inflation, and interest rates in forming expectations about the relevant variables. They do not incorporate foreign counterparts of such variables in their VAR. This is an assumption whose validity can be empirically tested in the model comparison section.

3. Estimation

3.1. Data. Countries

AUSTRALIA

CANADA

IRELAND

NETHERLAND

AUSTRIA

NEW ZEALAND

I use quarterly data on output gap, the inflation rate, the nominal interest rate for the domestic small open economy and for the foreign economy, plus data on the real stock price gap for the foreign economy, and on the (log) terms of trade (in deviation from a natural level).

The vector $\Theta$ collects the coefficients that need to be estimated:

$$\Theta = \left\{ \gamma, \theta, n, \lambda^i, \rho^i, \chi_n^i, \chi_x^i, \chi_s^i, \chi_{q_F}^i, \tilde{g}^i, Q^i \right\}$$

The model is estimated by likelihood-based Bayesian methods to fit the output gap, real stock price gap, inflation, and Federal Funds rate series. The estimation technique follows Milani (2007), who extends the approach described in An and Schorfheide (2007) to permit the estimation of DSGE
models with near-rational expectations and learning by economic agents. The results may depend on the assumed learning process, if this is imposed a priori. Therefore, here, I instead estimate also the learning process (which depends on the constant gain coefficient) jointly with the rest of structural parameters of the economy. In this way, the best-fitting learning process is extrapolated from actual data along with the best-fitting preference and policy parameters.

I use the Metropolis-Hastings algorithm to generate draws from the posterior distribution. At each iteration, the likelihood is evaluated using the Kalman filter. I consider 500,000 draws, discarding the first 25% as initial burn-in.

The priors for the model parameters are described in Table 1.

The foreign economy in the estimation is modeled both structurally as described in equations (2.13) to (2.15) and exogenously as a VAR in the endogenous foreign variables. In this way no restriction that the wealth effects should be equal across countries is imposed.

[Estimation Results to be included]

4. Conclusions

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References