Abstract

We use a two-good dynamic optimizing small open economy model to formalize the economic intuition of Krugman about the explanation of the J-curve phenomenon in terms of habit persistence in consumption and sluggishness in capital adjustment. The results differ markedly depending on the permanence or temporary nature of the relative price change. A short-lived terms of trade worsening may lead to a once-for-all decrease in the marginal utility of wealth and to higher steady-state values of the habitual standard of living, the real expense and the net foreign assets through the combination of intertemporal speculation, inertia, and hysteresis effects. Investment and real expense follow non-monotonic transitional paths and current account dynamics is driven by new forces as the long-run intertemporal elasticity of substitution under time non-separable preferences. In accordance with recent empirical results, investment is procyclical, H-L-M effect holds, net foreign assets adjustment exhibits a J-curve, current account surplus is associated with a fall in real income.

Keywords: Current account; Habit Formation; Temporary shock; J-curve.
JEL Classification: F41, E22, E21, F32.
1 Introduction

The objective of this contribution is to provide a new explanation of the J-curve phenomenon by elaborating a framework which differs from the previous two-good intertemporal optimizing continuous time models interested in terms of trade shocks in four respects: [i] the framework combines time non-separable preferences in the consumption-side with adjustment costs in capital in the production-side, [ii] an explicit and a consistent two-step analytical procedure for studying short-run and long-run effects of a temporary terms of trade worsening has been applied, [iii] a comparison with stylized facts is operated, [iv] the analytical solutions allow to determine rigorously and accurately the factors which influence the current account dynamics. We show that the J-shape response of the net foreign assets following an adverse terms of trade shock depends crucially on [i] the degree of habit persistence in consumption, [ii] the installation costs of capital, [iii] the domestic contents of consumption and investment expenditure, [iv] the long-run intertemporal elasticity of substitution (under time non-separable preferences), and [v] the length of the shock. The four last factors determine in turn the strength of the smoothing and intertemporal speculation effects which influence the real expense and investment reactions once the perturbation hits the economy. Two articles have motivated this contribution: one shows evidence of a J-curve but rejects its standard interpretation, and the second suggests a new explanation in terms of habit persistence in consumption and capital adjustment costs but does not formalize it. From empirical results of Leonard and Stockman [2002], a new explanation of the J-curve phenomenon is needed.¹ To this end, a micro-founded model is elaborated which allows to identify the channels through which relative price perturbations affect short and long term changes of macroeconomic aggregates; in particular, we formalize the economic intuition of Krugman [1989] about the interpretation of non-monotonic adjustment of the current account.²

What are the optimal responses of macroeconomic aggregates following a terms of trade shock, particularly a transitory perturbation? This question has received a lot of attention in the eighties by abstracting from capital accumulation in infinite horizon or two-period models (see e. g. Obstfeld [1982], [1983], Svensson and Razin [1983], Ostry [1988]). Sen [1990], Karayalçın [1995], Mansoorian [1993], [1998] address this issue in the nineties by introducing real money balances in the utility, heterogeneity between the individuals, or by assuming an habit-forming behavior. These contributions share the common characteristic of considering a small open economy without capital accumulation and of focusing only on an unanticipated permanent terms of trade shock. Sen and Turnovsky [1989] and Servèn [1999] analyze transitory terms of trade perturbations by considering capital adjustment costs and by restricting the utility function to be time separable. Although the two papers explore the effects of an unanticipated transitory change in the relative price, the former applies a procedure which contains an inconsistency (see Schubert and Turnovsky [2002]). The latter use a framework which is unable to allow for consumption dynamics and therefore to explore its joint adjustment with net foreign assets accu-
mulation. Our analysis departs from these studies by allowing both consumption and investment dynamics in an optimizing-intertemporal model that captures the main characteristics of a small open economy. The formalization is sufficiently tractable to enable us to characterize in detail the dynamic of current account after permanent and transitory shocks. More particularly, we follow a recent literature strand that explores the implications of time non-separable preferences and explore the possibility of non-monotonic adjustment of net foreign asset position (see e.g. Karayalçın [1994], Ikeda and Gombi [1998]). We go beyond by drawing the potential importance of considering the effects of temporary shocks using a consistent and explicit analytical procedure in a two-good model. The added flexibility of our higher order system gives rise to some non-monotonic transitional paths which originate from new factors. It is formally shown that the initial current account response following an unexpected transitory terms of trade shock may be disentangled into five components: a softened response of saving and investment with respect to a permanent perturbation, a term reflecting the consumption smoothing effect, and the influences of investment and consumption responses due to the transitory fall of the investment- and consumption-based real interest rates. The specification of time non-separable preferences coupled with the analysis of temporary terms of trade shocks provides an interesting insight into the consumption-current account relation. The intertemporal speculation effect is driven by the long-run intertemporal elasticity of substitution which is higher than the long-run intertemporal elasticity of substitution under time separable preferences. A second factor softens the consumption response and determines the direction of temporal path of real expense depending on the shock’s length.

The justification for focusing on terms of trade disturbances is threefold. First, they constitute both supply-side and demand-side shocks. A change in the price of an intermediary input or in total factor productivity are typically supply-side perturbations as they affect the production function and restrict their influence on consumption decisions to the change in the equilibrium value of the marginal utility of wealth. Second a relative price disturbance gives rise to an income, a smoothing, an intratemporal, and an intertemporal speculation effects which enlarge the scope of economic aggregates’ responses. Third, regressions performed by Fischer [1993] and Easterly et al. [1993] show that terms of trade variations influence economic growth. According to panel data estimations of Loayza et al. [2000], terms of trade play a key role in explaining saving rates variations. Other empirical works show that a great part of real income, current account and net exports fluctuations can be attributed to terms of trade shocks, particularly for small open economies (see e.g. Fox et al. [2002], Otto [2003]).

The framework of Ikeda and Gombi [1998] is extended by considering that consumption and investment have an import content. Our analysis departs from the study of the authors in the kind of shock considered, which in turn, expand the effects at work and results in new conclusions both in the short-run and in the long-run. More importantly, it differs in the analytical method applied to study unanticipated transitory terms of trade shocks. This new solution procedure
has been recently proposed by Schubert and Turnovsky [2002] which corrects an inconsistency in the preceding solution method initiated by Sen and Turnovsky [1990]. Moreover, this framework is more suitable than its predecessors which study the current account response to terms of trade worsening. First, by assuming habit-forming consumers we allow for a slow adjustment of real expense following the relative price shock. This sluggish response of consumption in accordance with “excess smoothness in consumption” is no longer obtained by Obstfeld [1983] and Servèn [1999]. Second, by considering that investment has an import content, our study highlights relevant factors that determine optimal investment response following an unanticipated transitory terms of trade perturbation. Obstfeld [1982] and Mansoorian [1993], [1998] assume no capital accumulation and consider only permanent terms of trade shocks. Sen and Turnovsky [1989] incorporates a labor-leisure arbitrage and assumes that investment has zero import content. Third, the model is sufficiently tractable to investigate the factors that determine the consumption, investment, and external position reactions to an unanticipated temporary and permanent terms of trade deterioration. The international business cycle models like those of Mendoza [1995], Kose [2002] provide complex frameworks with imported capital goods, multiple sectors, intersectoral costs of reallocation of production factors, recursive preferences, but cannot be solved analytically. By taking advantage of the present model’s tractability to extract analytical results, we propose new insights about the effects of various terms of trade shocks on consumption, investment, and external position dynamics.

Number of empirical studies have shown that terms of trade changes have a large temporary component (see e. g. Reinhart and Wickham [1994] for developing countries, Cashin and Mc Dermott [2002] for developed countries) and that the duration of relative price shocks varies widely across countries (see Cashin, Mc Dermott, Patillo [2004]). Surprisingly, the analysis of transitory terms of trade perturbations in intertemporal optimizing continuous time models in an analytical way are scarce, particularly when capital accumulation is considered. When such an investigation is performed, some papers are not very explicit in their solution methods, or contain an inconsistency as emphasized by Schubert and Turnovsky [2002], or report only a numerical analysis (see for example Glenn [1997]). The study of an unanticipated temporary terms of trade worsening in the formal setup we present below allows to show a new effect at work by adopting the recent two-step approach recently developed by Schubert and Turnovsky [2002]. Following a transitory decline in the home goods’ relative price, real expense may rise in the short-run as the conventional time separable utility function predicts (see Obstfeld [1983]); but more importantly real expense may be persistently higher in the long-run in the present formal setup. The combination of a strong habit persistence in consumption, an intertemporal speculation effect, and an hysteresis phenomenon plays a key role in generating the long-run response of real expense.

The paper is organized as follows. In section 2, we present the framework of a two-good model of a small open economy, facing given terms of trade and world interest rate. In section
3, we analyze the equilibrium dynamics and the steady-state of the model. Section 4 explores in detail the effects of a permanent worsening of the terms of trade. In section 5, a consistent solution method for analyzing temporary shocks is applied to an adverse transitory relative price perturbation. Consumption, stock of habits, investment, and current account dynamics are studied and the results are compared to the effects of a permanent terms of trade deterioration. Conclusions and a short outlook on further research are contained in section 6.\(^3\)

2 The Framework

Consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. We normalize, without loss of generality, the number of households to one. There are four types of goods. The representative firm is completely specialized in the production of a final good that can be consumed domestically or exported. This good can also be transformed, at some cost, in capital. The domestic good is an imperfect substitute for an imported good which can be used for consumption or investment. The country is small in world good and capital markets and faces given terms of trade (price of the domestic good in terms of the foreign good), \(p\), and world interest rate, \(r^*\).

2.1 Structure of the Economy

Households

At each instant the representative household consumes domestic goods and foreign goods denoted by \(d\) and \(f\). The measure of utility of consumption at \(t\), \(c(t)\), is given by the relation:

\[
c(t) = c(d(t), f(t)),
\]

where \(c(\cdot)\) is a positive, increasing, concave and linearly homogeneous aggregator function. The representative household maximizes the objective function

\[
U[C(0)] = \int_0^\infty u[c(d(t), f(t)), s(t)] \exp(-\delta t) \, dt,
\]

where \(\delta\) is the consumer’s discount rate, and \(s(t)\) a distributed lag on past real expenditure as (see Ryder and Heal [1973]),\(^4\)

\[
s(t) = \sigma \int_{-\infty}^t c(d(\tau), f(\tau)) \exp(-\sigma (t - \tau)) \, d\tau.
\]

From (3), the dynamic equation of the habit stock is given by

\[
\dot{s}(t) = \sigma [c(t) - s(t)].
\]
Following Ryder and Heal [1973], the instantaneous utility function is assumed to be: [H1] increasing in current real expenditure, \( u_1 > 0 \); [H2] non-increasing in past real expense, \( u_2 \leq 0 \); [H3] increasing in a uniformly maintained real expense level, \( u_1 (c, c) + u_2 (c, c) > 0 \), which guarantees non-satiation; [H4] strictly concave in \( c \) and \( s \), \( u_{11} (c, s) < 0 \), \( u_{22} < 0 \), and concave in \( (c, s) \), \( u_{11} (c, s) u_{22} (c, s) - [u_{12} (c, s)]^2 \geq 0 \); [H5] \( \lim_{c \to 0} u_1 (c, s) = \infty \) and \( \lim_{c \to 0} [u_1 (c, c) + u_2 (c, c)] = \infty \).

Since \( c(.) \) is homothetic, the household’s maximization problem can be decomposed into two stages (see Frenkel and Razin [1987], chapter 6). At the first stage, the household minimizes the cost, \( z_e(t) = p(t)d(t) + f(t) \), for a given level of subutility, \( c(t) \), where \( p(t) \) is the relative price of the domestic good. For any chosen \( c(t) \), the optimal basket \( (d(t), f(t)) \) is a solution to

\[
p_c (p(t)) c(t) = \min_{\{d(t), f(t)\}} \{p(t)d(t) + f(t) : c(d(t), f(t)) \geq c(t)\}. \tag{5}
\]

The assumption that the subutility function \( c(.) \) is linear homogeneous implies that the total expense in consumption goods can be expressed as \( z_e(t) = p_c (p(t)) c(t) \), with \( p_c (p(t)) \) is the unit cost function dual (or consumption-based price index) to \( c \). Intra-temporal allocations between domestic goods and imports follow from Sheppard’s Lemma (or the envelope theorem) applied to (5):

\[
d(t) = p'_c (p(t)) c(t), \quad f(t) = [p_c (p(t)) - p(t)p'_c (p(t))] c(t), \tag{6}
\]

with (see Deaton and Muelbauer [1980]),

\[
p_c (p) > 0, \quad p'_c (p) > 0, \quad p''_c (p) < 0. \tag{7}
\]

In the second stage, consumers choose their real expense, \( c \), and rates of accumulation of consumption “experience” and traded bonds to maximize (2) subject to (3) and the flow budget constraint,

\[
\dot{b}(t) = r^* b(t) + [D(t) + w(t)] - p_c (p(t)) c(t), \tag{8}
\]

and initial conditions \( b(0) = b_0 \). Households’ income consists of interest earnings, \( r^* b(t) \), dividend payments on equity holdings, \( D(t) \); moreover, households inelastically supply one unit of labor services and receive the wage, \( w(t) \), per unit of time. The real stock of foreign assets held by the household, \( b(t) \), is denominated in terms of the imported good since we assume that external borrowing and lending are measured in units of the foreign good.

**Firms**

Perfectly competitive firms produce output, \( Y \), from labor, \( l \), and capital, \( k \), by means of a constant returns to scale production function, which is assumed to have the usual neoclassical properties of positive and diminishing marginal products. Like Abel and Blanchard [1983], the
installation cost function $\psi (I(t)/k(t))$, is assumed to have the following properties:

$$\psi(0) = 0, \quad \psi'(.) > 0, \quad 2\psi''(.) + \frac{I}{k} \psi'''(.) > 0.$$  \hspace{1cm} (9)

Following Gavin [1992] and Servèn [1999], we assume that domestic and imported goods are converted in an investment good according to a linearly homogeneous technology:

$$J(t) = J(J_D(t), J_F(t)),$$  \hspace{1cm} (10)

where $J_D$ and $J_F$ denote domestic and foreign inputs combined into the investment process. Since $J(.)$ is homogeneous of degree one, the investment decision can be done in two stages like consumption decision. The solution to the atemporal investment allocation problem can be written in the form

$$p_I(p(t))J(t) = \min_{\{J_D(t), J_F(t)\}} \{p(t)J_D(t) + J_F(t) : J(J_D(t), J_F(t)) \geq J(t)\},$$  \hspace{1cm} (11)

where the exact investment price index is a function of terms of trade and has the following properties

$$p_I(p) > 0, \quad p_I'(p) > 0, \quad p_{II}''(p) < 0.$$  \hspace{1cm} (12)

From Sheppard’s lemma, we obtain investment demand for the domestic and imported goods:

$$J_D(t) = p_I'(p(t)) J(t), \quad J_F(t) = \left[p_I(p(t)) - p(t)p'_I(p(t))\right] J(t).$$  \hspace{1cm} (13)

In the second stage, the representative firm maximizes the present value of anticipated future cash flow:

$$\max_{\{I(t), l(t)\}} \int_0^\infty D(t) e^{-r^* t} dt = \max_{\{I(t), l(t)\}} \int_0^\infty \left\{ pF(k, l) - w l - p_I(p) I \left[1 + \psi\left(\frac{I}{k}\right)\right]\right\} e^{-r^* t} dt,$$  \hspace{1cm} (14a)

subject to

$$\dot{k}(t) = I(t),$$  \hspace{1cm} (14b)

and the initial condition

$$k(0) = k_0.$$  \hspace{1cm} (14c)

2.2 Macroeconomic Equilibrium

To obtain the macroeconomic equilibrium, we first derive the optimality conditions for households and firms and combine these with the accumulation equations. This leads to the set of equations\textsuperscript{6}
\[ u_1(c, s) + \sigma \xi = p_c(p) \lambda, \quad (15a) \]

\[ p F_I(k, 1) = w, \quad (15b) \]

\[ q = p_I(p) \left[ 1 + \psi \left( \frac{I}{k} \right) + \left( \frac{I}{k} \right) \psi' \left( \frac{I}{k} \right) \right], \quad (15c) \]

\[ \dot{\lambda} = 0, \quad \text{i.e.} \quad \lambda = \bar{\lambda}, \quad (15d) \]

\[ \dot{\xi} = (\delta + \sigma) \xi - u_2(c, s), \quad (15e) \]

\[ \dot{q} = r^* q - \left[ p F_k(k, 1) + p_I(p) \left( \frac{I}{k} \right)^2 \psi' \left( \frac{I}{k} \right) \right], \quad (15f) \]

\[ \dot{b} = r^* b + p F(k, 1) - p_c(p) c - p_I(p) I \left[ 1 + \psi \left( \frac{I}{k} \right) \right], \quad (15g) \]

and dynamic equations (4) and (14b), and the transversality conditions

\[ \lim_{t \to \infty} \bar{\lambda} b \exp (-r^* t) = \lim_{t \to \infty} \xi s \exp (-r^* t) = \lim_{t \to \infty} q k \exp (-r^* t) = 0, \quad (16) \]

where \( \lambda, \xi, q \) are the co-state variables associated with dynamic equations (8), (3), and (14b).

The solution of the differential equation (15e) using (16) is given by

\[ \xi(t) = \int_{t}^{\infty} u_2(c(\tau), s(\tau)) e^{-(\delta + \sigma)(\tau-t)} d\tau. \quad (17) \]

The shadow price of habit stock is equal to the present discounted value of marginal disutility of consumption “experience”, \( u_2 \leq 0 \), which depreciates at the rate \( \sigma \).

Solving (15f) forward and ruling out “bubble trajectories”, we obtain

\[ q(t) = \int_{t}^{\infty} \left\{ p F_k[k(\tau), 1] + p_I(p) \left( \frac{I}{k} \right)^2 \psi' \left( \frac{I}{k} \right) \right\} e^{-r^*(\tau-t)} d\tau. \quad (18) \]

According to (18), the shadow price of capital is equal to the present discounted value of the sum of the marginal product of capital and the reduction of the marginal cost induced by an increase in the capital stock for a given flow of investment, both expressed in the foreign good.

The first static efficiency condition (15a) requires that along an optimal path the sum of marginal current utility of real expense and its marginal contribution to the future felicity stream derived from a higher habitual standard of living is equal to the marginal utility of wealth in the form of internationally traded bonds measured in terms of the domestic good, \( p_c \lambda \). The second static efficiency condition (15b) establishes the usual equality between the marginal productivity of labor and the real wage. Equation (15c) equates the ratio of market price of installed capital to the replacement cost of capital, i.e. the Tobin’s \( q \), to the marginal cost investment.
With a constant rate of time preference and an exogenous world interest rate, we require that
\[ \delta = r^*, \]  
(19)
in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, \( \lambda \), must remain constant over time (see (15d)), and gives rise to the zero-root property (see Sen and Turnovsky [1990]).

Finally, the first transversality condition of (16) rules out the possibility of running up infinite debt or credit and ensures that the nation remains intertemporally solvent.

3 Equilibrium Dynamics and the Steady-State

Equilibrium Dynamics

The static efficiency condition (15c) implies that the rate of investment is a function of Tobin’s \( q \):
\[ \frac{I}{k} = \kappa \left( \frac{q}{p_I(p)} \right), \quad \kappa' (.) > 0, \quad \kappa (1) = 0. \]  
(20)

From (20), the rate of investment rises when the market price of capital is higher than investment replacement cost, that is to say the Tobin’s \( q \), denoted by \( \nu \), is greater than one.

Total differentiation of equation (15a), substitution of (4) and (15e), and elimination of \( \xi \) using (15a) lead to the following dynamic equation of real expense:
\[ \dot{c} = \frac{1}{u_{11}} \left[ (\delta + \sigma) (u_1 - p_c (p) \bar{\lambda}) + \sigma u_2 - u_{12} \sigma (c - s) \right]. \]  
(21)

Inserting the short run static solution (20) in (14b) and (15f), linearizing these with dynamic equations (3) and (21) around the steady-state, and denoting \( \bar{x} = \bar{s}, \bar{c}, \bar{k}, \bar{q}, \bar{p}, \bar{c} \) the long-term values of \( x = s, \xi, k, q, p, c \), we obtain in a matrix form
\[ \begin{pmatrix} \dot{s}, \dot{c}, \dot{k}, \dot{q} \end{pmatrix}^T = J \begin{pmatrix} s(t) - \bar{s}, c(t) - \bar{c}, k(t) - \bar{k}, q(t) - \bar{q} \end{pmatrix}^T, \]  
(22)

where \( J \) is given by
\[ J = \begin{pmatrix} -\sigma & \sigma & 0 & 0 \\ \frac{\delta + 2\sigma}{u_{11}} u_{12} + \frac{\sigma}{\pi + 2\sigma} u_{22} & \delta + \sigma & 0 & 0 \\ 0 & 0 & 0 & \kappa' (1) \frac{k}{p_I(p)} \\ 0 & 0 & -pF_{kk} & r^* \end{pmatrix}. \]  
(23)

The Fisherian separation theorem implies that the matrix is block recursive. As we will see below, the number of predetermined variables equals the number of negative eigenvalues and
the number of jump variables equals the number of positive eigenvalues, so there exists a unit convergent path towards the steady-state.

The characteristic roots obtained from $J_{11}$ write as follows:

$$
\mu_i \equiv \frac{1}{2} \left\{ \delta \pm \sqrt{(\delta + 2\sigma)^2 + \frac{4\sigma (\delta + 2\sigma)}{u_{11}}} \right\} \geq 0, \quad i = 1, 2, \quad (24)
$$

where we let

$$
\Gamma = u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \leq 0. \quad (25)
$$

The sign of $\Gamma$ depends on $u_{12}$. If the marginal utility of real expense is sufficiently increasing in stock of habits, the preferences of the representative consumer display “adjacent complementarity” and $\Gamma$ is positive (see Ryder and Heal [1973]). If $u_{12}$ has a negative or a low positive value, $\Gamma$ is negative and preferences are said to display “distant complementarity”.

We denote respectively $\mu_1 < 0$ and $\mu_2 > 0$ the stable and unstable real eigenvalues satisfying

$$
\mu_1 < 0 < r^* < \mu_2, \quad (26)
$$

under the condition (see Obstfeld [1992])

$$
u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} < - \left( \frac{\delta + \sigma}{\delta + 2\sigma} \right) u_{11}, \quad (27)$$

so that the long-run equilibrium is a saddle-point in $(s, c)$-space. While the habit stock evolves always gradually, the real expense, $c$, can jump instantaneously in response to a new information.

Starting from an initial “consumption experience” $s(0) = s_0$, the stable dynamic time paths followed by $s$ and $c$ are given by

$$
\begin{align*}
    s(t) & = \bar{s} + A_1 e^{\mu_1 t}, \\
    c(t) & = \bar{c} + \left( \frac{\sigma + \mu_1}{\sigma} \right) A_1 e^{\mu_1 t},
\end{align*} \quad (28a, b)
$$

where $A_1 \equiv s_0 - \bar{s}$. When the stock of habits is expected to be higher, real expense and habitual standard of living co-vary in the same or in an opposite direction according to whether $(\sigma + \mu_1)$ is positive (adjacent complementarity, $\Gamma > 0$) or negative (distant complementarity, $\Gamma < 0$), i.e. according to whether the marginal current utility of real expense is sufficiently strongly increasing or decreasing (or weakly increasing) in future “consumption experience” with respect to the increase of marginal desutility of habits.

The two real characteristic roots obtained from $J_{22}$ write as follows:

$$
\chi_i \equiv \frac{1}{2} \left\{ r^* - \sqrt{(r^*)^2 - \frac{4\kappa'(1) \bar{p} F_{kk}}{p_{I}(\bar{p})}} \right\} \geq 0, \quad i = 1, 2. \quad (29)
$$
The stable and unstable eigenvalues satisfy

\[ \chi_1 < 0 < r^* < \chi_2. \]

(30)

Hence the dynamics describe a saddle-point in \((k, q)\)-space. The stable dynamic time paths followed by \(k\) and \(q\) are

\[ k(t) = \bar{k} + B_1 e^{\chi_1 t}, \]

(31a)

\[ q(t) = \bar{q} + \left( \frac{p_I \chi_1}{k'(1) k} \right) B_1 e^{\chi_1 t}, \]

(31b)

where \(B_1 \equiv k_0 - \bar{k}\). The solution (31b) indicates that the stable branch is downward-sloping.

Substituting the short-run solution (20), linearizing the dynamic equation of the foreign asset stock (15g) in the neighborhood of the steady-state, using the fact that \(\bar{p}_F k(\bar{k}, 1) = r^* p_I(\bar{p})\) at the steady-state, substituting the general solutions of \(c, k,\) and \(q,\) and finally invoking the transversality condition, one obtain the linearized version of the nation’s intertemporal budget constraint

\[ (b_0 - \bar{b}) = -p_I (k_0 - \bar{k}) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_0 - \bar{s}), \]

(32)

where the initial stocks, \(k_0, b_0, s_0\) are given. The stable solution for \(b(t)\) consistent with long-run solvency writes as follows

\[ (b(t) - \bar{b}) = -p_I (k(t) - \bar{k}) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s(t) - \bar{s}). \]

(33)

This equation describes the relationship between the stock of internationally traded bonds, the stock of physical capital, and the stock of habits along a stable path. Time derivatives of solutions (31a) and (33) and elimination of \(A_1 e^{\mu_1 t}\) and \(B_1 e^{\chi_1 t}\) allow to express the change of capital and foreign assets stocks in term of deviations from steady state as

\[ \dot{k}(t) = \chi_1 \left( k(t) - \bar{k} \right), \]

(34a)

\[ \dot{b}(t) = \mu_1 \left( b(t) - \bar{b} \right) + p_I (\mu_1 - \chi_1) \left( k(t) - \bar{k} \right), \]

(34b)

where the multiplier \((\mu_1 - \chi_1)\) can be positive or negative according to whether the speed of adjustment of consumption habits, \(|\mu_1|\), is lower or greater than this of capital stock, \(|\chi_1|\). The new differential equation system has two stable roots \(\mu_1 < 0\) and \(\chi_1 < 0\); hence the steady-state is a stable node in \((k, b)\)-space (see figure 2).

\textit{Steady-State}

The steady-state of the economy is obtained by setting \(\dot{c}, \dot{s}, \dot{k}, \dot{q}, \dot{b} = 0\) and is defined by the
following set of equations:

\[ u_1(\bar{c}, \bar{s}) + \frac{\sigma}{\delta + \sigma} u_2(\bar{c}, \bar{s}) = p_c(\bar{p}) \bar{\lambda}, \quad (35a) \]

\[ \bar{c} = \bar{s}, \quad (35b) \]

\[ \bar{q} = p_I(\bar{p}), \quad (35c) \]

\[ r^* p_I(\bar{p}) = \bar{p} F_k(\bar{k}, 1), \quad (35d) \]

\[ r^* \bar{b} + \bar{p} F(\bar{k}, 1) - p_c(\bar{p}) \bar{c} = 0, \quad (35e) \]

and the intertemporal solvency condition

\[ (b_0 - \bar{b}) = -p_I(k_0 - \bar{k}) - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} (s_0 - \bar{s}). \quad (35f) \]

From (35a), the marginal utility of consumption along a constant path, i.e. the sum of marginal current utility of real expense \( u_1 \) and the capitalized value of marginal utility of “consumption experience” \( u_2 / (\delta + \sigma) \) adjusted by the parameter \( \sigma \) is equal to the marginal utility of wealth in terms of the domestic good, \( p_c \bar{\lambda} \). The equation (35b) requires that the real expense is equal to the habit stock at the steady-state. Equation (35c) asserts that long-run investment is zero when the market price of installed capital is equal to its replacement cost, i.e. when the Tobin’s \( q \) is equal to one. Equation (35d) indicates that in the long-run the marginal product of capital, \( \bar{p} F_k \), is equal to its user cost, \( r^* p_I \), both expressed in terms of the foreign good. Equation (35e) implies that in the steady-state equilibrium, the current account must be zero, that is, the gross national product, \( r^* \bar{b} + \bar{p} Y \), must be equal to the total expenditure in consumption goods, \( p_c(\bar{p}) \bar{c} \). Finally, the linearized version of the nation’s intertemporal budget constraint (35f) implies that the steady-state depends on the initial stocks \( k_0, b_0, \) and \( s_0 \). This dependency upon initial conditions comes from the assumptions of infinitely lived maximizing agents having a constant rate of discount and facing perfect capital markets and leads to hysteresis effects, that is, temporary terms of trade disturbances have permanent effects (see Sen and Turnovsky [1989], [1990]).

System (35) may be solved for the steady-state values by applying the two-step solution method described by Schubert [2002] and Schubert and Turnovsky [2002]. We first solve equations (35a) to (35e) as functions of marginal utility of wealth expressed in terms of the foreign good, \( \bar{\lambda} \), and of the terms of trade, \( p \). This yields to the following functions

\[ \bar{s} = \bar{c} = t(\bar{\lambda}, p), \quad t_\lambda < 0, \quad t_p < 0, \quad (36a) \]

\[ \bar{k} = u(p), \quad u_p > 0, \quad (36b) \]

\[ \bar{q} = p_I(p), \quad p'_I > 0, \quad (36c) \]

\[ \bar{b} = v(\bar{\lambda}, p), \quad v_\lambda < 0, \quad v_p < 0. \quad (36d) \]

In the second step, we insert these functions into the economy’s intertemporal budget constraint
(eq (35f)), which may be solved for the equilibrium value of the marginal utility of wealth:

\[ \bar{\lambda} = g(s_0, k_0, b_0, p), \quad \lambda_s \geq 0, \quad \lambda_k < 0, \quad \lambda_b < 0, \quad \lambda_p < 0. \] (37)

Substituting then \( \bar{\lambda} \) into the other steady-state functions (36) gives the conventional steady-state values of the economy as functions of the terms of trade and the initial conditions, \( k_0, b_0 \) and \( s_0 \) (except for production-side variables, \( k \) and \( q \)).

4 A permanent Deterioration of Terms of Trade

We investigate the effects of a permanent decrease in the relative price of the domestic good (denoted by the subscript \( \text{perm} \)), \( p \), from \( p_0 \) to \( p_1 \), which occurs at time \( t = 0 \), where the economy is originally in steady state. Like Obstfeld [1983] and Sen and Turnovsky [1989], we assume that the small open economy is a net exporter of the domestic good at the steady-state, that is \( (\bar{Y} - \bar{d}) > 0 \). All agents perfectly understand the permanence of the terms of trade deterioration, but its occurrence at time \( t = 0 \) is unanticipated. Because of perfect foresight assumption, the transitional dynamics are affected by the expected long-run state of the economy.

Steady-State Effects of an Unanticipated Permanent Terms of Trade Deterioration

The long-run effects after a permanent change in the relative price of the home good, consistent with long-run solvency, are obtained from the total differential of the equilibrium system (35):

\[
\begin{align*}
\frac{d\bar{c}}{dp}\bigg|_{\text{perm}} &= \frac{d\bar{s}}{dp}\bigg|_{\text{perm}} = \frac{\sigma (\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} (\bar{Y} - \bar{d}) > 0, \\
\frac{d\bar{k}}{dp}\bigg|_{\text{perm}} &= -\frac{F_k (1 - \alpha I)}{pF_{kk}} > 0, \quad \frac{d\bar{q}}{dp}\bigg|_{\text{perm}} = p'_I (p) > 0, \\
\frac{d\bar{\lambda}}{dp}\bigg|_{\text{perm}} &= \frac{\sigma (\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} \left[ u_{11} + \frac{\delta + 2\sigma}{\delta + \sigma} \right] - \frac{\alpha c_{\bar{\lambda}}}{p} < 0, \\
\frac{d\bar{b}}{dp}\bigg|_{\text{perm}} &= -\frac{(\sigma + \mu_1)(\bar{Y} - \bar{d})}{\mu_1 (\sigma + r^*)} + \frac{p_t F_k (1 - \alpha I)}{pF_{kk}} \leq 0, \\
\frac{d\bar{x}}{dp}\bigg|_{\text{perm}} &= -r \frac{d\bar{b}}{dp}\bigg|_{\text{perm}} = (\bar{Y} - \bar{d}) + pF_k \frac{d\bar{k}}{dp}\bigg|_{\text{perm}} - p_c \frac{d\bar{c}}{dp}\bigg|_{\text{perm}},
\end{align*}
\]

where the net exports expressed in terms of the foreign good are denoted by \(nx\); they are defined as the difference between the domestic output and absorption, the latter being equal to the sum of consumption and investment expenditure, i.e. \(nx(t) = pF [k(t), 1] - p_c c(t) - p_I J(t)\).

A permanent decrease in \( p \) reduces the long-term values of real expense, \( \bar{c} \), and of habits, \( \bar{s} \), of the same amount (see (38a)). These changes are proportional to the loss\(^{11}\) in the purchasing
power of exports in terms of consumption goods, \( \left[ (Y - d) / p_c \right] dp < 0 \). The difference with the conventional time separable utility function is that the modification of \( c \) in the long-run is greater (in absolute value) the higher is habit persistence in consumption, i.e. the lower is \( |\mu_1| \). This can be explained in an intuitive way; a drop in \( p \) implies a fall in real income which incites households to decrease their real expenditure. When preferences display adjacent complementarity, i.e. habit persistence in consumption is strong, consumers reduce initially \( c \) but less than the drop in real income which implies a decumulation of financial wealth and hence amplifies the long-term effects of the negative income shock on consumption (compared to standard preferences). This process can be unstable if condition (27) is removed (see Becker and Murphy [1988]).

From expression (38c), the permanent decrease in \( p \) has two influences which work in the same direction on marginal utility of wealth expressed in terms of the foreign good. First, a fall in \( p \) lowers domestic real income which induces a drop in consumption. This income effect is proportional to \( (Y - d) \) (see the first term on the right hand side of eq (38c)). Second, a negative change in \( p \) favors the consumption of the domestically produced good, \( d \), and incites households to reduce the level of their consumption in the foreign good. This substitution effect is proportional to \( \alpha_c \), the share of domestic goods in consumption expenditure (see the second term on the right hand side of eq (38c)). These two adjustments lead to an increase in marginal utility of wealth expressed in terms of the foreign good.

When the expenditure in capital goods has an import content, a decrease in \( p \) leads to a drop in the marginal product of capital expressed in the foreign good greater in absolute value than the reduction of the user cost of capital. Adjustment in the long-run calls for a lower capital stock, \( \bar{k} \). At the new steady-state, the shadow price of installed capital is definitely reduced to the level \( \bar{q}_1 = p_I (p_1) \) and Tobin’s \( q \) is equal to unity.

The change in the foreign assets stock, \( \bar{b} \), is the result of two forces. First, in response to an adverse permanent terms of trade perturbation, financial wealth increases or decreases according to whether households’ preferences display distant \( ((\sigma + \mu_1)^{\text{dist}} < 0) \) or adjacent complementarity \( ((\sigma + \mu_1)^{\text{adj}} > 0) \), i.e.

\[
\frac{d\bar{a}}{dp}\bigg|_{\text{perm}} = -\frac{p_c (\sigma + \mu_1) d\bar{s}}{\sigma (\mu_1 - r^*)} \bigg|_{\text{perm}} \leq 0 \quad \text{according to} \quad (\sigma + \mu_1) \leq 0,
\]

where the non-human wealth, \( a = b + qk \), is the sum of the value of internationally traded bonds and the value of domestic equities, both expressed in terms of the foreign good. Second, a drop in the relative price of domestic goods discourages investment which affects positively the stock of foreign bonds. Finally, we assume that under adjacent complementarity, the savings flow dominates the investment flow, i.e the long-run value of net foreign assets decreases following a permanent terms of trade deterioration. The H-L-M effect according to which a terms of trade worsening leads to a current account deficit depends on this assumption that can be expressed
in a more formal way:\textsuperscript{12} 
\[ \frac{\text{adj}}{\partial \bar{d}_b}{p_{\text{perm}}^p} = -p_1 \frac{\text{adj}}{\partial \bar{d}_p}{p_{\text{perm}}^p} - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} \frac{\text{adj}}{\partial \bar{s}}{p_{\text{perm}}^p} > 0, \tag{39} \]

where the notation \text{adj} means that we refer to the adjacent complementarity case. At the new steady-state, the current account must be zero, \( \bar{c}a = 0 \); the trade balance must mirror the balance of net interest earnings, i.e. \( \bar{n}x = -r^*\bar{b} \). Under adjacent complementarity, the economy must raise net exports to compensate the losses in interest earnings, i.e. \( \bar{n}x_1 = -r^*\bar{b}_1 > \bar{n}x_0 = -r^*\bar{b}_0 \). This can be achieved through the decline in the real expense by an amount greater than the fall in the real income due to a lower relative price (deflator effect) and a smaller capital stock (see (38e)).

\textit{Transitional Dynamics}

We assume that the economy is initially in an “old” steady-state. Due to the zero-root property, the marginal utility of wealth jumps instantaneously at time \( t = 0 \) to its new steady-state value, \( \bar{\lambda} \), and remains constant from thereon. Whereas \( s_0 \) is predetermined, real expense is free to jump at time \( t = 0 \) to situate the economy on the stable branch:

\[ \frac{\text{adj}}{\partial \bar{c}(0)}{p_{\text{perm}}^p} = \left[ 1 - \frac{\sigma + \mu_1}{\sigma} \right] = -\frac{\mu_1}{\sigma} \frac{\text{adj}}{\partial c}{p_{\text{perm}}^p} \gtrsim \frac{\text{adj}}{\partial c}{p_{\text{perm}}^p} \quad \text{according to} \quad (\sigma + \mu_1) \leq 0, \tag{40} \]

where we differentiated the stable solution (28) evaluated at time \( t = 0 \) with respect to \( p \). With conventional time separable preferences, a permanent deterioration of terms of trade leads to a decline in real expense equal to the drop in real income. The economy jumps immediately to the new steady-state and saving is not affected (see Obstfeld [1983], p. 139). When preferences of habit-forming consumers display adjacent complementarity, a negative income shock calls for a (decreasing) gradual response of \( c \). This sluggish consumption adjustment follows from an initial jump of real expense less important than its decline in the long-run (see eq (40)). Agents choose \( c(0) \) by assigning a positive weight to the initial stock of habits, \( s_0 \), and a weight less than one to the annuity value of total wealth deflated by the consumption price index, \( r^*[b_0 + W(0)]/p_c \), i.e. the permanent income. Therefore, households will reduce initially their consumption but less than the drop in real income.

When the representative household tries to maintain his habits, he decumulates financial wealth after the permanent deterioration of terms of trade. The economic intuition behind this result rests on the degree of habit persistence in consumption determined by \( \mu_1 \) which in turn depends on the value of \( u_{12} \). With adjacent complementarity (\( \Gamma > 0 \)), a consumption experience decrease in the long-run implies an initial rise in the rate of time preference, i.e.:

\[ \frac{\text{adj}}{\partial \rho(0)}{p_{\text{perm}}^p} = -\frac{u_{11} (\delta + \sigma)}{p_c \lambda} \left[ \left( \frac{\sigma + \mu_1}{\sigma} \right)^{\text{adj}} + \frac{\delta + 2\sigma}{u_{11} (\delta + \sigma)} \Gamma \right] \frac{\text{adj}}{\partial \bar{s}}{p_{\text{perm}}^p} < 0. \tag{41} \]
where the expression in square brackets is negative. In words, the high positive value of the cross-partial derivative of the felicity function, \( u_{12} \), implies that the marginal utility of current real expense is greater than the marginal utility of future real expense, so that consumption is always biased toward the present. Figure 1 depicts the adjustments of \( c \) and \( \rho \) following an unanticipated permanent deterioration of terms of trade. When intertemporal preferences display adjacent complementarity, the real expense and the stock of habits monotonically decrease toward their new long-run values, \( \bar{c}_1 = \bar{s}_1 \). In the same time, the rate of time preference falls toward the fixed discount rate, \( \delta \), as \( s \) converges to its new steady-state value.

Whenever investment expense has an import content, a fall in the home goods’ relative price leads to a greater decline in the marginal product of capital than in its real user cost. With \( k_0 \) being predetermined, the market price of capital jumps instantaneously from \( \bar{q}_0 \) down to \( q(0) \). The depressing effect on investment from the decrease in \( q \) is greater the larger is the gap between the short-run decline of the market price of capital, \( q(0) \), and the new investment price index, \( p_I (p_I) = \bar{q}_1 \), i.e the higher is the import content of capital goods.\(^{13}\) Figure 3 depicts the decumulation of capital stock, \( \dot{k}(t) < 0 \), which slows down, \( \ddot{k}(t) > 0 \), as economy approaches the steady-state. The decline of \( k \) raises its marginal productivity over time, and therefore its market price at a decreasing rate. Whenever one abstract from capital good imports, the marginal product of capital shifts by the same amount than the user cost and the domestic capital stock remains unaffected. If investment has an import content as we may expect in a small open economy, an unanticipated permanent terms of trade deterioration induces a fall in

Figure 1: The real expense and the rate of time preference dynamics after an unanticipated permanent terms of trade worsening
Figure 2: An unanticipated permanent terms of trade worsening and net foreign assets adjustment in the \((k, b)\)-space: the J-Curve - \(\mu_1 < \chi_1\) - Adjacent complementarity.

the long-run value of the equipment goods’ stock.

Having discussed the dynamic transition of real expense and investment toward the new long-run equilibrium of the economy, we turn now to the current account and net exports adjustment following a worsening of the terms of trade. The transitional dynamics can be best described by the use of phase portraits for the stable manifold. As the stable manifold for the stock of foreign assets is two-dimensional, its speed of convergence is a weighted average of the speeds of adjustment of saving and investment. The flexibility provided by the additional eigenvalue allows the system to match the non-monotonic convergence of current account featured by the data. Because we are interested in providing a new explanation of the J-curve phenomenon, we restrict the study by assuming that [i] the representative household’s preferences exhibit adjacent complementarity \(((\sigma + \mu_1)^{adj} > 0)\), [ii] the adjustment speed \(|\mu_1|\) of habits is higher than the adjustment speed \(|\chi_1|\) of capital, and [iii] inequality (39), according to which a permanent terms of trade worsening reduces the stock of foreign assets in the long-run, holds. Transitional dynamics can be disentangled in two phases by noting that there exists a date \(t = \tilde{t}\) (with \(ca(\tilde{t}) = 0\)) such that the stock of foreign assets overshoots along the stable trajectory (see figure 2). To summarize, we find

\[
\begin{align*}
ca(t) &= \dot{b}(t) \leq 0, \quad \text{for } t \leq \tilde{t}, \\
ca(t) &= \dot{b}(t) > 0, \quad \text{for } t > \tilde{t}.
\end{align*}
\]
Figure 3: An unanticipated permanent terms of trade worsening: saving and investment dynamics

An unanticipated permanent worsening of terms of trade leads to an immediate decumulation of foreign bonds and a deterioration in the trade balance, the initial level of $b$ being predetermined.\(^{15}\)

\[
\dot{b}(0) = ca(0) = \left[ p_I \chi_1 \frac{d\hat{k}}{dp}_{perm} + \mu_1 \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} \frac{d\hat{s}}{dp}_{perm} \right] dp = - \left( \frac{p_c}{\mu_1 - r^*} \right) \hat{c}(0) - p_I \hat{k}(0) < 0,
\]

where we differentiated (33) with respect to time, then evaluated the expression at time $t = 0$, and assumed a permanent fall in $p$. The initial current account deficit is a consequence of the fact that the initial negative flow of saving, $\dot{a}(0) = S(0) < 0$, more than outweighs the influence of the initial negative flow of investment which gives rise to the H-L-M effect (see figure 3). The assumption of an habit-forming behavior implies a decline in real expense less than proportional to the reduction in permanent income, i.e. an “excess smoothness” of consumption, and therefore a fall in financial wealth. At the same time, high installation costs imply a slow adjustment speed of capital stock, and then a small initial decumulation of equipment goods. The greater is the habit persistence in consumption, the higher are the installation costs and the lower is the import content of investment, the strongest is the negative initial response of saving and the smallest is the positive impact of investment on current account.

During the first phase $0 < t \leq \tilde{t}$, consumption experience and physical capital decrease
smoothly (see figure 2). Since the representative household reduces its financial wealth faster than the representative firm decumulates its capital stock, the non-human wealth $a$ approaches its steady-state faster than $k$. The fact that habit persistence effects dominate investment discouraging effects until date $\tilde{t}$ gives rise to an overshooting of net foreign assets’ adjustment, i.e. $b(\tilde{t}) < \tilde{b}_1 < \tilde{b}_0$.

At some point of time, $t = \tilde{t}$, the trajectory cuts the demarcation line $\dot{b}(t) = 0$ (see figure 2). From point $B$, the deterioration in the net foreign asset position is then followed by a current account surplus. Over the period $t > \tilde{t}$, investment influence becomes dominant over the habit effects. The rate at which saving falls turns to be lower than the rate at which investment decreases. This exercises a rise in the stock of foreign assets which converges in direction to its new steady-state level $\tilde{b}_1$ at point $E_1$. Turning to the trade balance adjustment, the effect of a decline in capital goods on the net exports operates through three channels: [i] the reduction of real output, [ii] the deceleration of the negative investment flow as $k$ approaches its new steady-state value, and [iii] the rise in marginal installation costs for a given investment flow. During the second phase, net exports decline over time but remain above their steady-state level.

Finally, at the new steady-state, investment has ceased, capital and hence output have fallen. Levels of habits and real expense are lower than at the initial steady-state. As decumulation of foreign bonds out weighs their accumulation during the second phase, $\dot{b}$ is definitively reduced. Since current account must be balanced in the long-run, the losses in interest earnings from abroad must be compensated by an improvement of the trade balance. Short-term and long-term effects in the present formal setup following an unanticipated terms of trade worsening may provide some explanations of recent empirical results. First, in accordance with estimations of Leonard and Stockman [2002], we find that the second phase of current account adjustment is associated with a fall in the real income, $p_1 \dot{Y}(t) = p_1 F_1 k(t) < 0$, which results from the decline in both the home goods’ relative price and the stock of physical capital. Second, investment and saving co-vary along the stable adjustment path, i.e. $\dot{a}(t) < 0$ and $\dot{k}(t) < 0$, as we depict on figure 3 (see Tesar [1991], Mendoza [1995]). Third, investment and real expense are procyclical, $\dot{k}(t) < 0$ and $\dot{c}(t) < 0$, and positively correlated with the terms of trade (see Mendoza [1995]).

5 An Unanticipated Temporary Deterioration of Terms of Trade

The study of an unanticipated transitory terms of trade perturbation in the present formal setup allows to provide some new insights of the real expense and net foreign asset position transitional dynamics by introducing an habit-forming behavior as we shall see below. In addition, the analytical techniques developed in this paper differ considerably from previous studies in that they allow to determine formal solutions for investment and consumption blocks, and then for current account, without retaining functional forms. Finally, it permits to emphasize
the relevant factors for investment and consumption decisions, and therefore current account dynamics, following an unanticipated temporary terms of trade worsening in an intertemporal general equilibrium model.

We suppose that terms of trade decrease unexpectedly at time $t = 0$ from the original level $p_0$ to level $p_1$ over the period $0 \leq t < T$, and they revert back at time $T$ permanently to their initial level $p_T = p_2 = p_0$. The temporary nature of the perturbation needs to consider two periods, say period 1 ($0 \leq t < T$) and period 2 ($t \geq T$). We have retained the two-step analytical procedure proposed by Schubert and Turnovsky [2002] and impose one single intertemporal solvency constraint which links the period 1 with period 2 through the new initial conditions. During the transition period 1, the economy accumulates capital, foreign assets, and habit stocks. Since this period is by its very nature unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy’s intertemporal budget constraint. At the same time, the zero-root property requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hit the economy. So $\lambda$ remains constant over the periods 1 and 2. The aim of the two-step method is to calculate the deviation of $\lambda$ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions $k_T$, $b_T$, and $s_T$ prevailing when the shock ends and accumulated over the unstable period. The consistency of two-step procedure rests on its capacity to link the agents’ decisions over the two phases and calculate the initial jump of the marginal utility of wealth such that the single national intertemporal budget constraint holds.

5.1 Production-Side Dynamics

Although we adopted a formal setup for the investment-side closely related to the framework specified by Gavin [1992] and Servèn [1999] and our results are qualitatively the same, the solution method we apply to study temporary terms of trade disturbances effects yield to some new formal solutions for temporary shocks which allow for an analytical comparison with formal solutions obtained with permanent shocks and for a rigorous study of transitional dynamics.\textsuperscript{17}

Since steady-state values (36b) and (36c) of capital stock and its market price are only function of terms of trade and do not depend on marginal utility of wealth, a transitory home goods’ relative price disturbance has not permanent effects on production-side aggregates. Therefore, $k$ and $q$ revert back to their initial levels when the shock ends. The dynamic evolutions of $k$ and $q$ when the terms of trade disturbance is active are governed by a couple of equations which contain an explosive part, say terms with $e^{\mu_2 t}$; that is why we say that period 1 is unstable. After an unanticipated temporary drop of $p$, the market price of equipment goods jumps instantaneously
in the same direction as after a definitive reduction of \( p \) but is softened by a scale down factor:

\[
\frac{dq(0)}{dp} \bigg|_{\text{temp}} = (1 - e^{-\chi_2 T}) \frac{dq(0)}{dp} \bigg|_{\text{perm}} > 0.
\] (44)

The fall is moderated by the factor \( 0 < (1 - e^{-\chi_2 T}) < 1 \) which is an increasing function of the length \( T \) of the shock and a decreasing function of capital installation costs. As the Tobin’s \( q, \nu \), is the ratio between the capital market price and the investment price index, investment flow may be positive or negative depending on the strength of the initial fall of \( q \). From the solutions prevailing in the case of a temporary terms of trade shock, it can be shown that the market price of capital may jump under or above the transitory lower level of the investment price index, \( p_I(p_1) = \bar{q}_1 \):

\[
q(0) - \bar{q}_1 = \frac{p_I}{p} \left[ \frac{r^* (1 - \alpha_I)}{\chi_2} \frac{1}{(1 - e^{-\chi_2 T})} \begin{cases} \text{capital profitability effect} & \text{capital profitability effect} \\ \text{intertemp. spec. effect} & \text{intertemp. spec. effect} \end{cases} \begin{cases} (+) & (+) \\ (-) & (-) \end{cases} \right] dp \leq 0.
\] (45)

On the one hand, a drop in the home goods’ relative price discourages investment by lowering capital marginal productivity below its user cost, as long as investment expenditure has an import content. Since the negative shock is transitory, \( q \) declines less compared to its reaction to a negative permanent shock: this is the transitory capital profitability effect. On the other hand, as the relevant real rate of interest is relatively low while firms expect a rise in home goods’ relative price, \( r^I = r^* - \alpha_I \dot{p}/\dot{p} < r^* \), there is an incentive to raise investment because the price of future real investment in terms of present real investment is transitorily higher. In words, firms perfectly understand the temporary nature of the perturbation and perfectly expect a terms of trade improvement at time \( T \) which is reflected by a transitory lower real user cost of capital. This is the intertemporal speculation effect.

From (45), a transitory unfavorable terms of trade movement, \( dp = p_1 - p_0 < 0 \), implies an initial jump of \( q \) below or above its period 1’s steady-state value, i.e. \( \nu(0) < 1 \) or \( \nu(0) > 1 \), according to whether the first term or the second term in square brackets predominates. The former represents the capital profitability effect which is an increasing function of \([1]\) the share of imported capital goods in investment expenditure, \( 1 - \alpha_I \), and \([2]\) the shock’s length, \( T \). The latter represents the intertemporal speculation effect which works in an opposite direction on investment. The shorter-lasting is the shock, the less the present value of future marginal product is affected and therefore the smaller is the fall in the market price of installed capital; the higher is the domestic content in investment expenditure, \( \alpha_I \), the greater is the fall in the investment-based real interest rate \( r^I \) below the world interest rate \( r^* \). This last effect encourages the domestic firms to benefit from the transitorily low level of the investment price index.

There exists a length of the relative price disturbance \( \hat{T} \) such that the initial value of the Tobin’s \( q, \nu (0) \), is equal to unity. Whenever the negative terms of trade perturbation lasts only
a short time period and/or the share of home goods in investment expenditure is important, the *intertemporal speculation* effect outweighs the *capital profitability* effect. This implies that the second term of the expression (45) dominates the first term. Investment dynamics depicted on figure 5 indicate that the capital stock adjustment over time exhibits a non-monotonic behavior in transition to the steady-state equilibrium. The moderated initial jump of the capital market price implies a Tobin’s *q* higher than unity and a positive flow of investment, *I*(0) > 0. Over the first phase, 0 < t < *T*, the capital increases, *k*(t) > 0, at a growing rate, *k*(t) > 0, that is the positive value of trajectory’s slope in the (*k*, *q*)-space becomes higher. Although the economy moves along an unstable and explosive path, all domestic firms perfectly anticipate that the negative terms of trade worsening will end in the near future and that the marginal product will rise at time *T*. Because the marginal productivity of capital is transitory low and the rise of the stock of capital goods intensifies it, the no-arbitrage condition calls for increases in the market price of capital (" (46)"). Since *k* rises at a growing rate, the successive increases of *q* must be higher (i.e., *q* becomes bigger). At time *T*, the temporary terms of trade disturbance ends and the economy must be situated on the stable path. Now, the market price of installed capital is below unity since *q*(*T*+) < *p*∧(*p*2) = *q*2. Over period *t* ≥ *T*, the investment flow is negative as the capital stock must be restored to its initial level, *k*2 = *k*0.

### 5.2 Consumption-Side Dynamics

We describe now the short-term and long-term effects of an unanticipated temporary terms of trade worsening on the real expense and the stock of habits. In a two-good model with conventional time separable preferences or recursive preferences, an unanticipated temporary terms of trade worsening leads to a fall in the marginal utility of wealth which is a scaled-down factor, of the response to a permanent shock, say d*λ* = *λ* *dp* (1 – *e*−*r*∗*T*) *dp*.19 This result is not longer obtained in a two-good model whenever consumers have habits as we can see from the equation below:

\[
\frac{d\lambda}{dp}\bigg|_{\text{temp}} = \left(1 - e^{-r^*T}\right) \frac{d\lambda}{dp}\bigg|_{\text{perm}} - \frac{p_1}{p_c} \frac{\lambda r^*}{\mu_1} \left(e^{-r^*T} - e^{-\mu_2T}\right) \leq 0
\]

(46)

where \( p_1/p_c \equiv \alpha_c/p \), and \( e^{-r^*T} - e^{-\mu_2T} \) is positive because \( \mu_2 > r^* \). From expression (46), an adverse transitory relative price disturbance exerts on \( \lambda \) two possibly offsetting effects. The *wealth* effect comes from the fall of the real income which reduces the present value of wealth. The first term on the right-hand side of (46) indicates that the change of the marginal utility of wealth equals the change after a permanent terms of trade shock, \( \lambda_p \), scaled-down by the term \( 0 < (1 - e^{-r^*T}) < 1 \). The *intertemporal speculation* effect coupled with an *inertia* effect work in an opposite direction of the *wealth* effect. This phenomenon reflected by the second term, i.e., \( -\frac{\alpha \lambda p_1}{\bar{p}} \frac{r^*}{\mu_1} \left(e^{-r^*T} - e^{-\mu_2T}\right) > 0 \), may outweigh the *wealth* effect if the shock’s persistence is not too high (i.e., a low value of \( T \)) and/or the share of domestic goods in consumption expenditure
Figure 4: An unanticipated temporary terms of trade worsening and real expense dynamics: the wealth, smoothing, intertemporal speculation, inertia, and hysteresis effects

is important (i.e. a high value of $\alpha_c$) and/or the habit persistence in consumption is strong (i.e. a low value of $|\mu_1|$).

These effects can be best commented from the investigation of the expression of real expense’s initial response given by:20

$$
\left. \frac{dc(0)}{dp} \right|_{\text{temp}} = -\frac{\mu_1}{\sigma} \frac{dc}{dp} \left|_{\text{perm}} \right(1 - e^{-r^*T}) + \frac{\mu_2}{\sigma} \left( e^{-r^*T} - e^{-\mu_2 T} \right) \geq 0, \quad (47)
$$

The first term on the right-hand side of (47) captures the wealth effect dampened by the smoothing effect. The latter is greater the shorter is the shock’s length reflected by the parameter $T$, that is the higher is the deviation of current income from permanent income. Following an adverse terms of trade perturbation, the fall of $c$ is moderated by the factor $(1 - e^{-r^*T})$ compared to a permanent decrease of $p$. In words, since agents know that the decrease in $p$ is only temporary, the present value of the necessary reduction in real expense to satisfy the intertemporal budget constraint is less than for an equal but permanent decline in $p$. The second term on the right-hand side of (47) reflects the intertemporal speculation effect coupled with the inertia effect which work against the wealth effect. Intuitively, since agents know that the decline in the relative price $p$ and hence the advantage of a lower cost of consumption goods last only temporarily, they wish to benefit from it by consuming at higher rates.21 Under the conditions of a low value of $T$, a high share of domestic goods $\alpha_c$ in consumption expenditure, and a strong habit persistence, the domestic households may be encouraged to raise initially their real expense (i.e. $c(0) > \bar{c}_0$).
If the *intertemporal speculation* effect predominates, agents may raise in the short-run and more importantly in the long-run their real expense as they adjust smoothly their consumption. The latter conclusion differs markedly from Obstfeld’s result. Such dynamics emerge since households wish to maintain their new higher standard of living induced by a greater purchasing power in terms of consumption goods. The *inertia* effect makes possible the rising temporal path of real expense after a negative perturbation through the fall by a sufficient amount of the time preference rate below the consumption-based real interest rate. An habit-forming behavior departs from the behavior prevailing with time separable preferences since agents raise their consumption only gradually. As agents expect to be accustomed to the greater level of consumption experience, their financial wealth accumulation behavior tends to amplify the *intertemporal speculation* effect in the long run. The dynamics of real expense over the unstable (period 1) and stable (period 2) periods affect the accumulation of internationally traded bonds which in turn influence the once-for-all jump of the marginal utility of wealth. By assuming an habit-forming behavior, \( \lambda \) may be permanently lower and \( c \) definitively higher after a transitory decline in \( p \) depending on the strength of the *intertemporal speculation* effect.

Since multiple cases may emerge and we are only interested in providing a new explanation to the *J-curve* phenomenon, we shall restrict our discussion to adjacent complementarity preferences and a short-lived negative terms of trade perturbation. Figure 4 graphs real expense dynamics when the *intertemporal speculation* effect combined with the *inertia* effect outweigh the *wealth* effect softened by the *smoothing* effect. This implies higher habits at the steady state of period 1. In the adjacent complementarity case, a higher consumption experience in the long-run implies that the marginal utility of real expense in the future is greater than in the present. The rising habitual standard effect leads to a growing path over time for real expense while the value of the time preference rate is transitorily below the real rate of interest, \( r_c = r^* - \alpha_c \dot{p}(t)/p(t) < r^* \).

At some date, the value of the time preference rate becomes greater than \( r_c \) and the path of real expense is decreasing until date \( T \). The real expense dynamics over period 1 can be clarified from the following dynamic equation:

\[
\dot{c}(t) = -\frac{p \lambda}{u_{11}} \left( r^* - \alpha_c \dot{p}(t)/p(t) - \rho(t) \right) \geq 0, \quad 0 < t < T,
\]

where the gap between \( r_c \) and \( \rho(t) \) influences the consumption adjustment over the period \( 0 < t \leq T \). According to (48), the greatest is the fall of the time preference rate, the highest is the domestic content in consumption expenditure, and the most likely a positive change in real expense following its upward initial jump. Over the unstable period 1, the time preference rate increases as the stock of habits rises. At a date \( t = \hat{t} \), the time preference rate is equal to the real rate of interest and the real expense stops increasing, that is \( \dot{c}(\hat{t}) = 0 \) (see figure 4). As \( \rho(t) \) keeps on increasing since the consumption experience rises over the entire period 1, the time preference rate becomes higher than \( r_c \) which in turn leads to a decreasing temporal path of \( c \) until time \( T \). The economic intuition behind this result may be explained as follows. Since the
period 1’s steady-state, \((\bar{s}_1, \bar{c}_1)\), is not viable in solvency terms, the economy cannot raise real expense over the entire period 1 and must establish the level of consumption habits at time \(T\) to a level that satisfies the overall intertemporal budget constraint.

At time \(T\) when the negative perturbation is ended, there are no news effects since all agents perfectly anticipated that change in domestic relative price and the marginal utility of wealth remains constant in the neighborhood of time \(T\). Only the improvement in the terms of trade, that is \(p_1 - p_2 < 0\), constitutes a disturbance effect and exercises an impact on the real expense. The long-run value of \(c\) falls because of the rise of the marginal utility of wealth measured in terms of the domestic good, \(p_c(p_2)\lambda > p_c(p_1)\lambda\), and establishes to the level \(\bar{c}_2 < \bar{c}_1\). The expiration of the terms of trade disturbance leads to an abruptly rise in the consumption-based real interest rate, that is \(r^c(T^+) = r^*\), which remains constant during the convergence toward the new steady-state. When preferences display adjacent complementarity, the marginal utility of real expense is strongly increasing with the habitual standard of living \((\Gamma > 0)\), and the expectation of a higher value of the habit stock in the final steady-state implies that the time preference rate is below its long-run value, \(\delta = r^*\). The explanation comes from the marginal utility of future real expense which is higher than the marginal utility of current real expense. This, in turn, encourages to forgo present for future real expense. The temporal path of \(c\) is monotonically increasing over period 2

\[
\dot{c} = -\frac{p_c}{u_{11}}(\lambda(r^* - \rho(t))) > 0, \quad t \geq T.
\]

As the consumption experience increases, the time preference rate rises and converges toward the psychological time discount rate. Since the representative consumer smooths the change in real expense, this behavior makes possible the accumulation of financial wealth when the terms of trade are favorable (over period 2).

5.3 Current Account Dynamics: a J-Curve?

We show now that current account dynamics may exhibit a sequence of deficit-surplus after an unanticipated transitory terms of trade worsening. The specification of time non-separable preferences induces a departure from the conventional time separable utility function as the temporal path of real expense is no longer flat over respectively periods 1 and 2. Instead of the recursive preferences specification, transitory terms of trade shocks have permanent effects through the hysteresis effects and the stock of foreign assets may reach a permanently higher level. It differs also from the conclusions obtained within a one-good small open economy model with habits since real expense and investment may rise after a transitory negative disturbance, driven by the intertemporal speculation effect. The fact that our dynamic system can generate more flexible dynamic paths and may lead to a multiplicity of possible adjustments comes at price and requires to restrict the scope of possibilities. Although it would be interested to investigate
all of them, we set some assumptions for reasons of space and in order to emphasize the new transitional paths, and to provide a new explanation of the J-curve adjustment of the current account in accordance with the recent empirical results obtained by Leonard and Stockman [2002].

Since the relative price perturbation has no permanent effects on $k$, and therefore on the real domestic product, a fall in $\lambda$ implies a permanent rise in real expense which in turn must be exactly outweighed by higher interest revenues in order to guarantee the equilibrium of the current account at the new steady-state. Formally, in the new long-run equilibrium, the gross national product must be equal to the absorption, that is

$$p_2 F(\bar{k}_2, 1) + r^* \bar{b}_2 = p_c(p_2) \bar{c}_2,$$

where $p_2 = p_0$ and $\bar{k}_2 = \bar{k}_0$. When $\bar{c}_2 > \bar{c}_0$, the long-run foreign assets stock must be higher, $\bar{b}_2 > \bar{b}_0$, in order to allow the small open economy to reach the steady-state.

Formally, after tedious computations, the initial response of the current account can be
calculated as
\[
\frac{dca(0)}{dp} \bigg|_{\text{temp}} = \frac{dS(0)}{dp} \bigg|_{\text{perm}} \left(1 - e^{-r^*T}\right) - \frac{dI(0)}{dp} \bigg|_{\text{perm}} \left(1 - e^{-\chi^2 T}\right) + p_1 \frac{\kappa'(1) \bar{k} - e^{-\chi^2 T} \alpha_1}{p} e^{-r^*T}.
\]

The sign of the expression (51) depends on five key factors: [i] the length of the shock (T), [ii] the import content of investment and consumption expenditure (\(\alpha_I\) and \(\alpha_c\)), [iii] the marginal installation costs reflected by \(\kappa'\) (and measured by \(\chi^2\)), [iv] the degree of habit persistence in consumption, \(|\mu_1|\), and [v] the long-run intertemporal elasticity of substitution (under time non-separable preferences) as it is shown below. The first two terms on the right-hand side of (51) indicate that saving and investment responses to a temporary shock are moderated by the factors \((1 - e^{-r^*T})\) and \((1 - e^{-\chi^2 T})\) compared to their reaction to a permanent perturbation. The shorter-lasting is the adverse terms of trade perturbation, the less are the fall in present values of real income and of the capital revenues, the lower are the responses of saving and investment, and therefore their initial impact on the current account. Since \(\chi^2 > r^*\), the dampening factor is smaller for investment than for saving, i.e. \((1 - e^{-\chi^2 T}) > (1 - e^{-r^*T})\). The third and fourth terms represent the intertemporal speculation effect that affects respectively investment and consumption decisions. The higher are the shares of domestic goods in investment and consumption expenditure, and the less persistent is the adverse terms of trade perturbation, the stronger is the real interest rate effect, and the more probably the current account deteriorates.

Concerning investment choices, the higher is \(\kappa'(1)\), the lower are marginal installation costs and the more investment is responsive to a terms of trade shock (see the third term of (51)). Let us now turn to the consumption reaction to a transitory fall of the consumption-based real interest rate, \(r^c\), below the world interest rate, \(r^*\). With time separable preferences and a CRRA utility functional, the fourth term reduces to \(\frac{de^{-r^*T}}{\theta}\) where the intertemporal elasticity of substitution, denoted by \(\eta_{\text{habd}}\), is equal to \(1/\theta\) (see Cardi [2004]).\(^{24}\) Allowing for habits implies that the representative agent cares not only about the level of current consumption but also to his consumption experience. When the utility function is supposed to be of the CRRA form as assumed by Carroll et al. [2000], the long-run intertemporal elasticity of substitution is no longer equal to \(1/\theta\) but equal to \(\eta_{\text{hab}} = 1/(|\gamma + \theta (1 - \gamma)|)\) where \(\gamma\) is the weight attached to accumulated consumption experience, \(s\).\(^{25}\)

The long-run intertemporal elasticity of substitution under time non-separable preferences is greater than this obtained under conventional preferences whenever \(\theta > 1\) that is if \(u_{12} > 0\) (see Alvarez-Cuadrado et al. [2004], Carroll et al. [2000]). The fourth term on the right-hand side of (51) can be rewritten as follows \(\frac{d\mu_2 \eta_{\text{hab}}}{\sigma} (1 - e^{\mu_1 T}) e^{-r^*T}\). Like with time separable preferences, the intertemporal speculation effect increases with the import content of consumption.
expenditure, $\alpha_c \equiv (p\tilde{d}) / (p_c\tilde{c})$ (see Cardi [2004], Serven [1999]). The novel result is that consumption habits amplify the intertemporal speculation effect compared to the conventional time separable case by raising the long-run intertemporal elasticity of substitution above the inverse of the coefficient of relative risk aversion. In response to a temporary fall in the real cost of consumption following a transitory terms of trade worsening, habits make consumers more willing to reallocate expenditure towards the present and against the future. The dampening term $-e^{\mu_1 T}$ reflects the fact that habit-forming agents dislike large and rapid changes in consumption and prefer to smooth the variation of their real expense over time. The more increasing is the marginal utility of real real expense with respect to the stock of habits, the lower is the absolute value the characteristic root, $|\mu_1|$, and the smoother is the real expense’s reaction. If the shock is sufficiently short, it may be desirable for households to have a temporal decreasing path of their real expense immediately after the upward jump. Finally, the last and fifth term represents the impact of the consumption smoothing effect on the current account. This term is decreasing with the shock’s persistence. This can be explained as follows. A temporary shock results in a larger impact on current than on permanent income, and this gap is as greater as the perturbation is short-living. Therefore, the current account will be more affected the shorter-lasting is the perturbation.

The discussion on transitional dynamics rests on the assumption that the intertemporal speculation effect predominates, in particular one consider that the relative price change is short-living. Starting off from an initial steady-state (period 0), the initial increase in real expense and investment expenditure leads to an increase in domestic absorption. In the same time, as the initial stock of physical is predetermined, the decline in the relative price of home goods induces a fall in the real income. Therefore, the trade balance deteriorates and the current account turns negative at time $t = 0^+$ (see figure 5). Over the period 1, the transition does not take place along a path converging toward a viable steady-state. This unsustainable long-run equilibrium $(\tilde{b}_1, \tilde{k}_1, \tilde{s}_1)$ would ultimately lead the small open economy to intertemporal insolvency. At time $t = \ell < T$, real expense begins its decreasing and investment keeps on increasing. As the growth of capital goods raises the real income and saving becomes less negative, the current account remains negative but may eventually improve. At time $T$, the relative price of home goods is restored to its original level, $p_2 = p_0$. Since the switch of terms of trade was perfectly anticipated, no new information is diffused. The levels of economic aggregates remain unchanged in the neighborhood of time $T$. The consumption and investment-based real rates of interest increase abruptly to the level of the world interest rate, that is $r^c (T^+) = r^I (T^+) = r^*$, and remain constant as the economy converges toward the final steady-state. The rise in the marginal utility of wealth measured in terms of the domestic goods induces a fall in the real expense steady-state from the level $\tilde{c}_1$ to the level $\tilde{c}_2$. At the same time, the absence of hysteresis effects on production-side aggregates implies that the stock of physical capital must return to its initial level, $\tilde{k}_2 = \tilde{k}_0$. The real income reaches its maximum at time $T^+$, that is $p_2 F [k(T^+), 1]$. The
current account becomes unambiguously positive as the terms of trade improve. The economy switches back to a sustainable transition, consistent with intertemporal solvency, that satisfies the unique and overall intertemporal national budget constraint
\[ b_T - \bar{b}_2 = -p_I (k_T - \bar{k}) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_T - \bar{s}_2), \tag{52} \]
where the economy’s unstable period 1 transition is contained in stocks \( k_T, s_T, \) and \( b_T \) serving as new initial conditions. According to the intertemporal solvency condition (52), the stocks of habits and of capital goods move respectively in the same and in an opposite direction with the stock of foreign assets.

Since the rate of change in the stock of habits is continuous, that is \( \dot{s} (T^-) = \dot{s} (T^+) > 0, \) the consumption experience keeps on increasing. The adjustment calls for a growth of real expense as households’ preferences exhibit adjacent complementarity. The consumption and capital goods adjustments deteriorate the trade balance by respectively raising absorption and lowering real income. As the fall in \( k \) decreases in absolute value, that is \( \dot{I}(t) > 0, \) the net exports are declining without ambiguity over the stable period 2. Thanks to the inertia effect, the real expense rises slowly and progressively as the level of terms of trade is favorable, a stable transition calls for rising wealth through the accumulation of internationally traded bonds. Since the real income is above its long-run equilibrium value, and the investment is negative, and the real expense is below its steady-state, the current account remains positive along the stable path. Once the small open economy reaches the final steady-state, the stock of equipment goods is restored to its initial level and the market price of capital is equal to the original investment price index. Real expense and the habitual standard of living reach a higher level in the new long-run equilibrium. This result departs markedly from the conclusions of Obstfeld [1983] and Servèn [1999] who assume time separable preferences and obtained by Ikeda and Gombi [1998] who restrict their analysis to a one-good small open economy model.

6 Conclusion

Our contribution is twofold. From an economic viewpoint, we provide a new explanation of the J-curve phenomenon in terms of habit persistence in consumption and capital adjustment costs. The non-monotonic adjustment of the net foreign assets position displays a sequence deficit-surplus following a permanent or a (short-) medium-lived terms of trade worsening. In accordance with Leonard and Stockman’s empirical results, the surplus phase of current account (second period) is associated with a decline in the real income. Beyond the formalization of Krugman’s economic intuition, the present framework extends Obstfeld’s analysis by introducing an habit-forming behavior. It has been shown that real expense adjustment may exhibit a hump-shape response when the adverse transitory disturbance is at work and may eventually reach a higher
level at the new steady-state. From an analytical viewpoint, we propose a two-good extension of the Ikeda and Gombi framework and apply a new formal procedure to investigate short-term and long-term effects of an unanticipated temporary terms of trade worsening. The two-step method suggested by Schubert and Turnovsky to study the effects of transitory perturbations guarantees that arbitrage relationships are respected and allows for an analytical comparison with permanent shocks. At the opposite of the international business cycles, we obtain some analytical solutions from which the transitional dynamics have been accurately and fully characterized, and the underlying forces that drive real expense, investment, net exports, and current adjustments have been highlighted. Using the Schubert and Turnovsky’s procedure to analyze the effects of temporary shocks shows its capacity to deal in a consistent and explicit way with a complex dynamic model in continuous time and one can expect that further research in international macroeconomic dynamics may take advantage from this attractive mathematical tool.

The new explanation of the J-curve phenomenon suggested in this paper calls for new empirical studies which may focus on an extension of the structural model initiated by Glick and Rogoff [1995] to a two-good framework with habit-forming consumers. The introduction of habit persistence in consumption in the Glick and Rogoff discrete-time formalization has been performed by Gruber [2002] and leads to encouraging results. The work needs to be pursued by considering that the consumption and investment expenditures have an import content and by exploring the effects of relative price changes in an empirical way.

The complexity of the model elaborated in the present paper restricts the possible extensions. If consumption durable goods are introduced, the analytical tractability is no longer maintained and a numerical analysis is necessary. A future interesting study would be to incorporate some consumption durable goods by abstracting from an habit-forming behavior. We may expect that the adjustment would be close to the one obtained by assuming distant complementarity in preferences. Although after a terms of trade permanent worsening, a sequence deficit-surplus is not possible, the current account may exhibit a J-curve adjustment (under some conditions) after an unanticipated transitory terms of trade worsening. A two-country model will be also an interesting way to extend our model as it would endogenize the world interest rate as well as the relative price. Such a formalization has been recently performed by Gombi and Ikeda [2003] by abstracting from capital accumulation in a one-good framework.

Notes

1 “Our evidence supports the J-curve in the data but not its common explanation” (Leonard and Stockman [2002], p. 485).

2 “As recent experience has confirmed, the response of trade flows to the exchange rate take years, both because
consumers are slow to change habits and, even more important, because many changes in supply and sourcing require long-term investment decisions" (Krugman [1989], p. 33).

The parameter $\sigma \geq 0$ determines the relative weight of real expenditure at different times. Performing the differentiation of (3) with respect to time, one obtains a motion equation of the stock of habits where $\sigma$ is the coefficient (or speed) of adjustment. For example, taking a value of $\sigma = 0.6$ in the line of empirical results of Constantinides [1990], then the time required to close 95% of the discrepancy between $s(t)$ and $c(t)$ by changes in $s(t)$ following a change in $c(t)$ is five years (because $e^{-0.6t} = 0.05$ for $t \approx 4.99$).

The unit cost dual function, $p_c(\cdot)$, is defined as the minimum total expense in consumption goods, $z_c$, such that $c = c(d(t), f(t)) = 1$, for a given level of terms of trade, $p$. The minimized unit cost function depends on the terms of trade and is expressed in terms of the foreign good.

The dynamic equation of the foreign assets’ stock is obtained by the substitution of the dividend flow expression, $D(t)$ (see (14a)).

We refer to $J_{11}$ and $J_{22}$ as the submatrices composed respectively by the two first and two last lines and columns of matrix $J$.

When intertemporal preferences display distant complementarity, inequality (27) is always satisfied. This inequality is necessary to ensure that adjacent complementarity is not too strong and that the dynamic system exhibits a saddle point stability (see Becker and Murphy [1988]).

The left hand side of (35b) is the Volterra derivative applied to the functional (2) expressed in current value and obtained along a constant path (see Ryder and Heal [1973]).

The dependency on initial conditions can be avoided thanks to the assumptions of an endogenous psychological rate of discount (see e. g. Obstfeld [1982], Ikeda [?]), imperfect world capital markets (see e. g. Fisher and Terrell [2000]), finite life (see e. g. Matsuyama [1987]) or a continuum of infinitely lived agents with different birth dates (see e. g. Weil [1989]).

Following Sen and Turnovsky [1989] and Servèn [1999], we assume that the small open economy is a net exporter of the home good, and symmetrically a net importer of the foreign good at the steady-state. This assumption is formalized as follows:

$$p (\bar{Y} - \bar{d}) = (\bar{f} - r^* \bar{b}) > 0.$$

Recent empirical studies find strong support of the H-L-M effect (see e. g. Otto [2003]).

To see it, evaluate the stable solution (31b) at $t = 0$ letting $d\bar{k} \equiv \bar{k} - k_0$, and substitute (38b) to obtain

$$q(0) - \bar{q}_1 = \left(\frac{p_1 \chi_1}{\kappa'(1)k}\right) \frac{F_k(1 - \alpha I)}{pF_{kk}} dp.$$

From the expression above, the gap between the short run shadow value of capital, $q(0)$, and its new steady state value, $\bar{q}_1 = p_1(p_1)$, is greater, i. e. the decline in Tobin’s q is larger, the higher is the share of import goods in investment expenditure, $(1 - \alpha I)$.

Non-monotonic convergent paths of net foreign assets are also obtained by Matsuyama [1987] in a finite horizon model with capital adjustment costs, by Karayalçın [1994], [1995] respectively in an infinite horizon model with recursive preferences and installation costs and in a small open economy model allowing heterogeneity across

15 Business cycle properties documented by Mendoza [1995] and Senhadji [1998] for developed countries and developing countries, more particularly the negative contemporaneous correlation between current account and the relative price of imports, suggest the presence of a H-L-M effect. It finds also a strong empirical support in Otto [2003] by using a structural vector autoregression model and rests on the strength of saving decline with respect to negative investment flow in our framework (see eq (43)).

16 Mansoorian [1993], [1998] studies exclusively the impact of a permanent terms of trade deterioration by introducing habit formation behavior in an economy without capital accumulation. Sen and Turnovsky [1989] analyze the response of economic aggregates after a temporary terms of trade disturbance by allowing for a labor-leisure choice and capital adjustment costs. They do not consider habit-forming consumers and capital good imports. The framework of Ikeda and Gombi [1998] is relatively close to ours but we allow for two consumption and capital goods. Moreover, the authors apply the solution procedure proposed by Sen and Turnovsky [1990] which contain an inconsistency (see Schubert [2002], chapter 2, and Schubert and Turnovsky [2002]). Finally, our study differs from Servèn’s analysis in retaining dependent intertemporal preferences and in the analytical method we have chosen to study an unanticipated transitory terms of trade shock.

17 We have retained a formalization of the supply-side analogue to these specified by Gavin [1992] and Servèn [1999] who extend Abel and Blanchard’s model to an open economy by allowing international trade in goods and assets. Gavin [1992] studies temporary terms of trade disturbances only graphically and Servèn [1999] applies the solution procedure proposed by Buijten [1984].

18 To clarify this point, remember that the dampening term \( (1 - e^{-\chi T}) \) is an increasing function of the unstable eigenvalue of \( \chi > 0 \) which in turn raises with the derivative of the inverse function \( \kappa'(1) \) evaluated at the steady-state; the value of the latter is greater the smaller are capital installation costs. In conclusion, the term \( (1 - e^{-\chi T}) \) is a decreasing function of adjustment costs.

19 See Cardi [2004] for a formal treatment through the application of the two-step analytical approach to the zero-root property case.

20 The calculation of impact effect on real expense follows from the stable solution of \( c \) prevailing at period 1. All analytical results for temporary terms of trade shocks are available from the author upon request.

21 As emphasized by Obstfeld [1983], following an unanticipated fall in \( p \) which lasts only a short time period, the transitory low value of the consumption index price, \( p_{c}(p_{1}) < p_{c}(p_{2}) \), makes an incentive for the households to benefit from the temporary decline of the consumption-based real rate of interest, \( r^c < r^* \).

22 See for example Servèn [1999] who examines the impacts of permanent and transitory terms of trade shocks.

23 See for example Karayalçın [1994] who investigates the effects of permanent and temporary productivity shocks.

24 The subscripts “add” and “hab” refer to time separable and time non-separable preferences.

25 In the long-run, the stock of habits is equal to real expense, that is \( \bar{s} = \bar{c} \). This leads to a long-run intertemporal
elasticity of substitution, denoted by $\eta_{hab}$, equal to

$$\eta_{hab} = -\frac{\bar{u}_1 + \frac{\sigma}{\sigma + \mu_2} u_2}{\bar{u}_{11} + \frac{\delta + 2\sigma}{3\sigma} \bar{e}},$$

where the superscript $^\ast$ means that variables are evaluated at the steady-state point. As in Carroll et al. [2000], if the utility is assumed to be of the CRRA form

$$u(c, s) = \frac{1}{1 - \theta} \left( \frac{c}{\bar{c}^{\gamma}} \right)^{1-\theta},$$

where $\theta$ is the coefficient of relative risk aversion and the parameter $\gamma \in [0, 1]$ the strength of habits, the long-run intertemporal elasticity of substitution reduces to

$$\eta_{hab} = \frac{1}{\gamma + \theta (1 - \gamma)} > \eta_{add} = \frac{1}{\theta} \text{ if and only if } \theta > 1.$$

As the cross-partial derivative of the felicity function, $u_{12}$ is equal to $-\gamma (1 - \theta) c^{-\theta} h^{-\gamma (1 - \theta) - 1}$, its sign is positive if and only if $\theta > 1$. This in turn implies that the long-run intertemporal elasticity of substitution under an habit forming-behavior is greater than the intertemporal elasticity of substitution under time separable preferences. The reason for the discrepancy between the long-run horizon elasticities is remarkably explained by Carroll et al. [2000] (see p. 347). For a given rise in the real expense in the long run, the marginal utility of real consumption in the new steady state results in a loss in utility, which in turn raises the intertemporal elasticity of substitution.

26To see it more formally, differentiate the formal solution (period 1) with respect to time, evaluate at time $t = 0$, and differentiate with respect to $p$:

$$\frac{dc(0)}{dp} \bigg|_{temp} = \mu_1 \left( \frac{\sigma + \mu_2}{\sigma} \right) + \mu_2 \left( \frac{\sigma + \mu_1}{\sigma} \right),$$

$$= (\sigma + \mu_1) \frac{dc(0)}{dp} \bigg|_{temp} + \frac{\mu_1 \mu_2}{\sigma} t p e^{-\mu_2 t},$$

where the second expression indicates that households may raise their real expense immediately after the shock ($c(0) > \bar{c}_0$) and decrease $c$ thereafter ($\dot{c}(0) < 0$); whenever intertemporal preferences display adjacent complementarity, the first and second term on the right hand-side have opposite signs. As the second term is decreasing with the shock’s length, we may deduce that a very weak persistent shock may be associated with a temporal decreasing path of real expense.

27The unstable nature of period 1’s transitional path can be seen from the solution for $b(t)$:

$$b(t) = \bar{b}_1 + \left[ \frac{Y - \bar{d}}{\bar{r}} + \frac{\mu_2}{\bar{r}} \frac{p_0 (\sigma + r^*)}{\sigma (\mu_1 - r^*)} t_{\mu} \right] e^{-r(1-\gamma)\mu} dp - p_1 \left( B_1 e^{\chi_{1t}} + B_2 e^{\chi_{2t}} \right) - \frac{p_0 (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A_{1e^{\mu_{1t}}} - \frac{p_0 (\sigma + \mu_2)}{\sigma (\mu_2 - r^*)} A_{2e^{\mu_{2t}}},$$

where the term in brackets represents the explosive (unstable) component due to the presence of the term $e^{-\gamma t}$. This term is an increasing function of time $t$ and is composed of two terms which both exert a negative effect on the current account: the first term refers to the smoothing effect and the second term to the intertemporal speculation effect that affects consumption optimal choices.

References


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