A Two Sector Small Open Economy Model.  
Which Inflation to Target?  

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Abstract  
This paper analyses welfare-improving monetary policy reaction functions in the context of a new-Keynesian small open economy model with a tradables and a non-tradables sector. The model is estimated for the case of Canada and used to evaluate the welfare gains of alternative specifications of the feedback nominal interest rate rule, such as allowing for different coefficients on the inflation rate of the two sectors, traded and non-traded.  

We find reasonable estimates for the deep parameters of the model as well as reasonable quantitative responses of sectorial and aggregate variables to local and foreign shocks. We find welfare gains in responding somehow more aggressively to inflation deviations from target than it has been the case in the last three decades and substantial welfare losses if the Bank of Canada aimed at stabilizing output more aggressively. We compute the welfare gain of the optimal parameterization of a Taylor rule where the reaction to deviations with respect to target of the the non-tradable, the home-consumed tradable and the imported inflation rates are allowed to differ and we find substantial welfare gains of being more aggressive on imported inflation, which is the one which has a higher level of estimated stickiness.  

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1 Introduction

This paper analyses welfare-improving monetary policy reaction functions in the context of a new-Keynesian small open economy model with a traded and a non-traded sector estimated for the case of Canada. The model belongs to the class of open-economy dynamic general equilibrium models with explicit microfoundations, nominal rigidities and imperfect competition that constitute the so-called New Open Economy Macroeconomics (NOEM), pioneered by Obstfeld and Rogoff (1995) and that has become a substantial literature, part of whose results are summarized in Lane (2001), among others.

The main objective of this paper is twofold. Firstly, we want to evaluate quantitatively the response of the Canadian economy to domestic sectoral and aggregate shocks and to foreign shocks. For example, we want to be able to answer questions such as what are the sectorial reallocation effects of a real appreciation of the Canadian dollar such as the one observed in the recent past? Several dynamic stochastic general equilibrium models have been estimated for Canada (Ambler, Dib, Rebei (2003), Murchison (2004))**, none of which in a multisectoral setting.

Secondly, we want to know which would be the monetary policy reaction function that would deliver higher welfare given the estimated model. In particular, we compare the welfare gain of the optimal standard Taylor rule with alternative specifications of the feedback nominal interest rate rule, such as allowing for different coefficients on the inflation rate of the traded and non-traded sectors. To the best of our knowledge, this issue hasn’t been explored yet in the context of a multi-sector small open economy NOEM models.1

The main features of the model are that (i) monopolistic competition and Calvo-type price stickiness are assumed in the traded and non-traded sectors but are not imposed to be of equal intensity, (ii) labor and capital are mobile across sectors and each sector has its own technology process, (iii) traded goods are priced to market and (iv) the systematic behavior of the monetary policy is represented by the standard Taylor rule where nominal interest rates respond to deviations of overall inflation from target and to the output gap.

The model is estimated using Bayesian techniques for quarterly Canadian data. Our estimates are compatible with other small open economy estimated models in the NOEM literature such as Bergin (2003) or Ambler, Dib and Rebei (2003) for the Canadian case.

Our main findings are that the estimated degree of price rigidity is significantly different across sectors. Import prices are the more sticky (with an average duration of prices of over 7 quarters), then prices of non-tradables and prices of tradables are those found more flexible in Canada (with average duration of prices of 5 months)

1Kollman (2002) and Smets and Wouters (2002) are recent examples of papers where the issue of optimal monetary policy is investigated for small open economy NOEM models.
Section 2 describes the model. Section 3 describes the estimation method and discusses the parameter estimates. Section 4 discusses the quantitative implications of the model in terms of variance decomposition and impulse responses to sectoral and aggregate shocks as well as to foreign shocks. Particular attention is paid to the responses to a shock to the exogenous foreign interest rate, which directly translates into a shock to the exchange rate. Section 5 discusses the optimal parameterization for the monetary policy rule under alternative specifications. Section 6 concludes.

2 The model

2.1 Households

The representative household chooses consumption $c_t$, investment $i_t$, money balances $M_t$, hours worked $h_t$, local riskless bonds $Bd_t$, and foreign bonds $Bd^*_t$ that maximize its expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left( c_t, \frac{M_t}{P_t}, h_t \right)$$

where $\beta \in (0,1)$, $E_0$ is the conditional expectations operator, $M_t$ denotes nominal money balances held at the end of the period and $P_t$ is a price index that can be interpreted as the consumer price index (CPI). The functional form of time $t$ utility is given by

$$U(\cdot) = \frac{\gamma}{\gamma - 1} \log \left( c_t^{\frac{\gamma - 1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left( \frac{M_t}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) + \eta \log (1 - h_t),$$

where $\gamma$ and $\eta$ are positive parameters. Total time available to the household in the period is normalized to one.

The $b_t$ term is a shock to money demand. It follows the first-order autoregressive process given by

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt},$$

with $0 < \rho_b < 1$ and where the serially uncorrelated shock $\varepsilon_{bt}$ is normally distributed with zero mean and standard deviation $\sigma_b$. The household’s budget constraint is given by:

$$P_tC_t + P_t i_t (1 + CAC_t) + M_t + \frac{Bd_t}{R_t} + \frac{e_t Bd^*_t}{\kappa_t R^*_t} \leq$$

$$W_t h_t + R_t k_t + M_{t-1} + Bd_{t-1} + e_t Bd^*_{t-1} + T_t + D_t$$

where $CAC_t = \frac{1}{2} \left( \frac{t+1}{k_{t+1}} - \delta \right)^2 k_t$ is the cost faced each time household adjust his stock of capital $k_t$, $i_t$ is the investment, $W_t$ is the nominal wage rate, $R_t$ is the nominal interest on rented capital, $Bd_t^*$ and $Bd_t$ are foreign-currency and
domestic-currency bonds purchased in \( t \), and \( e_t \) is the nominal exchange rate. Domestic-currency bonds are used by the government to finance its deficit. \( R_t \) and \( R^{*}_t \) denote, respectively, the gross nominal domestic and foreign interest rates between \( t \) and \( t + 1 \). The household also receives nominal profits \( D_t = D^T_t + D^{NT}_t + D^M_t \) from domestic producers of tradable and non-tradable goods and from importers of intermediate goods, as well as nominal lump-sum transfers from the government \( T_t \).

\( \kappa_t \) is a risk premium that reflects departures from uncovered interest parity. It depends on the ratio of net foreign assets to domestic output:

\[
\log(\kappa_t) = \varphi \left[ \exp\left( \frac{e_t B d_t^*}{P_t y_t} \right) - 1 \right] + \varpi_{\kappa t}
\]

By following this functional form, the risk premium ensures that the model has a unique steady state. If domestic and foreign interest rates are equal, the time paths of domestic consumption and wealth follow random walks.\(^2\) We allow for an exogenous shock on the risk premium whose law of motion is

\[
\log(\varpi_{\kappa t}) = (1 - \rho_{\kappa}) \log(\varpi_{\kappa}) + \rho_{\kappa} \log(\varpi_{\kappa - 1}) + \epsilon_{\kappa t},
\]

with serially uncorrelated disturbance \( \epsilon_{\kappa t} \) normally distributed with zero mean and standard deviation \( \sigma_{\kappa} \), and with with \( 0 < \rho_{\kappa} < 1 \).

The foreign nominal interest rate, \( R^{*}_t \), is exogenous and evolves according to the following stochastic process:

\[
\log(R^{*}_t) = (1 - \rho_{R^{*}}) \log(R^{*}) + \rho_{R^{*}} \log(R^{*}_{t - 1}) + \epsilon_{R^{*} t},
\]

with \( 0 < \rho_{R^{*}} < 1 \) and where the serially uncorrelated shock, \( \epsilon_{R^{*} t} \), is normally distributed with zero mean and standard deviation \( \sigma_{R^{*}} \).

Households also face a no-Ponzi-game restriction:

\[
\lim_{T \to \infty} \left( \prod_{t=0}^{T} \frac{1}{\kappa_t R^{*}_t} \right) B d^{*}_T = 0.
\]

The first order conditions are as follows

\[
\frac{c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{1}{\gamma}}} = \lambda_t
\]

\[
\frac{b_t^{\frac{1}{\gamma}} m_t^{-\frac{1}{\gamma}}}{c_t^{-\frac{1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{1}{\gamma}}} = \lambda_t \left( 1 - \frac{1}{R_t} \right)
\]

\(^2\)For an early discussion of this problem, see Giavazzi and Wyplosz (1984). Our risk premium equation is similar to the one used by Senhadji (1997). For alternative ways of ensuring that stationary paths exist for consumption in small open-economy models, see Schmitt-Grohé and Uribe (2003).
\[
\frac{\lambda_t}{R_t} = \beta E_t \lambda_{t+1} \frac{1}{\pi_{t+1}} \tag{10}
\]

\[
s_t E_t \frac{\pi_{t+1}}{\kappa_t R_t} = E_t \frac{s_{t+1}}{R_t} \pi_{t+1} \tag{11}
\]

\[
\eta \frac{h_t}{1 - h_t} = \lambda_t w_t \tag{12}
\]

\[
\begin{align*}
\lambda_t & \left[1 + \chi \left( \frac{i_t}{k_t} - \delta \right) \right] = \\
& \beta E_t \lambda_{t+1} \left[1 + r_{k_{t+1}} + \chi \left( \frac{i_{t+1}}{k_{t+1}} - \delta \right) - \delta + \frac{\chi}{2} \left( \frac{i_{t+1}}{k_{t+1}} - \delta \right)^2 \right] \tag{13}
\end{align*}
\]

## 2.2 Firms

Monopolistically competitive firms produce tradable and non-tradable goods. The tradable goods are either imported or produced domestically, which in turn can either be sold home or exported.

### 2.2.1 Non-tradable sector

There is a continuum of firms indexed by \( j \in [0, 1] \) in the non-traded sector. There is monopolistic competition in the market for non-traded goods, which are imperfect substitutes for each other in the production of the composite imported good \( y_t^N \), produced by a representative competitive firm. Aggregate non-traded output is defined using the Dixit and Stiglitz aggregator function

\[
y_t^N = \left( \int_0^1 y_t^N(j) \frac{\sigma_{N-1}}{\sigma_N} dj \right)^{\frac{\sigma_N}{\sigma_N-1}}
\]

where \( \sigma_N \) is the elasticity of substitution between differentiated non-traded goods. Given the prices \( P_t^N \) and \( P_t^N(i) \), the non-traded final good-producing chooses the production, \( y_t^N \), that maximizes its profits. The first order condition corresponds to the demand constraint for each intermediary firm \( j \)

\[
y_t^N(j) = \left( \frac{P_t^N(j)}{P_t^N} \right)^{-\sigma_N} y_t^N \tag{14}
\]

where the price index for the composite imported goods is given by:

\[
P_t^N = \left( \int_0^1 P_t^N(j)^{-\sigma_N} dj \right)^{1-\sigma_N} \tag{15}
\]
Each monopolistically competitive firm has a production function given by

\[ y_t^N(j) = A_t^N [k_t^N(j)]^{\alpha^N} [h_t^N(j)]^{1-\alpha^N} \]

where \( A_t^N \) is the non-tradable sector specific total factor productivity that follows the stochastic process

\[ \log(A_t^N) = (1 - \rho_{AN}) \log(A_t^N) + \rho_{AN} \log(A_{t-1}^N) + \varepsilon_{AN,t} \]

with \( \varepsilon_{AN,t} \) a non-serially correlated technology shock normally distributed with zero mean and standard deviation \( \sigma_{AN} \).

Firms face a nominal rigidity coming from a Calvo type contract on prices. When allowed to do so (with probability \( (1 - d_N) \) each period), the producer of non-traded good \( j \) sets the price \( \tilde{P}_t^N(j) \) to maximize its weighted expected profits. Therefore, each individual firm chooses \( k_t^N(j), h_t^N(j), \) and \( \tilde{P}_t^N(j) \) through solving

\[
\max_{\{k_t^N(j), h_t^N(j), \tilde{P}_t^N(j)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta d_N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \frac{D_{t+l}^N(j)}{P_{t+l}} \right]
\]

where \( \lambda_t \) is the marginal utility of wealth for a representative household, and time \( t + l \) profits of the firm changing price at time \( t \) are

\[
D_{t+l}^N(j) \equiv \tilde{P}_t^N(j) y_{t+l}^N(j) - W_{t+l} h_{t+l}^N(j) - R_{t+l}^N k_{t+l}^N(j)
\]

The first-order conditions are:

\[
\frac{W_t}{P_t} = \xi_t(j)(1 - \alpha^N) \frac{y_t^N(j)}{\tilde{P}_t^N(j)}
\]

\[
\frac{R_t^k}{P_t} = \xi_t(j)\alpha^N \frac{y_t^N(j)}{\tilde{P}_t^N(j)}
\]

\[
\tilde{P}_t^N(j) = \left( \frac{\theta^N}{\theta^N - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \xi_{t+l}(j) y_{t+l}^N(j)}{E_t \sum_{l=0}^{\infty} (\beta d_N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^N(j) \frac{1}{P_{t+l}}} \]

where \( \xi_t(i) \) is the Lagrange multiplier associated with the production function constraint. It measures the non-tradable sector firm’s real marginal cost.

### 2.3 Tradable sector

Domestic firms producing good in the tradable sector have very similar problem except the fact that each monopolistically competitive firm \( k \) produces two types of goods, \( y_t^T(k) \) that will be consumed in the domestic market and \( y_t^X(k) \) that will be exported, for \( k \in [0, 1] \).

The production function is as follows

\[ y_t^T(k) = A_t^T [k_t^T(k)]^{\alpha^T} [h_t^T(k)]^{1-\alpha^T} \]
where $A^T_t$ is the tradable-sector specific technology process

$$\log(A^T_t) = (1 - \rho_A) \log(A^T) + \rho_A \log(A^T_{t-1}) + \varepsilon_{A^T_t}$$

and $\varepsilon_{A^T_t}$ is the serially uncorrelated shock which is normally distributed with zero mean and standard deviation $\sigma_{A^T_t}$.

Each individual firm chooses $k_t^T(k)$, $h_t^T(k)$, $P_t^{Td}(k)$, and $P_t^X(k)$. We assume complete pricing to market for exports, i.e. $P_t^X(k)$ is labelled in US dollars. In addition, once the firm has the chance to update its price (with probability $(1 - d_T)$ each period) it will choose simultaneously $P_t^{Td}(k)$ and $P_t^X(k)$. Export prices are set in foreign currency, following the literature on Pricing to Market.

The problem

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The problem of each firm can be summarized by

$$\max_{\{k_t^T(k), h_t^T(k), P_t^{Td}(k), P_t^X(k)\}} E_t \left[ \sum_{t=0}^{\infty} (\beta d_T)^t \left( \lambda_{t+1} \frac{D_{t+1}^T(k)}{P_{t+1}} \right) \right]$$

where time $t + l$ profits of the firm changing price at time $t$ are

$$D_{t+1}^T(k) = \tilde{P}_t^{T_d}(k) y_{t+1}^{T_d}(k) + e_t^1 \tilde{P}_t^X(k) y_{t+1}^X(k) - W_{t+1} h_{t+1}^T(k) - R_{t+1}^k k_{t+1}^T(k)$$

under the constraints dictating the local and foreign demand for tradable goods:

$$y_t^{T_d}(k) = \left( \frac{P_t^{T_d}(k)}{P_t^X} \right)^{-\vartheta} y_t^T$$

and

$$y_t^X(k) = \left( \frac{P_t^X(k)}{P_t^{T_d}} \right)^{-\vartheta} y_t^X$$

where $\vartheta$ is the elasticity of substitution between differentiated traded goods.

The first-order conditions are:

$$\frac{W_t}{P_t} = \zeta_t(k) (1 - \alpha^T) \frac{y_t^T(k)}{h_t^T(k)}$$

$$\frac{R_t^k}{P_t} = \zeta_t(k) \alpha^T \frac{y_t^T(k)}{k_t^T(k)}$$

$$\tilde{P}_t^{T_d}(k) = \left( \frac{\vartheta_T}{\vartheta_T - 1} \right) E_t \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \zeta_{t+l}(k) y_{t+l}^{T_d}(k)$$

$$\tilde{P}_t^X(k) = \left( \frac{\vartheta_T}{\vartheta_T - 1} \right) E_t \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \zeta_{t+l}(k) y_{t+l}^X(k)$$

where $\zeta_{t+l}(k)$ is the tradable sector firm’s real marginal cost.
Similarly, the final tradable good-producing sector has the following aggregate functions:

\[ y_t^{T_d} = \left( \int_0^1 y_t^{T_d}(k) \frac{\theta^{1-\theta}}{\theta^{1-\theta}} \, dk \right)^{\frac{\phi^{T}}{\theta^{1-\theta}}} \]  

(29)

and

\[ y_t^{X} = \left( \int_0^1 y_t^{X}(k) \frac{\phi^{X}}{\phi^{X-1}} \, dk \right)^{\frac{\phi^{X}}{\phi^{X-1}}} \]  

(30)

with

\[ y_t^T = y_t^{T_d} + y_t^{X} \]  

(31)

where \( y_t^T \) is the total production in the tradable goods sector, \( y_t^{T_d} \) and \( y_t^{X} \) are tradable goods respectively for domestic and foreign markets.

The price indices for domestically consumed tradables and exports are as follows:

\[ P_t^{T_d} = \left( \int_0^1 P_t^{T_d}(k) \frac{1}{1-\theta} \, dk \right)^{\frac{1}{1-\theta}} \]  

(32)

\[ P_t^{X} = \left( \int_0^1 P_t^{X}(k) \frac{1}{1-\theta} \, dk \right)^{\frac{1}{1-\theta}} \]  

(33)

The foreign demand for locally produced goods is as follows:

\[ y_t^{X} = \left( \frac{P_t^{X}}{P_t} \right)^{-\mu} y_t \]  

(34)

where \( \frac{\mu-1}{\mu} \) captures the elasticity of substitution between the exported goods and foreign-produced goods in the consumption basket of foreign consumers and \( y_t \), and \( P_t^{X} \) are, respectively, foreign output and price index. Both variables are exogenously given and foreign output and inflation follow the stochastic processes:

\[ \log(y_t^*) = (1 - \rho_{y^*}) \log(y^*) + \rho_{y^*} \log(y_{t-1}^*) + \epsilon_{y^*t} \]

\[ \log(\pi_t^*) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \epsilon_{\pi^*t} \]  

(35)

with \( 0 < \rho_{y^*}, \rho_{\pi^*} < 1 \) and where the serially uncorrelated shocks, \( \epsilon_{y^*t} \) and \( \epsilon_{\pi^*t} \), are normally distributed with zero mean and standard deviation \( \sigma_{y^*} \) and \( \sigma_{\pi^*} \), respectively.
2.4 Imported-goods sector

Finally, there is a continuum of intermediate good-importing firms indexed by \( i \in [0, 1] \). There is monopolistic competition in the market for imported intermediates, which are imperfect substitutes for each other in the production of the composite imported good, \( y^M_t \), produced by a representative competitive firm. We also assume Calov-type staggered price setting in the imported goods sector in order to capture the empirical evidence on gradual exchange rate pass-through into import prices\(^3\). Thus, when allowed to do so (with probability \((1 - d_M)\) each period), the importer of good \( i \) sets the price, \( \tilde{P}^M_t(i) \), to maximize its weighted expected profits. It solves:

\[
\begin{aligned}
\max_{\{\tilde{P}^M_t(i)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \frac{D^M_{t+l}(i)}{P_{t+l}} \right] & \\
\end{aligned}
\]  

where time \( t + l \) profits of the firm changing price at time \( t \) are:

\[
D^M_{t+l}(i) = \left( \tilde{P}^M_t(i) - e_{t+l}P^*_t \right) \left( \frac{\tilde{P}^M_t(i)}{P^*_t} \right)^{-\vartheta^M} y^M_{t+l}
\]  

with \( \vartheta^M \) representing the elasticity of substitution across differentiated imported goods. Note that the marginal cost of the importing firm is \( e_tP^*_t \) \(^4\) and thus its real marginal cost is the real exchange rate \( s_t \equiv \frac{e_tP^*_t}{P_t} \). The first-order condition is:

\[
\tilde{P}^M_t(i) = \left( \frac{\vartheta^M}{\vartheta^M - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y^M_{t+l}(i)e_{t+l}P^*_t/P_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y^M_{t+l}(i)/P_{t+l}}
\]  

As in the other cases, aggregate imported output is defined using the Dixit and Stiglitz aggregator function

\[
y^M_t = \left( \int_0^1 y^M_t(i) \frac{\vartheta^M - 1}{\vartheta^M} di \right)^{\frac{\vartheta^M}{\vartheta^M - 1}}
\]  

and the price index for the aggregated good is

\[
P^M_t = \left( \int_0^1 P^M_t(i)^{1-\vartheta^M} di \right)^{\frac{1}{1-\vartheta^M}}
\]  

\(^3\)Campa and Goldberg (2001) find that they can reject the hypothesis of complete short-run pass-through in 22 of the 25 OECED countries of their study for the period 1975-1999, but they find complete long-run pass-through. Ghosh and Wolf (2001) argue that sticky prices or menu cost are a preferable explanation for imperfect pass-through since it’s compatible with complete long-run pass-through, while that’s not the case of explanations based on international product differentiation.

The evidence of incomplete exchange rate pass-through in Canada is well documented and seems to conclude that it’s moved towards almost zero pass-through in the recent past. See for example Kichian (2003).

\(^4\)For convenience, we assume that the price in foreign currency of all imported intermediates is \( P^*_t \), which is also equal to the foreign price level.
2.5 Final goods aggregators

The final domestically consumed good, $y^d_t$, is produced by a competitive firm that uses non-traded goods, $y^N_t$, and domestically consumed tradable goods, $y^T_d$, as inputs subject to the following CES technology

$$y^d_t = \left[ n^{\frac{\phi}{\sigma}} (y^N_t)^{\frac{\sigma-1}{\sigma}} + (1 - n)^{\frac{\phi}{\sigma}} (y^T_d)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma}}$$

(40)

where $n > 0$ is the share of non traded goods in the domestic goods basket at the steady state and $\frac{\sigma-1}{\sigma} > 0$ is the elasticity of substitution between non-traded and non-exported tradable goods. Profit maximization entails

$$y^N_t = n \left( \frac{P^N_t}{P^d_t} \right)^{-\phi} y^d_t$$

(41)

and

$$y^T_d = (1 - n) \left( \frac{P^T_d}{P^d_t} \right)^{-\phi} y^d_t$$

(42)

Furthermore, the domestic final-good price, $P^d_t$ is given by

$$P^d_t = [n(P^N_t)^{1-\phi} + (1 - n)(P^T_d)^{1-\phi}]^{1/(1-\phi)}$$

(43)

Finally, we aggregate domestic and imported goods using a CES function as follows

$$z_t = \left[ m^{\frac{1}{\nu}} (y^d_t)^{\frac{\nu-1}{\nu}} + (1 - m)^{\frac{1}{\nu}} (y^M_t)^{\frac{\nu-1}{\nu}} \right]^{\frac{1}{\nu}}$$

(44)

where $m > 0$ is the share of domestic goods in the final goods basket at the steady state; and $\frac{\nu-1}{\nu} > 0$ is the elasticity of substitution between domestic and imported goods. The first order conditions are

$$y^d_t = m \left( \frac{P^d_t}{P_t} \right)^{-\nu} z_t$$

(45)

and

$$y^M_t = (1 - m) \left( \frac{P^M_t}{P_t} \right)^{-\nu} z_t$$

(46)

The final-good price, $P_t$, which corresponds to the consumer price index or CPI, is given by

$$P_t = [m(P^N_t)^{1-\nu} + (1 - m)(P^M_t)^{1-\nu}]^{1/(1-\nu)}$$

(47)

Aggregate output is used for consumption, investment, and covering the cost of adjusting capital

$$z_t = c_t + i_t (1 + CAC_t)$$

(48)

The gross domestic product is $y_t = y^N_t + (y^T_d + y^X_t) - y^M_t$. Finally, sectoral hours and capital simply sum to the aggregate hours and capital offered by households (i.e $h^N_t + h^T_t = h_t$ and $k^N_t + k^T_t = k_t$).
2.6 The government

The government budget constraint is given by

$$T_t + Bd_{t-1} = M_t - M_{t-1} + \frac{Bd_t}{R_t}$$  \hspace{1cm} (49)

which is combined with the no-Ponzi-game restriction:

$$\lim_{T \to \infty} \left( \prod_{t=0}^{T} \frac{1}{R_t} \right) Bd_T = 0$$

We consider a simple decision rule for nominal interest rate such as the standard Taylor rule

$$\log(R_t/R) = \theta_R \log(R_{t-1}/R) + \theta_\pi \log(\pi_t/\pi) + \theta_y \log(y_t/y) + \varepsilon_{R_t}, \hspace{1cm} (50)$$

where $R$, $\pi$, and $y$ are the steady-state values of the gross nominal interest rate, CPI inflation, and real gross domestic output, and where $\varepsilon_{R_t}$ is a zero-mean, serially uncorrelated monetary policy shock with standard deviation $\sigma_R$.

3 Estimation

The above model is estimated using Bayesian estimation techniques that update prior distributions for the deep parameters of the model, which are defined according to a reasonable calibration, with the actual data. The estimation is done using recursive simulation methods, in particular the Metropolis-Hastings algorithm, which have been applied to estimate similar dynamic stochastic general equilibrium models in the literature, such as Smets and Wouters (2000****).

The model has 8 shocks processes: three common domestic shocks – monetary policy shocks $\varepsilon_{R_t}$, to the money demand $\varepsilon_{bt}$, and to the risk premium $\varepsilon_{rt}$ – two sector specific technology shocks – to the non-tradable sector $\varepsilon_{ANt}$ and to the tradable one $\varepsilon_{ANt-}$ and three foreign shocks – output $\varepsilon_{yt}$, inflation $\varepsilon_{\pi t}$ and nominal interest rate $\varepsilon_{Rt}$. In order to identify them in the estimation process we need to use the same number of actual series. We choose them to be as informative as possible. We use HP-filtered**** seasonally adjusted***** quarterly series for Canada for the period 1972q1-2003q4. The series are real exports, real imports, CPI inflation, services inflation, real M2 (deflated with the CPI), and US output, US CPI inflation and nominal US interest rate on 3-month T-bills.

Table 1 shows the prior distributions we have imposed for the deep parameters of the model as well as the median and 90 percent confidence interval for the posterior distributions. Figures 1 and 2 convey the same information by drawing the prior distributions, in red thick lines, together with the posterior ones, in thin blue lines.
We have borrowed some of the prior distributions from the literature but for those we didn't have references we have used our best common sense while trying to construct little restrictive priors. We have selected beta distributions for those coefficients we wanted to restrict to lie between 0 and 1, such as the autocorrelation coefficients of the shock processes or the share parameters. Gamma distributions are imposed when required to guarantee real positive values.

All three sectors, domestic tradable, imports and non-tradable and treated symmetrically. They are given the same degree of nominal rigidity, in the form of an average prior probability of not changing prices of 0.75 which corresponds to changing prices every 4 quarters on average. Elasticities of substitution between differentiated goods are also equal across sectors, corresponding to equal steady state markups across sectors.

Some parameter values are taken as fixed rather than given a prior distribution that will be updated with the data, and calibrated to values similar to the ones found in the literature. We have performed sensitivity analysis on their calibrated values and observed that the estimates of the rest of the model parameters were unchanged. These parameters are: the subjective discount rate, $\beta = 0.99$, which implies an annual real interest rate of 4 percent; the weight of leisure in the utility function, $\eta$, which is calibrated to yield a steady state share of time devoted to market activities of 30 percent; the depreciation rate of capital, $\delta = 0.025$, and the gross steady state markups in all sectors, $\frac{\sigma}{\sigma - 1} = 1.14$, which lies between the estimates of the empirical literature between 10 and 20 percent (see, for example, Basu (1995)).

We find that data is most informative for the adequate parameterization of the price stickiness, the Taylor rule and the shocks processes.

The prior of equal nominal rigidity across sectors does not hold, consistently with the findings of Bils and Klenow (2004), who document a high degree of heterogeneity in the frequency of price changes across retail goods and services. We find indeed significant heterogeneity in the degree of price stickiness across sectors, as shown in Table 1 by the fact that the 90 percent posterior confidence intervals for $d_M$, $d_N$ and $d_T$ do not even overlap. Figure 2 also shows how the equal prior distributions for $d_M$ and $d_N$ barely overlap with their respective posterior distributions, and do not at all in the case of $d_T$.

This is an important finding and will condition many of the model implications for the dynamics as well as for the welfare improvement of alternative specifications of the monetary policy reaction function. In particular, it is imported goods prices the ones showing more sluggishness, with a posterior median duration of prices of over 7 quarters, while the domestic traded sector faces the lowest price stickiness, with posterior median duration of just above 5 months. Non-tradables are in-between, but still more flexible than the usual assumption for aggregate price stickiness\(^5\), showing posterior median duration of prices of

\(^5\)Our sectoral estimates bridge the gap between usual estimates of around 4 quarters for the aggregate price level and the microeconomic evidence of average duration of prices at the individual firm level of around one quarter. In a back-of-the-envelope calculation, if we weight the sectorial posterior median durations by the posterior median estimates of the steady state
around 21/2 quarters.

In an estimated model for Canada with imported and domestic goods, without distinction between tradables and non-tradables, Ambler, Dib and Rebei (2003) also find that the maximum likelihood estimate for import price stickiness is higher than that of domestic goods and are of similar magnitude to what we find for our imported and non-tradable sectors. Smets and Wouters (2002) find in a similar model estimated for the Euro Area that import prices are about as sticky as domestic ones. The higher price stickiness in import prices found in Canada with respect to domestic price stickiness is consistent with the low degree of exchange rate pass-through typically found in the case of Canada, as discussed earlier. Compared to these two papers, the fact that this paper disaggregates further into tradables and non-tradables allows us to unveil further heterogeneity and to find estimates of lower price stickiness for the tradable sector.

The posterior estimates of the Taylor rule maintain the prior of high interest rate smoothing (although even higher, posterior median \( \rho_R = 0.9 \)) and inflation coefficient of \( \rho_y \) of 2 but find no significant reaction to the output gap, while in the prior average \( \rho_y \) is 0.2 (there is almost no overlap between the prior and posterior distributions in figure 2). The historical estimated Taylor rule, therefore, is of a strict inflation targeting with high sluggishness in the monetary policy instrument.

The actual data is also found very informative for estimating the volatility of shocks, which were given equal priors. Posterior estimates indicate that the technology shocks, especially in the non-tradable sector, are the more volatile, followed by shocks to the risk premium.

Data, however, is found little informative for some parameters whose posterior distributions are very coincident with their priors. In particular, this is the case of preference parameters such as the one governing the elasticity of substitution between consumption and real balances, \( \gamma \).

4 Quantitative Implications of the model

This section discusses the dynamics of the estimated model in terms of the variance decomposition of its main endogenous variables and in terms of their impulse responses to the shocks contemplated in the model. The mechanisms through which

4.1 Variance decomposition

Table 2*not 3!!** shows the decomposition of the long-run variance of the main endogenous variables of the model into the contribution of each of the eight shocks. The business cycle volatility of the output in each production sector, weights of the sectoral outputs in final consumption, we obtain an overall economy duration of prices of 3 1/2 quarters.
tradables and non-tradables, is mainly explained by its corresponding sector-specific technology shock. Aggregate inflation is explained in more than 84% by monetary policy shocks. Final spending, i.e. consumption and investment, are mainly explained by the non-tradable technology shock, which was the shock with higher estimated volatility, although the steady state share of the non-traded sector in final good is only $\frac{1}{3}$. The volatility of the real exchange rate are explained by the risk premium, technology and foreign monetary policy.

4.2 Responses to a foreign shock

Figure 3 represents the responses in terms of percentage deviations with respect to the steady state for all variables to a one-period increase of one per cent in the monetary policy instrument of the foreign economy, the U.S.

The uncovered interest parity yields a nominal and real impact depreciation of the Canadian dollar (2.5 percent posterior median depreciation on impact of the real exchange rate, $s_t$). The real depreciation rises directly the marginal cost of the importing firms and is therefore translated into a higher import prices and lower imports, $y_t^m$. It is important to note, however, due to the high estimated sluggishness of import prices the exchange rate pass-through is far from complete and imports inflation rises only by 0.15 percent or by 0.15% same for $R$, $R^*$, and all infl.

The tradable sector benefits from the depreciation. Because exports are priced in the foreign currency but tradable-sector firms maximize their profits in Canadian dollars, the depreciation by itself increases the benefits from the part of the production that is exported. Because of that, traded-sector producers lower export prices and increase their exports on impact. The overall production in the tradable sector, therefore, rises.

The small increase of imports inflation makes aggregate inflation rise, which causes a monetary policy contraction. That in turn decreases demand (${c_t}$ and $i_t$) that further reduces imports demand but also decreases demand of non-tradables and of tradables to be consumed domestically.

The fact that only the tradable sector is reacting positively to the shock, due to exports growth, causes a reallocation of resources from the non-tradable to the tradable sector. An impact real depreciation of 2.5 percent increases $y_t^T$ by 0.15 per cent ($h_t^T$ by 0.06 per cent) but decreases $y_t^{NT}$ by 0.13 percent ($h_t^{NT}$ by 0.04 percent).

4.3 Responses to a sectorial shock

Figure 4 represents the responses to a positive one-period technology shock in the non-tradable sector only of 1 per cent.

Increased production in the non-tradable sector rises demand all throughout.

---

As is well known in the sticky price literature, sticky prices prevent the 1 percent increase in total factor productivity to be fully transformed into a 1 per cent increase in $y_t^{NT}$. Since capital is predetermined, the only possible way is by reducing hours worked on impact, which
the economy and therefore increases output in the tradable and imports sectors.

Prices in the non-traded sector fall on impact leading to a mild fall in overall inflation, which in turn causes an expansionary reaction of the monetary policy that feeds into further increase of demand and also causes an impact nominal and real depreciation.

Increased demand increases imports as well as import inflation, which helps undo the impact fall of aggregate inflation quite quickly.

As before, the depreciation increases the profits of the exported production in the tradable sector but exports demand does not rise (foreign output being exogenous). Thus, tradable-sector profit maximization makes firms lower export prices fixed in US dollars (pricing to market) and increase exports.

4.4 Responses to a common domestic shock

Figure 5 represents the responses to a temporary monetary policy contraction. The nominal interest rate shock increases by 1 per cent for one-period. On impact, the monetary policy instrument rises by less than 1 per cent because of the immediate fall in inflation. In fact, nominal interest rates rise by only \( \frac{1}{6} \) of the 1 percent shock. Inflation falls on impact due to the impact decrease in demand and consequently in activity in every sector, tradables, non-tradables and imports.

The monetary policy contraction causes a nominal and real impact appreciation of the Canadian dollar. Exports prices being set in US dollars, the appreciation reduces exporters’ profits and thus export prices rise, which causes a fall of exports. The impact reaction depends on the magnitude of the exchange rate appreciation.

5 Optimal Monetary Policy

Once the structural parameters of the model are estimated, we optimize the unconditional welfare of households over the parameters of the Taylor rule. This implies:

\[
\max_{\rho_{\pi}, \rho_{\mu}} E \{ u(c_t, m_t, h_t) \},
\]

where \( \rho_{\pi} \) reflects the reaction of the monetary authority to the deviation of each inflation from its long run level.

Implicitly, we focus on identifying the welfare measure of with the unconditional expectation of lifetime utility, therefore, we don’t look at the initial state of the economy issue. Schmitt-Grohé and Uribe (2003) adopt the conditional welfare optimization in their framework and they consider the non-stochastic steady state as an initial state of the economy. From our point of view, transition costs are crucially dependant on that initial state especially if the real state is observed in figure 4. \( h_t^{NT} \) falls on impact but increases after 4 quarters.
of the economy is never at the deterministic level. In addition, Schmitt-Grohé and Uribe (2003) show that the optimal rule is robust to which definition of welfare is considered, however, the welfare improvement could be different in the sense that it is higher in the case of unconditional welfare given that no short term transition costs are incurred.

We measure the welfare gain associated with a particular monetary policy by means of the compensation variation. We measure the percentage change in consumption given the equilibrium with the historical values of the coefficients in the Taylor rule that would give households the same unconditional expected utility as in the aforementioned scenario. The compensating variation is defined as follows:

$$E \{u(c_t(1 + \text{compensation}), m_t, h_t)\} = E \{u(\tilde{c}_t, \tilde{m}_t, \tilde{h}_t)\},$$

where variables without tildes are obtained under the historical rule, and variables that have tildes are under the optimized Taylor rule.

The next step is to understand the origin of welfare improvement, then, we decompose the compensating variation in consumption to a level effect and a stabilization effect. It is possible to summarize the extent to which the gains in welfare come from the effects of the changes in policy regarding the average levels of consumption, real balances, and leisure versus changes in their volatilities. We can approximate the difference between welfare under optimal policy and the estimated values of the Taylor rule coefficients as follows:

$$E\{u(\tilde{z}_t)\} - E\{u(z_t)\} = u(z) + u_z E\left(\tilde{z}_t\right) + \frac{1}{2} E\left(\tilde{z}_t\right)' u_{zz} \tilde{z}_t - u(z) - u_z E\left(\hat{z}_t\right) - \frac{1}{2} E\left(\hat{z}_t\right)' u_{zz} \hat{z}_t + O(3),$$

where $z_t \equiv (c_t, m_t, h_t)$ is the vector of arguments of the utility function, $z$ is the value of these arguments in the deterministic steady state, and variables with hats measure deviations from their levels in the deterministic steady state. This implies:

$$E\{u(\tilde{z}_t)\} = E\{u(z_t)\} + u_z E\left(\tilde{z}_t - \hat{z}_t\right) + \frac{1}{2} E\left(\tilde{z}_t - \hat{z}_t\right)' u_{zz} \left(\tilde{z}_t - \hat{z}_t\right).$$

This allows us to decompose the gains in welfare from optimal monetary policy into a level effect and a stabilization effect. We define the level effect as:

$$E \{u(c_t(1 + \text{compensation}_L), m_t, h_t)\} = Eu(\tilde{z}_t) + u_z E\left(\tilde{z}_t - \hat{z}_t\right),$$

and we define the stabilization effect as:

$$E \{u(c_t(1 + \text{compensation}_S), m_t, h_t)\} = Eu(\tilde{z}_t) + \frac{1}{2} E\left(\tilde{z}_t - \hat{z}_t\right)' u_{zz} \left(\tilde{z}_t - \hat{z}_t\right).$$

The last two rows of Table 3 show the results. The overall effect in all cases is such that, approximately:

$$(1 + \text{compensation}) \approx (1 + \text{compensation}_L)(1 + \text{compensation}_S).$$
In our search for the optimal rule, we limit attention to the rules implying the existence of a unique and stable equilibrium in the neighborhood of the deterministic steady state.

We restrict our search to optimal reactions to inflation gap and output gap, we do this by keeping the degree of nominal interest smoothing, $R_B$, unchanged and equal to the historical median value. Nonetheless, it is fairly known that in this type of models the optimal rule lacks inertia in order to have better control on inflation and output fluctuations.

5.1 CPI inflation rate targeting

First, we consider the case where a central targets the CPI inflation. Figure ? shows the welfare surface with respect to $\rho_x$ and $\rho_y$ holding constant the degree of policy inertia. The welfare measure corresponds to a second order approximation of $Eu_t = E \left\{ \gamma \left( \frac{2}{\gamma - 1} c_{t+1}^{\gamma - 1} + b_{t+1}^4 m_{t+1}^{\frac{4}{\gamma - 1}} \right) + \eta \log(1 - h_t) \right\}$. The welfare surfaces appear to be piecewise smooth in $\rho_x$ and $\rho_y$. All combinations of $\rho_x$ and $\rho_y$ exhibit cases where equilibrium is determinate and the optimized Taylor rule is far from the indeterminacy region: $\rho_x = 9.20$ and $\rho_y = 0.20$. The inflation gap parameter seems to be quite high (1 per cent increase of inflation should lead to a nominal interest rate higher than its steady state by 9 per cent), however, this result is common for this type of models, and most importantly starting from the neighborhood of the value 3 for $\rho_x$ and 0.1 for $\rho_y$, the utility function becomes very flat and welfare improvements become insignificant.

The historical rule entails a welfare cost of 0.0758 per cent of the lifetime consumption associated with the optimal rule (see Table 4). Most of the welfare improvement by choosing $\rho_x = 9.20$ and $\rho_y = 0.20$ is obviously coming from the first level effect on average levels of consumption, real money balances, and hours, which is evaluated to 0.0887 per cent of average consumption per period. The optimal Taylor rule implies more instability in the utility which is reflecting a negative second order effect. This can be explained by the relatively high volatility of consumption as shown in Table 5. Despite the increase in the average of the real money balances, the effect on welfare is not significant given the very low weight on this variables in the utility function.

An important result coming from the lecture of Figure 8 is that not reacting sufficiently enough or very aggressively to the output gap can be very damageable in terms of welfare losses. This arises especially for less aggressive decision towards inflation gap where the welfare cost of the suboptimal rule is monotonically increasing in $\rho_y$.

5.2 Various inflation rates targeting

Now we turn the attention to the optimal reaction of the monetary policy to sectorial inflation rates: imports, tradables, and non-tradables. Here we ask the following question: Is there any gain for targeting individual inflation rates
rather than the aggregate CPI inflation, and if any gain, how much is the welfare improvement for households?

The main objective is to compare welfare for different combinations of the monetary authority reactions to $\pi^m_t$, $\pi^N_t$, and $\pi^Td_t$. Again policy inertia is set to the estimated value, and we set $\rho_y = 0.20$ corresponding to the optimal value with CPI inflation Taylor rule. We do so in order to diminish considerably the time of optimizing the monetary rule with different inflation rates.\footnote{The examination of the welfare function shape shows ... which motivates our choice of finding the optimal interest rate rule using a grid search over the policy parameters rather then relying local optimizing routines.}

Figure 9 shows the shape of the unconditional utility for different combinations of values of $\rho_{\pi m}$, $\rho_{\pi N}$, and $\rho_{\pi Td}$ changing through the interval (0.40, 5.00). Again welfare function looks very smooth with respect to all parameters. Results suggest that monetary authority should react mildly to non-tradables’ inflation, $\rho^opt_{\pi N} = 1.00$, aggressively to tradables’ inflation, $\rho^opt_{\pi Td} = 2.60$, and more aggressively to the import sector inflation, $\rho^opt_{\pi m} = 5.00$. Note, on one hand, that welfare gains are less significant starting from close to 5.00 for $\rho_{\pi m}$. On the other hand, reacting too little or very aggressively to non-tradables’ inflation has important welfare losses, also reacting weakly to the inflation of the tradable goods sector can generate the same important utility deteriorations.

Consequently, the central bank should react more aggressively towards imported goods inflation movements. This seems consistent with the point estimates of price stickiness in the three sectors which exhibit more price contract duration for the imports sector. As shown by Smets and Wouters (2002) sticky import prices generates low pass-through which makes exchange rate channel less effective in terms of adjustment to foreign shocks. As a result more of the adjustment needs to be led by the domestic interest rate movements which is, indeed, translated to domestic demand creating more fluctuations in output and consumption. Once the monetary authority can control all inflation rates, the reaction to inflation in the imports sector will be higher given the important distortion present in this market compared to the tradables and non-tradables sectors. Doing so the monetary authority can realize higher welfare improvement than the CPI inflation targeting case, and the households consumption can raise by 0.1545 per cent permanently compared to the historical case. Most of the additional welfare is coming from a first level improvement, 0.1757 per cent, it self is tributary to a permanent average increase for consumption even though its volatility is also increasing.

5.3 The impact of sticky prices: A counterfactual exercise

In the following we analyze the effect of the main sources of distortion in the model, mainly, we focus on the individual impact of sectorial price rigidity on the level of welfare. In the case where we set $d_N \simeq 0$, only price stickiness in the tradables and imports sectors are in action, in addition to the imperfect competition in all goods’ markets. Rows corresponding to ‘Flexible $N$’ in Table ? show that the historical rule, in addition to both versions of the optimal
rule, don't improve welfare compared to the deterministic case where the only distortion present in the model is coming from the monopolistic competition. Consequently, the distortion generated by the non-tradables sector is playing a minor role. In contrast, when we set prices in the tradables sector to be flexible, we are able to generate positive welfare compared to the deterministic case. In other words, targeting inflation lead to overcome the distortions caused by price stickiness in the non-tradables and importing sectors in addition to the monopolistic competition. Surprisingly, in the case for fully flexible imports prices we observe a welfare deterioration. This result can be related to two possible channels. The first, and most obvious, is the cost of changes in the exchange rate that carries a high relative price volatility in the import sector. The second is less obvious and it relates to Bergin and Tchakarov (2004) findings. Particularly, in an open economy where asset markets are characterized by 'original sin,' where the country is unable to issue debt in the international market denominated in its own currency, saving decisions and the reallocation of welfare are affected significantly away from the country suffering from original sin towards the rest of the world. Once we set the risk premium parameter, $\varphi$, high enough, meaning that the foreign bonds tend to be negligible, welfare deterioration reported in Table 7 tend to dramatically drop.

Finally, the fully flexible price version of the model exhibits high levels of welfare increasing in comparison to the steady state case where all shocks fluctuations are set to zero. This is explained by the fact that targeting inflation in any case will lead to partially correct for the monopolistic competition impact by increasing sectorial productions and then increasing households unconditional average utility.

6 Conclusions
References


Table 1: Parameter Estimation Results

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<th>Std. error or df</th>
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<td>$\alpha_{N}$</td>
<td>Beta</td>
<td>0.34</td>
<td>0.05</td>
<td>0.3571</td>
<td>[0.2744, 0.4777]</td>
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<tr>
<td>$\alpha_{T}$</td>
<td>Beta</td>
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<td>0.05</td>
<td>0.4715</td>
<td>[0.3992, 0.5299]</td>
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### Table 2: Variance Decomposition

<table>
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<tr>
<th>Variable</th>
<th>( A_t^N )</th>
<th>( A_t^T )</th>
<th>( R_t )</th>
<th>( b_t )</th>
<th>( R_t^N )</th>
<th>( y_t^N )</th>
<th>( \pi_t^N )</th>
<th>( \omega_t )</th>
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<tbody>
<tr>
<td>( y_t^N )</td>
<td>[93.96, 98.36]</td>
<td>[1.67]</td>
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<td>[0.00]</td>
<td>[0.25]</td>
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<td>[0.03]</td>
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<tr>
<td>( y_t^T )</td>
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<td>[66.48]</td>
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<td>[0.11]</td>
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<tr>
<td>( y_t^T )</td>
<td>[14.73, 29.07]</td>
<td>[23.65]</td>
<td>[1.45]</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[9.21]</td>
<td>[3.79]</td>
<td>[0.00]</td>
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<tr>
<td>( y_t^m )</td>
<td>[31.93, 48.32]</td>
<td>[16.59]</td>
<td>[5.64]</td>
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<td>[6.07]</td>
<td>[6.08]</td>
<td>[0.00]</td>
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<tr>
<td>( c_t )</td>
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<td>[14.29]</td>
<td>[1.28]</td>
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<td>[4.59]</td>
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<td>[31.17]</td>
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<tr>
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<td>[15.85]</td>
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<td>( i_t )</td>
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<td>( \pi_t )</td>
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<td>[11.42]</td>
<td>[83.78]</td>
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<td>[30.64]</td>
<td>[23.06]</td>
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<tr>
<td>( \pi_t^m )</td>
<td>[38.16, 50.85]</td>
<td>[28.57, 34.90]</td>
<td>[16.94, 30.66]</td>
<td>[0.00]</td>
<td>[0.00]</td>
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<td>[0.19]</td>
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<tr>
<td>( \pi_t^x )</td>
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<td>[5.12]</td>
<td>[10.24]</td>
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<td>[0.70, 25.74]</td>
<td>[14.42, 20.69]</td>
<td>[0.39]</td>
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<td>[0.00]</td>
<td>[0.00]</td>
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<tr>
<td>( R_t )</td>
<td>[20.22]</td>
<td>[33.93]</td>
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Table 3: Welfare Implication for Different Taylor Rule Specifications

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<tr>
<td></td>
<td>TE</td>
<td>FLE</td>
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<tr>
<td><strong>Sticky (N, T_d), and (M)</strong></td>
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<tr>
<td>Historical</td>
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<td>-0.2919</td>
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<td><strong>Flexible (N)</strong></td>
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<td>-0.3064</td>
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<td><strong>Flexible (T_d)</strong></td>
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Table 4: Impact of the Taylor Rule’s Coefficients on First and Second Moments

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<td>sd</td>
<td>Average</td>
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