The real interest rate: Is it mean-reverting? If yes, how long does it take to mean-revert?

Sofiane H. Sekioua*

University of Newcastle upon Tyne Business School, Room 16, 3rd Floor, Ridley Building, Claremont Road, Newcastle upon Tyne, NE1 7RU, UK
Tel: +44(0)1912227574; Fax: +44(0)1912226548
Email: S.H.Sekioua@ncl.ac.uk

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Abstract

This paper examines the dynamics of four major ex-post and ex-ante real interest rates using long spans of data. The principal tenet of this study is that real interest rates do not contain any unit roots, although shocks impinging upon these rates are quite persistent. This result has a number of economic interpretations.

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1. Introduction

The past two decades have seen numerous empirical studies of the relationship between the level of interest rates and expected inflation. It has proved to be one of the most studied topics in economics. According to the Fisher (1930) hypothesis, nominal interest rates should vary, ceteris paribus, one-for-one with expected inflation in the long-run. Thus, the ex-ante real interest rate, which represents the difference between the nominal interest rate and the expected decline in the purchasing power of money, must be stationary if the Fisher relationship is to hold. However, the stationarity of real interest rate has been questioned in several studies including the provocative paper of Rose (1988). Indeed, Rose (1988) asked the following question: Is the real interest rate stable? Using post-war data from 18 OECD countries and the conventional Dickey and Fuller (1979) unit root test, he failed to reject the presence of a unit root in the real interest rate. This conclusion is reached without analysing the real interest rate; simply finding that the nominal interest rate and inflation have different orders of integration means that any linear combination of the two variables is a nonstationary process with a unit root. Rose (1988) points out that a unit root in the real interest rate has serious implications not only for the Fisher hypothesis but also for the validity of the consumption-based capital asset pricing model (CCAPM) attributed to Lucas (1978). This model is characterised by a representative agent that maximises periodic utility subject to an intertemporal budget constraint. More importantly, however, the first-order Euler equation derived from this model implies that the time-series properties of the real interest rate (or the real asset return) and the growth rate of consumption should be similar. Given that the latter variable is found not to contain a unit root, nonstationarity of the former represents a violation of the conditions for the CCAPM. Consequently, it seems important to assess if the real interest rate is stationary or if it exhibits unit root behavior.

Since Rose (1988), a great deal of effort has been devoted to verifying the statistical properties of the real interest rate. The two most common approaches used in the literature involve either testing for a unit root in the real interest rate or for cointegration in systems containing inflation and nominal interest rates. In this case, the real interest rate is stationary if the nominal interest rate and inflation are nonstationary and cointegrate with vector [1,-1]. This literature includes, inter alia, McDonald and Murphy (1989), Mishkin (1992), Wallace and Warner (1993), Crowder and Hoffman (1999), Koustas and Serlitis (1999) and Rapach and Weber (2004). The evidence presented thus far is somewhat mixed. Further, even when the evidence from cointegration tests is supportive of the notion that movements in nominal interest rates reflect movements in expected inflation, changes in the latter generally seem to have less than a one-for-one effect on the former, implying that inflation is non-neutral. Besides, the evidence is sensitive to choice of the periods and countries analysed. The results on cointegration thus provide not much support for mean-reversion in real interest rates (Lai, 1997). As for direct tests for a unit root which are mostly based on the conventional Dickey-Fuller tests, they also yield less than supportive evidence for mean-reversion (e.g. Goodwin

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1 The Fisher effect is the cornerstone of many theoretical models that generate inflation neutrality and is important for understanding the movements in nominal interest rates. The real interest rate affects all intertemporal savings and investment decisions in the economy and, as such, plays a central role in the dynamics of asset prices. Understanding the relationship between interest rates and other variables such as inflation is, therefore, a central issue in the study of financial markets.

2 A unit root in the real interest rate is also problematic from the standpoint of the canonical neoclassical growth model with explicitly optimising, infinitely lived agents (Rapach and Weber, 2004) and issues which are central to financial theory such as the Black-Scholes formula for pricing options which is based on the assumption of a constant rate (Garcia and Perron, 1996).

3 Within the cointegration framework, the Fisher effect can only be tested if inflation and nominal interest rates are I(1). However, Rose (1988) and Grier and Perry (1998) argue that inflation is I(0) whereas Banerjee et al. (2001) and Banerjee and Russell (2001) find evidence of a unit root. Others such as Conrad and Karanasos (2004) argue that it is a long-memory process. Results from cointegration analysis must be interpreted cautiously, therefore.
and Grennes, 1994; Phylaktis, 1999). A unit root in the real interest rate is confirmed even when using the powerful tests of Hansen (1999) and Romano and Wolf (2001) as documented by Rapach and Wohar (2004a). Finally, Lai (1997) and Tsay (2000) explore the potential presence of a fractional unit root in the real interest rate and conclude that it is mean-reverting, though rather persistent.

This paper takes a completely different approach and extends the literature in a new direction. It argues that the inability to find evidence of a long-run cointegrating relationship between inflation and the nominal interest rate is due to the low power of conventional tests to reject a false null hypothesis of a unit root in the real interest rate or no cointegration with a data span corresponding to the short length of the samples typically used. It does not help that the data are often sampled at high or low frequencies, as the power of the tests depends on the span of the data, rather than its frequency (Hakkio and Rush, 1991; Rapach and Wohar, 2002). To overcome this problem, we propose to increase the power of the tests by increasing the length of the sample period under examination. Specifically, we examine the mean-reverting properties of real interest rates by applying uniform tests to monthly data spanning four major countries and 70 to 125 years of experience. To our knowledge, this is the first study which looks at the real interest rate using such a long historical data set.

An important pitfall of the recent literature is that it is only concerned with the question of whether or not real interest rates contain a unit root. However, rejection of the unit root null is not necessarily evidence in favor of the Fisher effect as it is possible that the tests reject the nonstationarity hypothesis but the process is still persistent. Also, studies, which do not account for the persistence of real interest rates, may lead to the incorrect acceptance of the Fisher hypothesis. Therefore, instead of focusing on the unit root question only, we believe that a powerful test requires a more thorough examination of the persistence and the speed of mean-reversion of real interest rates based on the estimation of confidence intervals for the largest roots of autoregressive (AR) models and the half-life. In this paper, we bring a recent empirical innovation to the long span of data to investigate the dynamics of the real interest rate. The empirical innovation is the median unbiased estimation (MUE) method of Gospodinov (2004). This method is based on the construction of confidence intervals for the largest root of local-to-unity AR processes and is particularly useful for estimating the half-lives which have become the standard tool for measuring persistence. Further, impulse

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4 The low power problem has received a lot of attention in the purchasing power parity (PPP) literature and one response has been the use of long span data (Abuaf and Jorion, 1990; Lothian and Taylor, 1996; Taylor, 2002). These studies find support for PPP contrary to those using short data. Given this finding, it seems surprising that the literature on the real interest rate has not pursued this response to the power problem.

5 Conventional tests may fail to reject the null even when rates exhibit slow mean-reversion. This low power problem is magnified for small samples because a mean-reverting series could be drifting away from its long-run equilibrium in the short-run. In response to this, Diebold et al. (1991) showed that long samples are important for identifying mean-reversion in slowly decaying processes whilst Frankel (1990) argued that, with slow convergence speeds, the autoregressive parameter might be very close to unity and one would need long spans of data to reject the unit root null. Thus, we might have sufficient span to have a reasonably powerful test.

6 It is of course possible to increase the power of the tests with, for instance, panel unit root tests under cross section dependence, SUR-ECM or panel VAR. This is, in fact, the other response to the low power problem followed in the PPP literature but not yet pursued in the real interest rate literature. However, rejection of the unit root null in this case may happen only because one of the variables considered is I(0). Hence, if rejection occurs, then it may not be informative and certainly it cannot be concluded that this rejection implies evidence supportive of mean reversion. Using long spans of data means that this problem is avoided. Nonetheless, there is the potential problem of structural instability when using long data. In this paper, we follow the PPP literature that utilizes long spans of data and assume that the dynamics of the real interest rate are relatively stable over the sample period. In order to employ more powerful tests and long spans of data, we have to assume relatively stable dynamic processes over long periods.
response analysis is performed which constitutes a useful visual tool for investigating the speed with which shocks are eliminated.

Finally, the Fisher hypothesis is essentially an *ex-ante* condition; however, much of the extant literature tends to assume that expectations are formed rationally and that the difference between the *ex-ante* and *ex-post* real interest rates is equal to a stationary, zero mean forecast error term. This assumption allows researchers to use *ex-post* instead of *ex-ante* inflation data and, therefore, sidestep the difficult issue of specifying how expectations are formed. Nevertheless, in a recent paper, Evans and Lewis (1995) argue that the use of *ex-post* inflation data can produce substantial small-sample bias in estimates of the Fisher relationship due to the peso problem; rational anticipations of shifts in the inflation process which do not occur in sample. This would result in a persistent wedge between expected and realised inflation and may, as a result, create the appearance of permanent shocks to the real interest rate even when none are truly present. To investigate whether our results are sensitive to the measurement of real interest rates, we consider the properties of both *ex-ante* and *ex-post* real interest rates. The expected values of inflation used to construct the *ex-ante* rates are obtained by means of a signal extraction procedure based on a state space model and a Kalman filter.

The remainder of this paper is set as follows. Section 2 describes the analytical Fisher relationship and explains the econometrics of local-to-unity processes. In Section 3 we discuss the data and report the empirical results. Impulse response analysis is provided in Section 4. The last section concludes.

2. The real interest rate and empirical methodology

The *ex-ante* Fisher equation is defined as follows:

\[ i_t = r_t + \pi_t^e \]  

where \( i_t \) is the nominal interest rate, \( r_t \) is the real interest rate and \( \pi_t^e \) is the anticipated inflation rate. For the Fisher effect to be valid \( i_t \) and \( \pi_t^e \) have to be cointegrated with vector \([1,-1]\) or, equivalently, the real interest rate, \( r_t \), must be a mean-reverting stationary process.

The stationarity of the real interest rate can be verified by performing unit root tests to determine whether it contains a unit root. However, if unit root is rejected, but the true value of the largest root of the AR representation of \( r_t \) is close to unity, shocks will be slow to dissipate, and this stationary process may not be significantly different from a true unit root process in the economic sense. As a result, the emphasis should not be on whether real interest rates have a unit root it should instead be on measuring the economic implications of their behaviour. What market participants and monetary authorities care about is the degree of persistence in the real interest rate. One measure of persistence that has received a lot of attention in the literature is the half-life which is defined as the number of periods it takes for deviations to subside permanently below 50% in response to a shock.

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7 The expected values of inflation are obtained using a signal extraction procedure. This procedure is used to separate unobservable components, state variables or expected values in our case, from an observable variable containing noise. This is achieved through the application of the law of iterated projections by means of the Kalman filter technique. The estimated model is:

\[ y_t = \xi_t + \nu_t \]  
\[ \xi_{t+1} = \xi_t + \varsigma_t \]

where \( \xi_t \) is a vector of possibly unobserved state variables and \( \nu_t \) and \( \varsigma_t \) are vectors of mean zero, Gaussian disturbances. Equations (2) and (3) are the signal and state equations, respectively. There are other ways of obtaining data on expected inflation, however. For instance, Data resources incorporated, the Michigan survey of consumers and the Livingston Survey all provide data for inflation forecasts. Unfortunately, they do not provide data for the period under examination. In fact, data provided by these sources begin mostly in the 1970s. In addition, the core inflation rate is sometimes preferred as a proxy for the expected inflation. Unfortunately, we do not have these data either. Recently, St-Amant (1996) and Gottschalk (2001) use structural VARs to derive estimates of expected inflation. Though promising, this approach requires a number of theoretical identifying restrictions.
The point estimate of the half-life alone does not provide a complete description of the persistence of the real interest rate, however. It needs to be supplemented with confidence intervals in order to measure the precision of the estimates. The construction of intervals for the slope coefficient and the half-lives using ordinary least squares (OLS) poses a number of problems, nonetheless. These intervals are not valid under the unit root null and, even if the Fisher effect holds in the long-run, are biased downwards in small samples. Moreover, the estimate of the half-life $h_{0.5} = \ln(0.5)/\ln(\rho)$, which is based on an autoregressive model of order 1, assumes that shocks decay monotonically, but for higher order AR processes this may not be the case. To remedy this, Cheung and Lai (2000) recommend using impulse response analysis. Now, to address the problems associated with the estimation of the confidence intervals for the largest root, we use the MUE method proposed by Gospodinov (2004).

### 2.1 Median unbiased estimation

The method employed in this paper is due to Gospodinov (2004) and is based on inverting the likelihood ratio (LR) statistic of the largest root under a sequence of null hypotheses of possible values for the impulse response and the half-life. Starting from the following ADF regression:\(^8\)

$$y_t = \alpha y_{t-1} + \sum_{j=1}^{p-1} \psi_j y_{t-j} + \epsilon_t$$  

(4)

where $\alpha$ is a measure of the persistence of the series and is cast as local-to-unity ($\alpha = 1 + c/T$ and holding $c$ fixed as $T \to \infty$), $\varphi = (\alpha', \psi') \in \Xi \subset R^p$ and the maximum likelihood estimator of $\varphi$ is $\hat{\varphi}$. Suppose that we are interested in the null that the impulse response function at horizon $l$, $\theta_l$, is 0.5 (the half-life), versus the alternative $\theta_l \neq 0.5$, then this null or restriction can be written as $h(\varphi) = 0$, where $h \equiv \theta_l - 0.5$. $R^p \to R$ is a polynomial of degree $l$. Let $\tilde{\varphi}$ denote the restricted maximum likelihood estimator and $LR_T$ the likelihood ratio statistic of the null. Gospodinov (2004) shows that the restricted estimator of $\alpha$ converges at a faster rate than the unrestricted estimator and this helps obtain a consistent estimate of the nuisance parameter $c$ under the imposed restriction (null hypothesis). Moreover, the restricted estimation provides consistent estimates of the impulse response functions and the half-lives.\(^9\)

The restricted LR estimator of (4) under the null hypothesis $h(\varphi) = 0$ is:

$$LR_T \to \int_0^1 J^*_c(s) dW(s) / \int_0^1 J^*_c(s)^2 ds$$  

(5)

where $J^*_c(r) = J_c(r) - \int_0^1 J_c(s) ds$, $J_c(r) = \int_0^1 \exp \left\{ (r-s)c \right\} dW(s)$ is a homogenous Ornstein-Uhlenbeck process and $\Rightarrow$ denotes weak convergence. The limiting theory of $LR$ is dominated by the near nonstationary component and is not affected by the presence of stationary components as measured by the second term in regression $\sum_{j=1}^{p-1} \psi_j y_{t-j}$.

\(^8\) Because neither theory nor empirics support the idea of a trend in real interest rates, the tests performed in this paper are based on demeaned data.

\(^9\) The standard method for estimating (4) is OLS and the conventional asymptotic interval is based on the asymptotic $N(0,1)$ approximation to the $t$-statistic which is valid only if $|\alpha| < 1$. This approximation is poor in practice especially when the persistence parameter $|\alpha|$ is close or equal to unity. Specifically, if the true persistence parameter is not unity, OLS estimates are biased downwards and confidence intervals based on asymptotic methods have poor coverage properties. When persistence is unity, the coverage problems of the asymptotic intervals stem from the fact that the asymptotic distribution of $\alpha$ is non-standard. Bootstrap methods are also poor. This is because the percentile-$t$ bootstrap is based on the assumption that the bootstrap quantile functions are constant, which is false for the AR model. This nonconstancy persists in large samples if we cast $\alpha$ as local-to-unity as $\alpha = 1 + c/T$. In this case, the asymptotic distribution of the $t$-statistic depends on $\alpha$ through the nuisance parameter $c$ that is not consistently estimable. Thus, in the near unit setting, the interval does not properly control for Type I error (Basawa et al., 1991).
The method proposed by Gospodinov (2004) has many interesting features. First, contrary to standard asymptotic and bootstrap methods, which have been shown to have poor coverage properties, this method parameterizes $\alpha$ as a function of $T$ and is expected to yield better small-sample and coverage performance. Second, the $LR$ statistic does not require variance estimation for studentization. It is criterion function-based and is tracking closely the profile of the objective function. Also, the inversion of the $LR$ statistic appears to shift the confidence intervals away from the nonstationarity region much more often compared to methods based on inverting the OLS estimator of $\alpha$ such as the grid bootstrap of Hansen (1999). For this reason, Gospodinov’s is preferred to Hansen’s method. Further, using a series of Monte Carlo experiments, Gospodinov (2004) shows that the inversion of the $LR$ statistic appears to be controlling the coverage over a wide range of parameter configurations and across different forecasting horizons. Finally, this method is expected to yield tight confidence intervals, which makes them highly informative.

Another statistic which takes into account the restricted and the unrestricted estimates is also proposed:

$$LR_T^z = \text{sgn}[h(\hat{\phi}) - h(\bar{\phi})]\sqrt{LR_T}$$

where $\text{sgn}(.)$ is the sign of $[h(\hat{\phi}) - h(\bar{\phi})]$. This statistic can be used for constructing two-sided, equal-tailed confidence interval and median unbiased estimate. Finally, the $100\eta\%$ confidence interval for the half-life, which is based on impulse response analysis, is:

$$C_{\eta}(l) = \{ l \in L : LR_T \leq q_{\eta}(c) \}$$

where $q_{\eta}(c)$ is the $\eta^{th}$ quantile of the asymptotic distribution, $l$ is the lead time of the impulse response function and $\bar{\phi} = \arg \max_{\theta} I_r(\phi)$ subject to $\theta_r - 0.5 = 0$. The confidence interval for the half-life can be constructed using either $LR_T^z$ or $LR_T$.

3. Data and preliminary analysis

The data utilized in this analysis is extracted from the www.globalfindata.com database and includes monthly long-term bond yields and consumer price index (CPI) series for the US, UK, France and Japan spanning the period from 1876 to 2003\[10\]. The exact sample period for each country is: 1876:01 to 2000:06 for the USA; 1934:01 to 2003:07 for the UK; 1923:01 to 2001:08 for Japan; 1916:01 to 2003:07 for France. The end date was in each case dictated by data availability. The long-term bond yields are preferred to short-term rates because these rates are closely linked to the cost of long-lived capital. Also, firms do not usually make their investment decisions on the basis of short-term rates. Indeed, to the extent that firms borrow in bond markets, long-term yields will be more informative (Fujii and Chinn, 2000). The inflation rate is defined as the annualised growth rate of the CPI which was seasonally adjusted by taking the average for the previous 12 months. Also, some data points were missing for Japan’s yield during the 1940s and this was corrected using linear interpolation\[11\].

First, we test for the stationarity of real interest rates using the efficient generalized least squares (GLS) version of the Dickey-Fuller (DF) test due to Elliott et al. (1996) whose results

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\[10\] Since the aim is to measure the half-life, by using high frequency monthly data, we reduce the effect of the temporal aggregation bias highlighted by Taylor (2001). Indeed, Taylor showed that the half-life can be seriously over-estimated if adjustment takes place during a time frame that is shorter than the sampling frequency of the data. This may be important in the case of the real interest rate. In a recent paper, Tsay (2000) modelled the real interest rate using an autoregressive fractionally integrated moving average (ARFIMA) process and found it to be more persistent using quarterly than monthly data.

\[11\] Although such an interpolation may be ad hoc, it was considered necessary to give the empirical analysis a fair chance on this data. Without interpolation, any mean-reversion of the real interest rates for Japan would be missed and a bias against stationarity would result.
are reported in table 1. While most unit root tests are only concerned with testing the null that the largest root is unity against the alternative that it is less than one, the DF-GLS tests the null against a specific alternative $H_1: \alpha < 1$ where $\alpha = 1 + c/T$. Further, using a sequence of tests of the null hypothesis of a unit root against a set of stationary local alternatives, Elliott et al. (1996) showed substantial power gain over the conventional ADF test, that has low power against close alternatives so that the unit root null can seldom be rejected for highly persistent variables, could be obtained from using the DF-GLS test. The lag length is chosen using the modified AIC (MAIC) of Ng and Perron (2001) which produces the best combination of size and power. From table 1, one can see that the DF-GLS test rejects the unit root null hypothesis for all series at the 1% level of significance. This is an important result since it contrasts sharply with what has been reported in earlier studies on real interest rates which were essentially based on shorter samples and weaker statistical tests than those we are using.

3.1. Confidence intervals for the largest root and the half-life

Table 1 reports the median unbiased estimates of $\alpha$ and the 90% and 95% MUE confidence intervals for this measure of persistence. The intervals are constructed by inverting the acceptance region of the powerful DF-GLS test of Elliott et al. (1996). Whilst the methodology in Section 2.1 is based on an ADF regression, the extension of this method to the DF-GLS test is straightforward. Instead of working with the data in levels as in (4), we simply work with the GLS demeaned data in the DF-GLS regression. Moreover, the finite-sample distribution of the DF-GLS test is obtained using the grid bootstrap of Hansen (1999).

The median unbiased estimates of the largest root are indicative of strong persistence in real interest rates. The MUE estimates of $\alpha$ range from 0.9491 to 0.9924. This latter figure corresponds to the US ex-ante real interest rate and suggests a high degree of persistence. Also, none of the MUE confidence intervals for the largest root are found to contain unity as an upper bound; although, the limits are in most cases near the unit root boundary. This is consistent with the results of the DF-GLS test which rejected the unit root null hypothesis. It is interesting to note that the lower bounds are close to the point estimates and are never below 0.9345. The lower bounds which can be re-interpreted as upper bounds for the fastest speed of mean-reversion are therefore still consistent with the view that the variables under scrutiny are slow to mean-revert. On the whole, our findings are supportive of the idea that real interest rates are mean-reverting, albeit quite persistent and display near-unit-root behavior, precisely the type of behavior that will be difficult for standard tests to detect for short samples. Finally, the MUE confidence intervals from the powerful DF-GLS test appear to be rather tight and this demonstrates the potential for sharper inference from this test (Elliott and Stock, 2001; Gospodinov, 2004).

The MUE point estimates and confidence intervals for the half-life based on impulse response analysis are shown in table 2. The point estimates of the half-lives generally fluctuate between a low of 1.4048 to a high of almost 2.9 years. Apart from Japan, most estimates are less than 2 years, nonetheless. Although, the point estimates are useful in principle, what is most important is the upper bound of the confidence interval. These bounds

12 To check the robustness of our findings, we also report the outcome of the Ng and Perron (2001) unit root test. The results which are reported in table 1 but not analysed are quantitatively similar to those of the DF-GLS test. Essentially, the test also rejects the unit root null significantly.

13 The long lags selected by MAIC are not surprising, however, as it is designed to select long lag lengths in the presence of roots near unity and shorter lags in the absence of such roots. In addition, Ng and Perron (2001) showed that criteria which choose too few lags for the DF-GLS test are badly sized and may produce inappropriate rejections of the unit root null.

14 The OLS estimate of the largest root, $\alpha_{\text{OLS}}$, is also reported. This estimate is based on ADF regressions and, thus, does not optimally exploit the sample information in terms of power whereas $\alpha_{\text{MUE}}$ based on the DF-GLS test, does. Besides, $\alpha_{\text{OLS}}$ is normally treated cautiously as it is biased downwards in small samples. However, given the large size of our samples, the bias in the OLS estimate disappears almost completely.
are again indicative of a high degree of persistence with the ex-ante real interest rate for the UK being the rate with the highest upper limit, approximately 7.56 years. The lowest upper limit is found for the UK ex-post real interest rate. The fact the upper bound for the UK ex-post rate is so much lower than the corresponding ex-ante one is somewhat surprising. Further, while the upper bounds imply that it takes a few years for deviations to subside permanently below 50% in response to a unit shock in the level of the real interest rate, the lower bounds do not necessarily confirm the Fisher effect as they are always greater than one year, except for Japan which has lower bounds equal to 0.9824 (90%) and 0.9679 (95%) years. In other words, while it is true that the upper bounds are high, the lower limits are compatible with a time horizon in which convergence is still rather slow.

In addition, it transpires that the estimates are not critically dependent on how rates are measured, whether ex-post or ex-ante, since both rates are rather persistent. The only exception is the UK for which the ex-post rate is significantly less persistent than the ex-ante rate. A possible explanation is that for a long-run concept such as persistence, the distinction between ex-ante and ex-post rates does not matter so much. Also, if forecast errors are to blame for the failure to detect mean-reversion in small samples due to peso problems, then the fact that there is no substantial difference between ex-post and ex-ante rates in terms of the speeds of mean-reversion (apart from the UK), means that these errors are likely to be much smaller over long periods than over shorter periods so that the problem of measuring expectations is mitigated to some extent if the researcher uses very long spans of data.

To sum up, the previous literature on the real interest rate could not reject the hypothesis that these rates were realisations of unit root processes. A unit root, in this context, implies that shocks to the real interest rate are permanent and the half-life is infinity. In retrospect, we realise that these studies had shorter samples and used tests of low power such as the traditional ADF test. Nevertheless, whilst we have sufficient power in our tests to accept the stationarity hypothesis and reject a unit root, the confidence intervals obtained by inverting the aforementioned tests suggest that real interest rates are still persistent, a conclusion which has implications for our theoretical priors. In any case, compared to prior studies, our evidence means that the persistence of the real interest rate is less than we thought before.

4. Impulse response analysis

As a final exercise, we construct the 90% and 95% confidence intervals of the impulse responses derived from the inversion of the LR statistic. The graphs of the first 120 responses are displayed in figure 1. Whilst, impulse response analysis can be performed for even longer horizons, we report results up to 10 years since this is quite a close approximation to the infinite horizon. In all cases, real interest rates have zero long-run persistence, confirming the existence of long-run mean-reversion and, most importantly, the absence of a unit root. The upper limits of the confidence intervals of the impulse response functions suggest that one quarter of the adjustment is completed within 3 years. The process of convergence deserves further attention, however. This process appears to exhibit some nonlinearity in the direction of adjustment due to short-term overshooting and overreacting. Specifically, the impulse responses are all hump-shaped, with initial shock amplification before eventual dissipation. Given that the shock impact magnifies in the initial few months rather than diminishes, the maximum response cannot be felt until a few years as a consequence.

15 Aside from Japan, the upper bounds indicate that the ex-ante real interest rates adjust more slowly than ex-post rates. In spite of this result, both rates exhibit slow mean-reversion and, apart from the UK, the difference between them is not very large. This is not surprising, however. Lai (1997) uses fractional integration analysis and finds the evidence to be robust with respect to whether inflation forecasts or realised inflation rates are used. Evans and Lewis (1995) find that the coefficient of the inflation rate in equation (1) to be closer to unity when using inflation forecasts generated with a Markov-switching model than when using realised data, even though it is still less than 0.8.
Recently, Coakley and Fuertes (2002) and Maki (2003) have explored the possibility of nonlinearities in real interest rates. Coakley and Fuertes (2002) examine the dynamics of quarterly UK real interest rates between 1950 and 1999 using the Enders and Granger (1998) threshold autoregressive (TAR) unit root test. Maki (2003), on the other hand, tests for nonlinear cointegration between inflation and the nominal interest rate for Japan with Breitung’s nonparametric method\textsuperscript{16}. Nonlinearity, in this context, materializes itself by a variable speed of adjustment towards equilibrium: the larger the deviation, the faster it will be driven back to its equilibrium value. This variable speed of adjustment might be due to the presence of uncertainty, noise trading, opportunistic central bank behaviour or market imperfections such as transaction costs, among others. Transaction costs, in particular, create a band of inaction within which no adjustment for small deviations from the relationship linking inflation to the nominal rate of interest takes place, and deviations may follow a persistent or even a (near) unit root process, while outside the band, as the benefit of arbitrage exceeds the cost, the process switches abruptly to become mean-reverting towards the threshold band. In this framework, the real interest rate follows a nonlinear process that is mean-reverting towards the threshold or transaction cost band. This sluggish band of inaction may indeed explain some of the long tails observed in the impulse response functions.

5. Conclusion

The Fisher effect embodies the hypothesis that inflation and the nominal interest rate move one-for-one in the long-run so that the real interest rate is stationary. Starting with Fisher and extending to the present, this seemingly simple and intuitive concept has found limited empirical support. Typically, the literature cannot reject the hypothesis that the real interest rate is a realization of a unit root process. However, unit root tests may have low power to reject the unit root null hypothesis due to the relatively small number of observations that are normally used.

Recognising the importance of the real interest rate for the Fisher hypothesis, monetary policy and theoretical modelling, we have examined the time-series properties of four major monthly real interest rates using samples that range from 70 to 125 years. The expectation is that tests based on long data series will have more power to reject a unit root than those using short samples which ignore most of the available data. The use of long spans of data is also motivated by studies that find better support for long-run economic relationships using long spans of data\textsuperscript{17}. We have also investigated the persistence of real interest rates through the computation of median unbiased point estimates and confidence intervals for the half-lives of deviations for DF-GLS regressions with the methodology of Gospodinov (2004). The results indicate the absence of a unit root in \textit{ex-ante} and \textit{ex-post} real interest rates. In fact, the unit root null hypothesis is unequivocally rejected at the 1\% level of significance. This finding is noteworthy given the failure to reject the unit root null in the extant literature. Nonetheless, the confidence intervals for the half-life and the impulse response functions suggest that these rates are rather persistent and it may take some time for a shock to die out. Though, compared with prior studies, which could not reject the unit root null hypothesis such that shocks are permanent and the half-life is infinity, the persistence of the real interest rates is clearly less than we previously thought.

Notwithstanding the rejection of the unit root null, the fact that we cannot discount the possibility of relatively high half-lives as demonstrated by the upper limits of their confidence intervals makes it difficult to validate the Fisher hypothesis. In fact, we can only conclude that the evidence is weak. Persistence is problematic not just for the Fisher hypothesis and the

\textsuperscript{16} Although, Maki (2003) finds evidence of nonlinear cointegration between inflation and the nominal interest rate, no test for the restriction that the vector is $[1,-1]$ is provided. As a result, the possibility that his evidence is supportive of only a weak version of the Fisher effect cannot be ruled out.

\textsuperscript{17} See Sarno and Valente (2004) for evidence on PPP and Rapach and Wohar (2002) for tests of the monetary model of exchange rate determination with long spans of data.
determination of the effects of monetary and fiscal policies but also for the models outlined in the introduction. The CCAPM, in particular, implies that the growth rate of consumption and the real interest rate should have similar time-series characteristics (Rose, 1988). Still, the growth rate of consumption has been found to contain no unit root and does not exhibit the persistence apparent in real interest rates. This means that the Euler equation derived from the CCAPM cannot be expected to hold in the presence of a persistent real interest rate. Hence, in finding no unit root, the results might have been seen as resolving the puzzling irregularity as observed by Rose (1988), concerning the unit root behaviour of the real interest rate and its implications for the CCAPM, the observed persistence means that another irregularity emerges. Finally, recent research by Garcia and Perron (1996), Bai and Perron (2003), Rapach and Wohar (2004b) and Lai (2004) suggest that real interest rates are not unit root once a mean shift is allowed for. However, breaks pose the same problems as persistence since consumption growth does not show evidence of structural breaks (Rapach and Weber, 2004).
References


<table>
<thead>
<tr>
<th>sample period</th>
<th>rate</th>
<th>$k_{MAIC}$</th>
<th>MZ$_a$</th>
<th>MZ$_t$</th>
<th>DF-GLS</th>
<th>$\alpha_{OLS}$</th>
<th>$\alpha_{MUE}$</th>
<th>90$_{lower}$</th>
<th>90$_{upper}$</th>
<th>95$_{lower}$</th>
<th>95$_{upper}$</th>
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<tbody>
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<td>-16.5737</td>
<td>-2.8697</td>
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<tr>
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<tr>
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<td>13</td>
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<td>[0.0025]</td>
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<td>0.9924</td>
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<tr>
<td>UK 1934:01-2003:07</td>
<td>ex-post</td>
<td>12</td>
<td>-21.3114</td>
<td>-3.2554</td>
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</table>

Note: The median unbiased estimates and confidence intervals for the largest root are constructed with the grid bootstrap of Hansen (1999) using the efficiently demeaned DF-GLS statistic. The optimal lag lengths for the unit root test statistics are set according to the modified Akaike information criterion of Ng and Perron (2001). The relevant critical values for the MZ$_a$ and MZ$_t$ statistics are: -13.8 and -2.58 at 1%; -8.1 and -1.98 at 5%; -5.7 and -1.62 at 10% significance level. Figures in square brackets are $p$-values.
Table 2
Median unbiased confidence intervals for the half-lives.

<table>
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<th>sample period</th>
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<th>half-life</th>
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<th>90upper</th>
<th>95lower</th>
<th>95upper</th>
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<td>1.3243</td>
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<td>0.9824</td>
<td>3.5599</td>
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<td>1.0781</td>
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<td>1.5058</td>
<td>6.7020</td>
</tr>
<tr>
<td>UK 1934:01-2003:07</td>
<td>ex-post</td>
<td>1.5827</td>
<td>1.3032</td>
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<td>1.2724</td>
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<tr>
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<td>1.2687</td>
<td>5.7577</td>
<td>1.2274</td>
<td>7.5519</td>
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</table>

Note: The half-lives estimated from the impulse response functions are measured in years.
Figure 1
Median unbiased impulse response functions estimated from the DF-GLS regressions for the real interest rates. The unbroken line indicates the point estimates of the impulse responses. The dashed and dotted lines give the corresponding confidence bands.

USA.
Figure 1 (continued)

UK.
Figure 1 (continued)

France.
Figure 1 (continued)

Japan.