Asymmetry and nonlinearity in Uncovered Interest Rate Parity

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Abstract

This paper provides empirical evidence that the relationship between spot exchange rate change and the lagged forward premium display significant nonlinearities and asymmetry. The nonlinear dynamics is consistent with general implications of recent theories based on transactions costs, and/or limits to speculation in the foreign exchange market. Evidence of asymmetry in the uncovered interest rate condition is consistent with findings of some recent work on UIP. The reported evidence in the paper suggests that the uncovered interest rate parity (UIP) anomaly documented in the literature may not indicate major inefficiencies in foreign exchange markets. Implications of our empirical evidence for the UIP anomaly and the predictability of exchange rate returns on the basis of lagged forward premium are investigated through Monte Carlo experiments. Results indicate reconciliation of our empirical results with the extant empirical literature on the UIP anomaly as the experiments show that if the true data generating process of UIP condition was of the nonlinear form we consider, estimation of conventional linear UIP regression and predictability regression would generate the well known puzzles documented in the previous literature.

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1 Introduction

The uncovered interest rate parity (UIP) condition hypothesizes that the return on domestic currency deposit should be equal to the expected return from converting the domestic currency into foreign currency, investing it in a foreign currency denominated asset and then converting the proceeds back into the domestic currency at the future expected exchange rate. That is to say that the expected change in the domestic price of a foreign currency should be just offset by the opportunity cost of holding funds in one currency rather than the other, i.e. the interest rate differential. Assuming that the covered interest rate parity condition (i.e. approximately the interest rate differential should be equal to forward premium between two currencies) and the rational expectations, UIP implies that forward premium should be an unbiased predictor of the expected change in the spot nominal exchange rate. In particular, the slope coefficient from the regression of the change in spot rate on the lagged forward premium should be one.

The widespread empirical finding that the spot exchange rate change on most freely floating nominal exchange rates up until 1990s appear to be negatively correlated with the lagged forward premium or forward discount is referred to be the “forward premium anomaly” or “forward premium puzzle” (see, Baillie and Bollerslev 2000). More specifically, contrary to the prediction of UIP, a regression of exchange rate change on lagged forward premium indicates that a non-negative interest rate differential would, on average, result in an appreciating currency for the high interest rate country, (see, Fama 1984). This finding sometimes also referred as “forward bias puzzle” because it may be interpreted to imply that the forward rate is a biased predictor of the future spot rate. The UIP anomaly prompted a large literature and continues to spur new papers. As pointed out in Sarno and Taylor (2002), regardless of the increasing sophistication of the econometric techniques employed and of increasing quality of the data sets utilized, empirical findings indicate rejection of UIP among major currencies during free floating period. Over the years several possible explanations for the anomaly have been suggested, including a time-varying risk premia, (e.g., Hodrick 1987), peso problems and bubbles (e.g. Lewis 1995), irrational behavior of market participants and heterogenous trading behavior (e.g. Frankel and Froot 1987). Excellent surveys of the literature are provided by Hodrick (1987), Engel (1996), and Sarno and Taylor (2002, Ch. 2). While all of the potential explanations suggested in the literature appear fairly reasonable, the forward premium anomaly has not been convincingly explained and continues to puzzle the international finance profession.

This paper contributes to the literature on UIP by investigating potential nonlinearity and asymmetry in the relationship between expected exchange rate changes and forward premium. The extant empirical research on UIP has generally relied on linear framework. Recently, several papers
have shown that the relationship between expected exchange rates and interest rate differentials (and hence forward premium) may be nonlinear due to presence of transactions costs (Baldwin 1990, Dumas 1992, Hollifield and Uppal 1997), central bank intervention (Mark and Moh 2004), and presence of limits to speculation (Lyons 2001). For example, Baldwin (1990) develops a model with two investment possibilities, home and foreign currency denominated assets, and with risk neutral homogenous foreign exchange traders who face a small transaction cost of moving between two assets. Under this setup, he shows that presence of small (even third-order) costs will produce a band (which he calls the “hysteresis band”) within which no trade will take place and hence expected change in spot rate will not be affected by the interest rate differential. It is only when the interest rate differential falls outside the band that exchange rate changes will be related to the differential. The model presented in Mark and Moh (2004) on the other hand appeals to the intervention of central banks by adjusting the interest rate differential within a band. The limits to speculation hypothesis of Lyons (2001) refers to the idea that financial institutions only take up a currency strategy if the expected excess return per unit of risk is higher than that of the alternative strategy of say, a simple buy-and-hold equity strategy. This hypothesis defines a band of inaction where the deviation from UIP does not attract speculative capital and therefore, does not imply any glaring profitable opportunity until it generates high enough excess returns to attract speculative capital away from alternative trading strategies.

In a paper by Wu and Zhang (1996), it is argued that the bias in forward exchange rate is asymmetrical in the sense that UIP holds in periods when the forward US dollar is quoted at a premium but fails when it is quoted at a discount. Bansal (1997) and Bansal and Dahlquist (2000) show that the response of dollar exchange rate changes will be different for positive and negative values of interest rate differential. When the interest rate differential is positive, the slope coefficient in the regression of spot exchange rate change on lagged forward premium is negative and the UIP is rejected. When the interest rate differential is negative, the slope coefficient is positive, and UIP holds better. This paper contributes to the UIP literature by showing that the relationship between exchange rate change and lagged forward premium can be characterized by a nonlinear dynamic model which allows both nonlinearity and asymmetric dynamics. In particular, we provide empirical evidence that indicates that during periods when forward premia on US dollar is high enough (i.e. when forward premium is positive and large enough) adjustment towards UIP condition is fast and UIP holds. On the other hand, when the US dollar is quoted at a discount or when the premium on US dollar is small, then significant and persistent departures from market efficiency occurs, and UIP does not hold. In other words, whenever the premia is not high enough the deviations from UIP is not corrected and adjustment towards UIP does not follow. It is argued
that this type of nonlinear behavior is consistent with general implications of limits-to-speculative hypothesis which implies that for small forward premium, deviations from UIP will not be corrected as small premia will not attract speculative capital. It can also be argued that the results may be consistent with the implications of models with trading costs as for small premia, marginal cost of moving between two currencies may exceed the marginal benefit (at least for some investors) and hence preventing spot and forward rates to move in the direction of UIP.

After the first draft of this paper, we found out that Sarno, Valente, and Leon (2004) test the implications of the limits to speculation hypothesis by utilizing an exponential smooth transition regression framework. They report strong evidence of nonlinearity in the relationship between spot and forward exchange rates. Following Bansal (1997) we first present evidence on the asymmetric behavior of UIP relationship. Then similar to Sarno, Valente, and Leon (2004) we model the UIP relationship by a smoothly changing nonlinear model. Our model differs from theirs in that we utilize a logistic function instead of an exponential function. We name our model as dynamic logistic UIP regression model. Contrary to exponential function, our modeling framework allows us not only model nonlinear dynamics that is consistent with general implications of recent theories of exchange rate that are based on trading costs and limits to speculation arguments but also the asymmetric dynamics outlined in some of the empirical literature. Our findings uncover that the degree of adjustment towards UIP will not only depend on the absolute size of the forward premia (as Sarno, Valente, and Leon 2004 study documents) but also the sign/direction and magnitude of the forward premia. Our findings indicate that the UIP condition can be characterized by three regimes during the sample period we study; a lower regime, a middle(transition) regime and an upper regime. In the lower regime, where US dollar is quoted at a premium or discount that is less than a threshold level, the deviations from UIP is persistent and UIP relationship does not hold. We find that for almost all of the currencies we investigate, this regime takes place up until late 1980s. This explains partly why studies who utilized data from 1970s, and 1980s consistently rejected the UIP condition. In the middle regime the premium and/or the discount on the US dollar is relatively small and falls into a band, and thus UIP deviations are somewhat less persistent but still far from the UIP condition. This regime corresponds to early 1990s (i.e. 1990-1994) for most of the currencies in our sample. The third regime which can be called as the “UIP-regime” is characterized by relatively large US dollar premiums and corresponds to late 1990s for almost all currencies in our sample. This also explains why studies which investigated the UIP relationship by using data from late 1980s and early 1990s reported supportive evidence of UIP.

The rest of the paper is organized as follows. In section 2, we discuss briefly the UIP condition together with the literature that motivates the
study of the nonlinear dynamics in the UIP relationship. In section 3, we provide and discuss our econometric model, and in section 4, we provide our empirical findings. Section 5, gives the results of simulation study we conduct, to investigate the relevance of our modeling framework in matching the stylized facts of UIP relationship. The last section concludes and discusses our findings in relation to the most recent literature.

2 Uncovered Interest Rate Parity and Nonlinearity

2.1 Uncovered Interest Rate Parity

The central hypothesis of interest in this paper is the UIP condition, which states that,

\[ E_t(\Delta s_{t+k}) = (i_{t,k} - i^*_{t,k}), \]  

(1)

where \( E_t(.) \) denotes the mathematical expectation conditioned on the set of all relevant information at time \( t \), \( s_t \) is the logarithm of the spot exchange rate (domestic price of foreign currency) at time \( t \), \( i_{t,k} \) and \( i^*_{t,k} \) are \( k \)-periods to maturity nominal interest rates available on similar domestic and foreign assets respectively, \( \Delta s_{t+k} \equiv s_{t+k} - s_t \). Under the assumption that covered interest rate parity (CIP) holds, so that \( f_{t,k} - s_t = i_{t,k} - i^*_{t,k} \), where \( f_{t,k} \) is the logarithm of the \( k \)-period forward rate, equation (1) can be written as

\[ E_t(\Delta s_{t+k}) = (f_{t,k} - s_t) = (i_{t,k} - i^*_{t,k}). \]  

(2)

Hence the expected \( k \)-period rate of appreciation/depreciation should be equal to the current forward premium (or forward discount), \( f_{t,k} - s_t \).

Following Fama (1984), it has been common to test the UIP hypothesis by embedding (2) into a regression framework:

\[ \Delta s_{t+1} = \alpha + \beta(f_{t,1} - s_t) + u_{t+1}. \]  

(3)

where we have assumed \( k = 1 \) for simplicity, and \( u_{t+1} \) is a disturbance term (the rational expectations forecast error under the null hypothesis). It follows that irrespective of sampling frequency, \( k \), the UIP hypothesis implies that \( \alpha = 0 \) and \( \beta = 1 \), and if the sampling frequency is equal to the maturity time of the forward contract, so that \( k = 1 \), \( u_{t+1} \) must be serially uncorrelated.

This regression has produced some rather surprising empirical results. The forward premium anomaly concerns the fact that with spot and forward rates for the 1970s and 90s the estimated slope coefficients are invariably

\footnote{Empirical evidence indicates that CIP holds in the data, see Frankel and Levich (1975), Levich (1985), Clinton (1988) among others. For an excellent survey see Sarno and Taylor (2002), ch. 2.}
found to be negative indicating rejection of UIP and efficient markets hypothesis, see the references in the survey of Hodrick (1987), Engel (1996), and Sarno and Taylor (2002). Froot and Tahler (1990), for instance, report that the mean value of $\hat{\beta}$ across 75 published studies is -0.88. The empirical fact of negative slope coefficient in the UIP regression (regression equation 3) implies that on average, the more the foreign currency is at a premium in the forward market the less the home currency is predicted to depreciate (equivalently, the more domestic interest rates exceed foreign interest rates, the more the domestic currency tends on average to appreciate over the holding period, not to depreciate so as to offset on average the interest rate differential in favor of the home currency).

Note that subtracting the lagged forward premium from both sides of equation (3) we can obtain

$$s_{t+1} - f_{t,1} = \alpha + \delta(f_{t,1} - s_{t}) + v_{t+1},$$

where $\delta = \beta - 1$. This regression has been used to study the predictability of UIP deviations using forward premium as predictor variable. Note that under UIP, $\delta$ should be zero. Bilson (1981), Fama (1984) and Backus, Gregory and Telmer (1993) among others have shown that consistent with a negative estimate of $\beta$ in equation (3), the estimate of $\delta$ in equation (4) is significantly negative. This in turn implies strong predictability of UIP deviations from the lagged forward premium. Therefore, any explanation of UIP anomaly should also be able to provide an explanation to the predictability of UIP deviations from lagged forward premium. In section 5 we show through simulation that both the UIP anomaly and the predictability of UIP deviations on the basis of lagged forward premium can be explained, if the true data generating process for the UIP relationship is characterized by the nonlinear framework we provide in this paper.

### 2.2 Nonlinearity in UIP Relationship: A Motivational Review of Literature

Several papers have shown that there may be nonlinearities in the spot-forward relationship. Among others, Baldwin (1990), Dumas (1992), Hollifield and Uppal (1995), Obstfeld and Rogoff (2000), and Sercu and Wu (2000), Lyons (2001), Killian and Taylor (2003), and Mark and Moh (2004) suggest models of exchange rate determination in which implied relationship between nominal exchange rate and forward rates can be characterized by nonlinear dynamics.

Dumas (1992) develops a general equilibrium model of exchange rate determination in spatially separated markets with international trade costs. He shows that nominal exchange rate will depend nonlinearly on its fundamentals in a way that the larger the deviation from the parity condition the faster the reversion towards parity condition (see Dumas 1992, p.
Baldwin (1990) develops a partial equilibrium model with two assets (home and foreign currency denominated assets), with homogenous foreign exchange traders who face a small transaction cost of moving between two assets. He shows that small transaction costs and uncertainty imply that optimal-cross currency interest rate speculation is marked by a first-order hysteresis band as such whenever the interest rate differential falls in the band, expected spot exchange rate change will not be affected by the differential, while whenever the differential is outside the band, the expected exchange rate change will be a nondecreasing function of the interest rate differential, (see Baldwin 1990, pages 9-11, especially equation 3.8). Baldwin’s analysis imply that UIP may not hold whenever the interest rate differential is not high enough to induce investors to change their portfolio in a way that will cause exchange rates move in the direction of UIP condition. In other words, it is only when the return on a currency is high enough, the forward looking behavior will induce investors to move in and out of home and foreign currency denominated assets causing exchange rates move in the direction of UIP. Sercu and Wu (2000) show that in the presence of transactions costs, expected exchange rate changes and forward premia are imperfectly aligned even in the absence of a risk premium, inducing nonlinearity in the spot-forward relationship and implying that the slope coefficient in the UIP regression may be different depending on the size of the forward premium and/or deviation from UIP.

In a recent paper, Mark and Moh (2004) consider a continuous-time stochastic model for the exchange rate where the solution for the spot rate is a nonlinear function of the interest rate differential, modeled according to a jump-diffusion process regulated by occasional central bank intervention. Kilian and Taylor (2003), on the other hand, provide a different rationalization for the presence of nonlinearity in the relationship between nominal exchange rates and its fundamental value. They argue that in the presence of heterogenous foreign exchange traders; noise traders and rational speculators, noise traders’ demand for foreign exchange is affected by beliefs that are not fully justified by news about the fundamentals. Speculators on the other hand, form fully rational expectations about the return on holding foreign exchange and they sell foreign exchange when noise traders push prices up and buy when noise traders depress prices, thereby making a profit in the process. Killian and Taylor (2003) show that presence of heterogenous investors may cause deviations of nominal exchange rates from its fundamentals to behave in a nonlinear fashion as such this nonlinearity may be described by a smooth transition, in which the strength of reversion to fundamental level is an increasing function of deviations from the fundamental level.

Another rationalization for the nonlinear dynamics in the UIP condition comes from the limits-to-speculation hypothesis of Lyons (2001). The limits-to-speculation hypothesis indicates that financial institutions will only
take up a currency trading strategy if the strategy yields a Sharpe ratio at least equal to an alternative investment strategy, say a buy-and-hold equity strategy. The Sharpe ratios are defined as, \( \frac{E[R_s - R_{rf}]}{\sigma_s} \), where \( E[R_s] \) is the expected return on the strategy, \( R_{rf} \) is the risk-free interest rate, and \( \sigma_s \) is the standard deviation of the returns to the strategy. Given that over the last fifty years, the Sharpe ratio for a buy-and-hold strategy in U.S. equities has been about 0.4 on an annual basis, a currency trading strategy with a Sharpe ratio less than 0.4 would not be attractive. Lyons (2001) argues that if UIP holds exactly, (i.e. \( \alpha = 0, \beta = 1 \) in (3)) the Sharpe ratio of currency strategies is zero. As the slope coefficient depart from unity, the Sharpe ratio becomes positive. The arguments presented in Lyons (2001, pp.209-220) indicate that there exists an inaction band for the slope coefficient (and therefore for the Sharpe ratio) as such within the band financial institutions would have no incentive to take up the currency strategy since a buy-and-hold equity strategy would have a higher return per unit of risk. It is only when the currency strategy has a Sharpe ratio that is higher than a threshold level, the deviation from UIP will be high enough to be viewed as a glaring opportunity. Therefore, one shouldn’t expect the UIP hold when the Sharpe ratio is within an inaction band but reversion to UIP should increase with the Sharpe ratio of currency strategies. As shown in Lyons (2001, pp. 216), this in turn implies that for small values of forward premia/discounts \( (f_{t,k} - s_t) \) the intercept and slope coefficient will be away from implied values under the UIP condition, and for large premia, they will approach to the values consistent with UIP (i.e. \( \alpha = 0 \) and \( \beta = 1 \) in equation (3)). This follows because the numerator of Sharpe ratio for a currency strategy is a function of the intercept, slope parameter and the forward premium/discount. Therefore, for given \( \alpha, \beta \) and the denominator of the Sharpe ratio, a forward premium farther from zero implies a larger Sharpe ratio, this in turn attracts more speculative capital which induces adjustment in prices toward consistency with UIP. These arguments imply that the limits-to-speculation create a band of Sharpe ratios (and hence a band of forward premia) where UIP does not hold and spot and forward rates may be unrelated and even move in opposite directions, but reversion to UIP condition should be expected as the forward premia (and hence the Sharp ratios of currency strategy) becomes larger and larger.

There are several studies which have documented asymmetric and differential dynamics in the UIP relationship. Bilson (1981) showed that the forecasting power of outlier observations (defined to be the forward premia larger than ten percent in absolute value) of forward premia worse than the normal observations. On the other hand, Flood and Rose (1994) and Flood and Taylor (1996) provide evidence that outlier observations outperform normal observations in forecasting power. Huisman, Koedijik, Kool and Nissen (1998) report that UIP holds almost perfectly in periods characterized by...
large forward premia. Wu and Zhang (1996) find evidence that forward premium anomaly is asymmetrical in the sense that UIP holds in periods when the forward US dollar is quoted at a premium but fails when it is quoted at a discount. Zhou (2002) shows that UIP does not hold in any period between 1980 and 1998, and the forward premium anomaly exists only for one period, namely, the 1980-87 period, where both the forward premia and the changes in the spot rates of the majority of countries contain a significant time trend. Bansal (1997) and Bansal and Dahlquist (2000) provide evidence that indicates that dollar exchange rates changes respond differently to positive and negative interest rate differentials as such forward premium anomaly occurs during periods in which US interest rates falls short of foreign interest rates. Zhou and Kutan (2002) argue that the rejection of UIP does not depend on whether the US dollar is quoted at a premium or a discount documenting little empirical evidence of asymmetry in UIP anomaly. Sarno, Valente, and Leon (2004) provide empirical evidence that indicates significant nonlinearities in deviations from UIP condition that is consistent with limits-to-speculation hypothesis. This paper is part of this recent literature which emphasizes nonlinear dynamics in the UIP relationship.

The discussion above suggests that there is considerable theoretical rationalization for the consideration of nonlinear dynamics in the UIP condition as well as empirical evidence that indicate asymmetric behavior of spot-forward rates. Motivated by the literature on transactions costs, heterogeneous traders, and limits-to-speculation hypothesis, as well as the recent empirical literature, we investigate empirically the presence of potential nonlinearity and asymmetry in the relationship between exchange rate changes and forward premium in several major currencies. We empirically study the general implications of the theoretical arguments discussed in this section rather than directly testing these arguments. In the next section, we provide our empirical framework, which allows us to study the general implications of the theories based on transactions costs, (hysteresis hypothesis), presence of heterogenous traders, and limits-to-speculation hypothesis as well as the asymmetric dynamics.

3 Dynamic Logistic UIP Regression

We characterize nonlinear dynamics in the UIP regression which allows for smooth rather than discrete adjustment as well as asymmetric behavior in terms of a dynamic logistic smooth transition regression (LSTR) model. The LSTR model is based on the smooth transition autoregressive (STAR) models which are introduced in econometrics by Granger and Teräsvirta (1993) and by Teräsvirta (1994). For an excellent survey of STAR models see van Dijk et al. (2002). Similar to STAR models, in the LSTR model, adjustment takes place in every period and the speed of adjustment is governed by
the values of a transition variable. Since we use a logistic function to model
the dynamics of adjustment, we can model asymmetric behavior as such
for both large and small forward premium and discount the exchange rate
change may behave differently. The LSTR model we utilize in this paper
for the spot and forward rates can be written as follows:

\[
\Delta s_{t+1} = [\alpha_1 + \beta_1 (f_{t,1} - s_t)] + [\alpha_2 + \beta_2 (f_{t,1} - s_t)] F(z_t, \gamma, c) + u_{t+1},
\]

where \( u_{t+1} \) is a stationary disturbance term, \( F(.) \) is the transition func-
tion that determines the degree of reversion towards UIP condition. The
transition function we select to work with is the logistic function:

\[
F(z_t; \gamma, c) = \frac{1}{1 + \exp(-\gamma(z_t - c)/\sigma_{z_t})}
\]

with \( \gamma > 0 \),

where \( z_t \) is the transition variable, \( \sigma_{z_t} \) is the standard deviation of
\( z_t \), \( \gamma \) is a slope parameter and \( c \) is a location parameter. The parameter restriction
\( \gamma > 0 \) is an identifying restriction. The value of the logistic function (6),
which is bounded between 0 and 1, depends on the transition variable
\( z_t \) as follows. \( F(z_t; \gamma, c) \to 0 \) as \( z_t \to -\infty \), \( F(z_t; \gamma, c) = 0.5 \) for \( z_t = c \), and
\( F(z_t; \gamma, c) \to 1 \) as \( z_t \to +\infty \). When \( \gamma \to \infty \), \( F(z_t; \gamma, c) \) becomes a step
function, such that the LSTR model becomes effectively a threshold (TR)
model. Therefore, LSTR model nests a two-regime threshold model. For
\( \gamma = 0 \), \( F(z_t; \gamma, c) = 0.5 \) for all \( z_t \), in which case the model reduces to a linear
UIP regression model with parameters, \( \alpha = \alpha_1 + \frac{1}{2} \alpha_2 \), and \( \beta = \beta_1 + \frac{1}{2} \beta_2 \).
The exponent in (6) is normalized by dividing \( \sigma_{z_t} \) to make the parameter
\( \gamma \) approximately scale-free, which is useful for the initial estimates for the
nonlinear optimization used to estimate the parameters in (5).

The values taken by the transition variable and the transition parameter
\( \gamma \) will determine the speed of reversion to UIP. For any given value of \( z_t \), the
transition parameter \( \gamma \) determines the slope of the transition function and
hence the speed of transition between extreme regimes, with low values of \( \gamma \)
implying slower transition. The transition variable, \( z_t \) is assumed to be the
forward premium, \( z_t = (f_{t,1} - s_t) \). The parameter \( c \) can be interpreted as the
threshold between the two regimes corresponding to \( F(z_t; \gamma, c) = 0 \) and
\( F(z_t; \gamma, c) = 1 \), in the sense that the logistic function changes monotonically
from 0 to 1 as \( z_t \) increases, while \( F(c; \gamma, c) = 0.5 \). Note that the inner regime
corresponds to \( z_t = c \), where \( F(z_t = 0; \gamma, c) = \frac{1}{2} \) and equation (5) becomes a UIP regression of the form:

\[
\Delta s_{t+1} = [(\alpha_1 + \frac{1}{2} \alpha_2) + (\beta_1 + \frac{1}{2} \beta_2)(f_{t,1} - s_t)] + u_{t+1}.
\]

\(^2\)We also employed deviations from UIP in our empirical work. Although results are
found to be qualitatively similar, diagnostic tests in most cases selected the models with
forward premium. (Results with deviations from UIP can be obtained upon request.)
Note that since we divide \( z_t \) by its standard error, in practice, it can be thought that we
are utilizing the risk-adjusted forward premium as the transition variable.
The lower regime corresponds to for given $\gamma$ and $c$ to $\lim_{z_t \to -\infty} F(z_t; \gamma, c)$ where (5) becomes a standard linear UIP regression:

$$\Delta s_{t+1} = [\alpha_1 + \beta_1(f_{t,1} - s_t)] + u_{t+1}, \quad (8)$$

while upper regime corresponds to $\lim_{z_t \to +\infty} F(z_t; \gamma, c)$ where (5) becomes a different UIP regression:

$$\Delta s_{t+1} = [(\alpha_1 + \alpha_2) + (\beta_1 + \beta_2)(f_{t,1} - s_t)] + u_{t+1}. \quad (9)$$

Hence the model in (5) is quite general in that it nests three regimes with different dynamics. Note that contrary to the exponential function, $G(z_t; \gamma) = 1 - \exp(-\gamma(z_t^2))$ (with $z_t$ being the deviations from UIP) utilized by Sarno, Valente, and Leon (2004) the logistic function allows us to model not only the nonlinear dynamics suggested by the literature discussed in the previous section, but also the potential asymptotic behavior in the UIP relationship that is discussed in section 2. Note that the LSTR model nests the standard UIP regression, to which it would collapse in the absence of nonlinearity. Note also that, under the restrictions $\alpha_2 = -\alpha_1$ and $\beta_2 = 1 - \beta_1$, which are formally testable using standard statistical procedures, the specification in (5) captures the dynamics between exchange rate changes and forward premia which is implied by the theoretical considerations discussed in section 2 in that when the lagged forward premium is less than a threshold level, the slope coefficient may take values far from unity (even negative values) and when the premium exceeds a threshold level this may induce investors to take positions as such deviations from UIP becomes less persistent and the slope coefficient move towards the theoretical value of unity.

4 Empirical Results

4.1 Data and the conventional UIP Regression

Our data set comprises of monthly observations of spot and one-month forward exchange rates for Belgian Franc (BF), Canadian Dollar (CD), Dutch Guilder (DG), French Franc (FF), German Mark (GM), Italian Lira (IL), Japanese Yen (JY), Swiss Franc (SF) and UK Pound. All rates are per US Dollar. The data is provided by Bank of International Settlements. The sample period spans from December 1978 to December 1998 for Euro area currencies (BF, DG, FF, GM and IL) and from December 1978 to December 2002 for other currencies.

As a preliminary exercise, first panel of Table (1) reports the results from conventional UIP regression (3). The results are consistent with the forward premium anomaly in that, while the estimates of constant term $\alpha$ are pretty close to zero and in most cases statistically indistinguishable from zero, the estimates of slope coefficient, $\beta$ is negative in all cases except in cases of FF
and IL. Except from DG, in all of Euro area currencies, the slope coefficient estimates are all statistically insignificant. An interesting finding from the first panel of Table (1) is that for currencies not participating in the Euro, the estimates of slope coefficient are significantly negative. The t-statistic \( t_{\beta=1} \) tests the null of slope coefficient is equal to the theoretical value of unity against the alternative of different from unity indicates that for all currencies the null hypothesis is rejected at conventional significance levels except in the case of FF. Note that in the case IL test statistic rejects the null at 10 percent level. The negative sign of tests also indicate that the one would reject the null against the one-sided alternative hypothesis that the slope coefficient is less than one in most cases.

### 4.2 Asymmetry in UIP Condition

To gain some initial insight on the nonlinear behavior of the relationship between spot and forward rates in the second panel of Table (1), we report estimation results from a regression of spot exchange rate changes on a constant and positive and negative forward premiums as in Bansal (1997) and Bansal and Dahlquist (2000). In other words, we estimate the following UIP regression,

\[
\Delta s_{t+1} = \alpha + \beta^+(f_{t,1} - s_t)^+ + \beta^-(f_{t,1} - s_t)^- + u_{t+1},
\]

(10)

where

\[
(f_{t,1} - s_t)^+ = \begin{cases} 
(f_{t,1} - s_t), & \text{if } (f_{t,1} - s_t) > 0 \\
0, & \text{if } (f_{t,1} - s_t) < 0
\end{cases}
\]

and

\[
(f_{t,1} - s_t)^- = \begin{cases} 
0, & \text{if } (f_{t,1} - s_t) > 0 \\
(f_{t,1} - s_t), & \text{if } (f_{t,1} - s_t) \leq 0
\end{cases}
\]

The variables \((f_t - s_t)^+\) and \((f_t - s_t)^-\) separate the forward premium into periods of positive and negative premiums.\(^3\)

The reported results indicate that when the forward premium is positive (i.e. assuming CIP holds, when US interest rate falls short of foreign interest rates), the slope coefficients are positive in all cases except in the case of CD and UKP. In the case of UKP note that the estimate of slope coefficient is statistically insignificant while it is significant in the case of CD. When the forward premium is negative (i.e. when the US interest rate is higher than the foreign rates), the slope coefficients are all negative, and significantly different from unity in all cases except in the cases of CD and IL. Note that

\(^3\)Note that given CIP, \((f_t - s_t) = (i_t - i^*_t)\), with \(i^*_t\) defined to be the monthly US interest rate in all cases, these variables effectively sperate the interest rate differential into periods of positive and negative interest rate differentials. Given the way exchange rate is defined, a positive premium indicates that the US dollar is in premium and a negative premium indicates that the US dollar is at a discount in the forward market.
for UKP, the slope coefficient is more negative when the forward premium is negative than when it is positive and it is less negative for the CD when the forward premium is negative than it is positive. The $t-$ test for the hypothesis that slope coefficient is one rejected only in three cases (CD, JY, and UKP) at 5 percent significance level in the state where forward premium is positive (i.e. when US dollar is in premium), while the same null hypothesis is rejected in seven out of nine cases in the state when the forward premium is negative (i.e. when the US dollar is at a discount). The $t-$test indicates that in the state when forward premium is positive, slope coefficient is significantly less than unity only for CD and UKP, but greater than unity for JY. Therefore, we have evidence that shows that UIP holds better in the state when US dollar is quoted at a premium in the forward market (i.e. the US interest rate is lower than the foreign interest rates) and it is rejected when the US dollar is at a discount (i.e. when the US interest rate exceed the foreign interest rates).

Table (1) also reports the robust Wald test to test the equality of the slope coefficient for $(f_{t,1} - s_t)^+$ and $(f_{t,1} - s_t)^-$. For five out of nine cases this test sharply rejects the null hypothesis that the slope coefficient is equal across $(f_{t,1} - s_t)^+$ and $(f_{t,1} - s_t)^-$. This rejection implies that the relationship between expected exchange rate change and forward premium is significantly different across $(f_{t,1} - s_t)^+$ and $(f_{t,1} - s_t)^-$ for BF, DG, FF, GM, and JY. The test fails to reject the null in CD, IL, SF, and UKP implying that there is less evidence of differential dynamics in UIP condition across different states for these currencies. Also note that the adjusted $R^2$ values for the two-state regressions (Reg 2) are measurable compared to the conventional UIP regression (Reg 1).

Our findings from two-state UIP regression model are consistent with the findings of Wu and Zhang (1996), Bansal (1997) and Bansal and Dahlquist (2000), in that overall, we found that on average, UIP holds better when the US Dollar is quoted at a premium (i.e. via CIP, US interest rates are lower than the foreign interest rates) and UIP is significantly rejected when the US dollar is quoted at a discount (i.e. via CIP, US interest rates exceed foreign interest rates). These findings also reveal presence of important nonlinearity and asymmetric dynamics in the relationship between expected exchange rate changes and forward premia. In the next subsection, we explore these issues more formally, further by utilizing LSTR model discussed in section 2.

### 4.3 The Dynamic Logistic UIP Regression: Estimation Results

We estimate the nonlinear logistic UIP regression (5) by nonlinear least squares. The starting values are obtained from a grid search over $\gamma$ and $c$. Following the suggestion of Teräsvirta (1994), we standardized the transition
variable by dividing the sample standard error of the transition variable, \( \hat{\sigma}_{zt} \). The dynamic logistic UIP model (5) is estimated both under the restrictions, \( \alpha_2 = -\alpha_1 \) and \( \beta_2 = 1 - \beta_1 \) and without these restrictions. Since in all cases, we have failed to reject those restrictions, in Table (2), we report estimation results from the restricted nonlinear logistic UIP regression.

The reported results indicate that the logistic regression is highly nonlinear. The estimated transition parameter appears to be strongly significantly different from zero on the basis of asymptotic standard errors in all cases except in the case of BF. Note also that in four cases, the estimated threshold parameter, \( \hat{\hat{c}} \) is nonzero and significantly so in cases of CD, IL and UKP. The estimates of the slope parameters \( \beta_1 \) and \( \beta_2 \) indicates that in all cases the estimated value for \( \beta_1 \) is negative and a large positive value of \( \beta_2 \) such that UIP holds exactly when the transition function \( F(\cdot) = 1 \). The results indicate that when the risk adjusted premium on US dollar is high enough, the transition function takes values in the neighborhood of unity and hence UIP holds better.\(^4\) On the other hand, when the premium on the US dollar is low (and in some cases negative) the transition function takes values in the neighborhood of zero, hence UIP does not hold and we observe the so called forward premium anomaly. The estimation results reveal that for the UIP to hold, it is not enough for the US dollar to be quoted at a premium (as we found in the previous subsection and evidence reported in Wu and Zhang 1996, Bansal 1997, and Bansal and Dahlquist 2000) in the forward market but also the quoted premium should be high enough to induce the reversion to UIP. In other words, in order for the arbitrage to take place and be effective, the premium on the US dollar need to be higher than a threshold level. This can be seen by noting that for some currencies the estimated threshold parameters are significantly different from zero and from inspecting the estimated transition functions in Figure 1, where plots of the estimated transition functions against the threshold variable and time are displayed.

Inspection of upper panels of Figure 1 together with the estimation results in Table (2), reveals that the risk adjusted monthly forward premium on US dollar should approximately be at least as large as 0.2% in cases of BF and SF, 0.1% in cases of DG, GM, and JY, 0.35% in the case of CD, 0.5% in cases of FF and UKP, and 1% in the case of IL in order for the arbitrage to be effective and UIP to hold.\(^5\) Note that the results also indicate that the UIP does not hold when the discount on the US dollar is less than about 0.2% in cases of BF, and SF, 0.1% in cases of DG, GM, and JY, 0.5% in case of FF. In cases of CD, IL, and UKP on the other hand, the estimates reveal

\(^4\)Note that since we have divided the lagged forward premium by its sample standard error, the transition variable is the lagged risk adjusted forward premium.

\(^5\)These results can be read from the estimated transition function graphs by looking at the region for the risk adjusted premium for which the transition function takes value unity.
that even if the US dollar is quoted in premium the UIP may not hold unless the premium high enough to induce arbitragers to take actions so that spot and froward rates move in the direction of UIP condition. As can be observed from the estimated transition functions, if the risk adjusted premium on US dollar is approximately less than 0.2% in case of CD, 0.6% in case of IL, and less than 0.5% in case of UKP, the transition function will take values in the neighborhood of zero implying that spot and forward relationship will be far from the UIP condition. Note that for those currencies the estimated threshold parameter \( \hat{c} \) is statistically significantly different from zero.\(^6\) These results may be interpreted as indicating that for the foreign exchange traders to change their investment strategies and/or portfolios, incremental benefit from changing their position in a given period should be high enough to cover costs of such a move. This interpretation is consistent with the transactions cost as well as limits to speculation arguments.

The estimated transition functions imply well-defined nonlinear dynamics in the UIP condition in that the transition functions take values in between 0 and 1 as such the spot-forward relationship can be characterized by three regimes; lower, middle (transition-regime) and an upper regime. As can be seen from the upper panels of Figure 1, each transition function has the shape expected from a logistic function and each estimated function attains each regimes, namely, lower, middle, and the upper regimes. The estimated transition parameter for the UKP pound is very high (261.7) compared to the rest of the currencies, indicating that the transition between extreme regimes (lower and upper regimes) for the UKP is pretty fast as such within a short time period, once the risk adjusted premium on US dollar exceeds 0.5%, arbitrage takes place and UIP condition is obtained.\(^7\) Plots of the estimated transition functions also indicate that in most of the time in our sample, transition function takes values in the neighborhood of zero, hence most of the observations of forward premium are less than threshold levels necessary to induce reversion to UIP. In other words, in most of the time in our sample, US dollar either quoted at a discount or when it is quoted at a premium, the premium is not high enough for the transition function to take values near unity (in other words, the premium is not high enough to induce investors to change their positions in the forward market hence, the spot and forward rates do not tend to move towards the UIP condition). This can also be seen from the plots of transition function

\(^6\)For the FF, we were able to identify a threshold value of 0.003 which is not significant at conventional levels. For all other currencies, dynamic logistic UIP regression model did not identify a threshold value. Note also that our results are very different from that of Sarno, Valente and Leon (2004) as their model do not consider presence of a threshold parameter.

\(^7\)This result can be attributed to some factors that we might not see in the monthly data that we are employing here or liquidity of UKP-US Dollar market. This differential dynamics across currencies need to be studied further which is beyond the scope of current paper.
against time in the lower panels. The plots of transition functions against time reveal that for most exchange rates, the transition function attains values closer to unity between 1989 and 1994 and closer to zero between 1979 and 1989 and after 1994. This finding is interesting as approximately between 1989-1994, US interest rates were lower than the other countries interest rates in our sample, and US dollar were quoted in premium during this period. This finding may also explain why conventional UIP regressions tend to reject the UIP hypothesis less severely, when data from the 1990s used to test it (see for instance, Baillie and Bollerslev 2000, and Flood and Rose 2002). The estimation results and the plots of estimated transition functions reveal that whenever the forward premium is less than a threshold level, the spot and forward rates deviate from the UIP condition and as the premium increases, they tend to move in the direction of UIP and whenever the premium exceeds certain levels, the UIP holds exactly in the upper regime where the premium is high enough to induce investors in foreign exchange market to take actions in such a way that pushes spot and forward rates adjust along with the UIP condition. The results also uncover that the asymmetric nature of this adjustment process in that, the adjustment takes place only when the US dollar is quoted at a premium that exceeds a threshold level. For each currency this threshold value is different and for some of the currencies in our sample, it is significantly different from zero.

5 Reconciling the UIP Anomaly and Predictability: Simulation Evidence

Given our empirical findings on the nonlinear and asymmetric dynamics in the spot-forward relationship, it is useful to explore further to see if we can match the stylized facts of UIP regression using a DGP calibrated according to estimated dynamic logistic UIP models. For this purpose, we conducted a series of Monte Carlo experiments based on simulated data that are generated under the assumption that the true DGP is given by the dynamic logistic UIP regression (5). The DGP is calibrated on the estimates reported in Table (2), with independent and identically distributed Gaussian innovations. We initialized the data at zero, and generated 50,000 samples of 100 + 240 observations for BF, DG, FF, GM, and IL, and 50,000 samples of 100 + 276 observations for CD, JY, SF, and UKP. In each simulation, we discarded the first 100 observations to minimize the impact of initialization. For each artificial sample we estimated the standard linear UIP regression (3) and the predictability regression (predictability of UIP deviations from the forward premium) (4). Panels of Table (3) provides results of our Monte Carlo experiments. In Panel A of the table, estimates of $\alpha$ and $\beta$ obtained from the actual data are reported again (taken from Table (2). In Panels B and C we report the average (say $\bar{\alpha}_{sim}$, $\bar{\beta}_{sim}$) and median (say $\tilde{\alpha}_{sim}$, $\tilde{\beta}_{sim}$)
of the 50,000 estimates obtained from the UIP regression (2) together with their 5th and 95th percentiles from the empirical distribution \((\alpha_{5\%}^{\text{sim}}, \beta_{5\%}^{\text{sim}})\) and \((\alpha_{95\%}^{\text{sim}}, \beta_{95\%}^{\text{sim}})\). The last rows of Panels B and C report the value of the \(t\)− statistic for testing the null hypothesis that \(\bar{\alpha}^{\text{sim}} = \alpha\) and \(\bar{\beta}^{\text{sim}} = \beta\) respectively.

The reported results from Panels B and C of Table (3) reveals that, if the true DGP for the UIP condition were indeed of the nonlinear form (5) and if we estimated the standard UIP regression, the estimates of \(\alpha\) and \(\beta\) on average will be close to the ones estimated on actual data. The estimates of \(\alpha\) and \(\beta\) based on the actual data are in the interval between 5th and 95th percentiles of the empirical distribution of \(\alpha^{\text{sim}}\) and \(\beta^{\text{sim}}\) obtained from estimating the UIP regression on the simulated data. Furthermore, the 5th and 95th percentiles of \(\beta^{\text{sim}}\) are quite wide indicating that if the true DGP is in the form of (5), we can observe both large negative and large positive values of slope parameter estimates if we run the standard UIP regression with positive probability. The values of the \(t\)− test reported in the last rows of the Panels B and C indicate that the estimates of \(\alpha\) and \(\beta\) obtained from the actual data are indeed statistically insignificantly different from the average estimates \(\bar{\alpha}^{\text{sim}}\) and \(\bar{\beta}^{\text{sim}}\) from the UIP regression on the simulated data. The evidence reported in Panels B and C may explain why the literature on the UIP have repeatedly produced slope estimates that were different from unity. Indeed since the unity is inside the 5th and 95th percentiles of \(\beta^{\text{sim}}\), even obtaining a slope estimate of unity may not be as informative as we hope. In other words, a slope parameter estimate of unity may not be considered to be evidence in favor of UIP condition if the true DGP is of nonlinear form (5).

First two rows of Panel D of Table 3 report the estimate of \(\delta\) and the value of the \(t\) statistic for testing the null hypothesis that \(\delta = 0\) obtained from the actual data for each currency. Consistent with earlier empirical evidence, the reported results indicate that the estimates of \(\delta\) are more negative than the estimate of \(\beta\) for all cases and they are significantly different from zero for all cases except FF and IL at conventional significance levels. The last column of Panel D reports the values of the \(t\)− statistic for testing the null hypothesis of \(\bar{\delta}^{\text{sim}} = \delta\). These values suggest that the estimate of \(\delta\) obtained from the actual data is not different from the average of the \(\delta\) that is obtained from simulated data.

The simulation exercise indicates that estimate of the slope parameter and constant term in the conventional UIP regression and regression of UIP deviations on a constant and lagged forward premium by using artificial data, on average statistically indistinguishable from the estimates obtained by using actual data. Thus our simulation experiments suggest that if the true DGP characterizing the relationship between spot and forward exchange rates were of the nonlinear form studied in this paper, estimation of the conventional UIP regression (3) and the predictability regression (4)
would lead us to reject UIP and to find evidence of predictability of excess returns on the basis of lagged forward premium.

6 Conclusions and Discussion

Empirical findings in this paper demonstrate that the relationship between exchange rate returns and lagged forward premium can be characterized by a nonlinear model which allows time-variation in UIP relationship and nonlinear reversion towards UIP for nine major bilateral US dollar exchange rates. The nonlinear dynamics that we document in the paper is characterized by mainly three regimes depending on whether the premium on US dollar is high enough, small and within the neighborhood of a band, and is negative and large enough (i.e. the US dollar is quoted at a large enough discount). The lower regime is characterized by persistent deviations from UIP, the middle regime which can be thought of a transition regime between the two extreme regimes is somewhat less persistent, and the upper regime where UIP holds exactly. The nonlinear dynamics we uncover also indicate that the spot-forward relationship is asymmetric in the sense that adjustment toward UIP condition occurs only when US dollar is quoted at a premium that is large enough to attract speculative capital and/or induce traders to take decisions (change their portfolios or move between US dollar denominated and foreign currency denominated assets) that will cause adjustments in the direction of UIP. The estimated models suggest that the relationship between spot and forward exchange rate can be characterized by an equilibrium in which whenever the premium on US dollar is smaller than a threshold level, there is statistically significant deviations from UIP and hence UIP anomaly occurs, while when the premium is higher than a threshold, the deviations from UIP dissipates with the size of the premium. This is consistent with the general implications of the recent theoretical work on the characteristics of exchange rate dynamics in the presence of transactions costs and/or presence of limits to speculation in the foreign exchange market. Our findings also indicate that up until late 1980s, and after approximately 1994, the UIP relationship is better characterized by persistent deviations while in between the UIP holds better. This in turn, partly explains why studies use data up until late 1980s have found strong rejections of UIP and those with data from 1990s have found weaker rejections of UIP. We also showed, by Monte Carlo experiments calibrated on the estimated LSTR model, that if the true DGP for the UIP relationship were of the nonlinear form we consider in this paper, estimation of conventional UIP regressions would generate well known UIP anomaly (or forward premium puzzle) and empirically documented predictability of foreign exchange excess returns on the basis of lagged forward premium.

The nonlinear dynamics we document is consistent with general implica-
tions of the models with transactions costs, heterogenous traders, and limits to speculation hypotheses in the sense that foreign exchange traders will take decisions that will induce adjustment towards UIP only when deviations from UIP/forward premium is high enough. Although, our results are consistent with implications of “band of inaction” type of arguments developed in the literature, we do not claim to test directly these hypotheses. In this sense, this paper can be thought of an empirical effort to characterize the UIP relationship motivated by limits to speculative hypothesis and/or hysteresis hypothesis. Moreover, findings in this paper indicate the potential relevance of limits to speculative and hysteresis type of arguments in understanding the nature of foreign market efficiency condition.

Our findings are consistent with those of Bansal (1997) Bansal and Dahlquist (2000), and Wu and Zhang (1996) who report similar asymmetric dynamics in the UIP relationship. Our results are also consistent with that of Sarno, Valente and Leon (2004) in that they also report presence of nonlinear dynamics in a different data set than ours. Their findings indicate that the deviations from UIP can be characterized by two regimes, an inner regime with persistent deviations from UIP and an outer regime where UIP holds. Our findings contrast with their in that their model of UIP deviations imply that for both positive and negative deviations (or for both positive and negative premia) adjustment towards UIP will take place. In other words, the exponential model they utilize implies that the deviations from UIP will be corrected in a symmetric manner while ours implies asymmetric dynamics which is consistent with the evidence reported in the empirical literature. Overall, both our findings and Sarno, Valente and Leon (2004)’s findings indicate that the dynamics of UIP may be more complicated than one would think of under a linear specification and further work may need to be done in order to better understand these complications.
References


## Table 1: UIP Regressions

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Table 2: Logistic UIP Regression Estimation Results

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<td>( LR )</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
<td>0.010</td>
<td>0.003</td>
<td>0.020</td>
<td>0.302</td>
</tr>
<tr>
<td>( LM(4) )</td>
<td>5.529</td>
<td>1.051</td>
<td>4.414</td>
<td>4.326</td>
<td>5.131</td>
<td>6.004</td>
<td>4.327</td>
<td>6.234</td>
<td>4.601</td>
</tr>
<tr>
<td>( LM(8) )</td>
<td>3.504</td>
<td>1.012</td>
<td>2.294</td>
<td>2.250</td>
<td>2.718</td>
<td>3.139</td>
<td>2.656</td>
<td>3.175</td>
<td>2.355</td>
</tr>
<tr>
<td>( pJB )</td>
<td>0.908</td>
<td>0.648</td>
<td>0.861</td>
<td>0.093</td>
<td>0.176</td>
<td>0.117</td>
<td>0.011</td>
<td>0.015</td>
<td>0.001</td>
</tr>
<tr>
<td>( pRNL )</td>
<td>0.160</td>
<td>0.185</td>
<td>0.231</td>
<td>0.264</td>
<td>0.564</td>
<td>0.696</td>
<td>0.372</td>
<td>0.004</td>
<td>0.110</td>
</tr>
<tr>
<td>Sample</td>
<td>240</td>
<td>276</td>
<td>240</td>
<td>240</td>
<td>240</td>
<td>240</td>
<td>276</td>
<td>276</td>
<td>276</td>
</tr>
</tbody>
</table>

Notes: The table shows the results from the nonlinear Logistic UIP regression, 
\[ \Delta s_{t+1} = \alpha_1 + \beta_1 (f_t - s_t) + \alpha_2 + \beta_2 (f_t - s_t) F(z_t, \gamma, c) + u_{t+1}, \] where \( \alpha_2 = -\alpha_1, \beta_2 = 1 - \beta_1 \) and \( F(\cdot) = \frac{1}{1+\exp(-\frac{1}{\gamma}(z_t-c))} \). In all cases the transition variable, \( z_t \), is indicated in the last row. \( F - test \) is the \( F \)-statistic for the null hypothesis that \( \alpha_2 = -\alpha_1, \beta_2 = 1 - \beta_1 \). The test statistic is \( F(T-k) \) distributed with \( T \) being the sample size and the \( k \) being the number of parameters estimated under the alternative. \( pNRNL \) denotes the \( p \)-value for the test of no remaining nonlinearity in the residuals, constructed as in Eitrheim and Terasvirta (1996).
Table 3: Simulation Results: Matching the Empirical Facts of UIP Regression

<table>
<thead>
<tr>
<th></th>
<th>BF</th>
<th>CD</th>
<th>DG</th>
<th>FF</th>
<th>GM</th>
<th>IL</th>
<th>JY</th>
<th>SF</th>
<th>UKP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.001</td>
<td>0.002</td>
<td>-0.002</td>
<td>0.001</td>
<td>-0.002</td>
<td>0.001</td>
<td>-0.010</td>
<td>-0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.814</td>
<td>-1.132</td>
<td>-1.598</td>
<td>0.032</td>
<td>-0.894</td>
<td>0.448</td>
<td>-2.728</td>
<td>-1.395</td>
<td>-2.526</td>
</tr>
</tbody>
</table>

| $\bar{\alpha}_{sim}$ | -0.003 | 0.001 | -0.008 | -0.001 | -0.006 | 0.004 | -0.007 | -0.006 | 0.005 |
| $\bar{\beta}_{sim}$ | -0.004 | 0.001 | -0.008 | -0.001 | -0.006 | 0.004 | -0.007 | -0.005 | 0.006 |
| $\alpha_{5%}$ | -0.113 | -0.111 | -0.125 | -0.115 | -0.128 | -0.177 | -0.164 | -0.138 | -0.118 |
| $\alpha_{95%}$ | 0.108 | 0.086 | 0.109 | 0.116 | 0.116 | 0.185 | 0.149 | 0.128 | 0.126 |
| $t_{\alpha}$ | -0.060 | -0.015 | -0.086 | -0.022 | -0.058 | 0.028 | 0.024 | -0.018 | -0.016 |

| $\bar{\beta}_{sim}$ | -3.710 | 0.178 | -2.877 | -1.091 | -1.731 | -0.295 | -1.515 | -0.870 | -2.160 |
| $\tilde{\beta}_{sim}$ | -3.656 | 0.057 | -2.982 | -1.081 | -1.847 | -0.381 | -1.483 | -0.893 | -2.248 |
| $\beta_{95%}$ | 35.831 | 63.703 | 36.815 | 28.891 | 34.976 | 33.810 | 38.355 | 29.872 | 41.097 |
| $t_{\beta}$ | -0.120 | 0.034 | -0.053 | -0.062 | -0.038 | -0.016 | 0.050 | 0.028 | 0.014 |

| $\delta$ | -1.824 | -2.123 | -2.598 | -0.968 | -1.894 | -0.552 | -3.728 | -2.395 | -3.526 |
| $t_{\delta}$ | -2.601 | -5.654 | -3.551 | -1.491 | -2.825 | -0.737 | -5.687 | -3.954 | -4.309 |
| $\tilde{\delta}_{sim}$ | -4.710 | -0.821 | -3.877 | -2.091 | -2.731 | -1.295 | -2.515 | -1.870 | -3.160 |
| $\tilde{\delta}_{sim}$ | -4.656 | -0.868 | -3.982 | -2.081 | -2.847 | -1.381 | -2.483 | -1.893 | -3.248 |
| $t_{\tilde{\delta}_{sim}}$ | -0.456 | -0.007 | 0.155 | -0.071 | -0.029 | -0.030 | 0.050 | 0.035 | 0.055 |

Notes: $\alpha$ and $\beta$ are estimates from the conventional UIP regression taken from (1). $\bar{\alpha}_{sim}$, $\bar{\beta}_{sim}$ denote the mean and median of the empirical distribution (based on 50,000 replications) of the coefficients $\alpha$ and $\beta$ respectively, obtained from estimating the UIP regression of the form (3) using simulated data under a true DGP of logistic UIP regression form as given in (5). ($\alpha_{5%}$, $\beta_{5%}$) and ($\alpha_{95%}$, $\beta_{95%}$) are the 5th and 95th percentiles of the empirical distribution of the parameters $\alpha_{sim}$, $\beta_{sim}$ respectively. $t_{\alpha}$ and $t_{\beta}$ are the $t$-values for the null hypothesis that $\bar{\alpha}_{sim} = \hat{\alpha}$ and $\bar{\beta}_{sim} = \hat{\beta}$, respectively. $\delta$ is the estimate of the slope parameter in a regression of excess return ($s_{t+1} - f_t$) on a constant and forward premium ($f_t - s_t$). $t_{\delta}$ is the value of the $t$-statistic to test the null hypothesis that $\tilde{\delta}_{sim} = \delta$. 

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Figure 1: Estimated transition functions

BF

CD

DG

FF