Monetary Policy and Welfare in a Small Open Economy

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Abstract

This paper characterizes welfare in a small open economy and derives the optimal monetary policy rule. It shows that the utility-based loss function for a small open economy is a quadratic expression on domestic inflation, the output gap and the real exchange rate. In contrast to previous works, this paper demonstrates that a small open economy, completely integrated with the rest of the world, should be concerned about exchange rate variability. Therefore, the optimal policy in a small open economy is not isomorphic to a closed economy and does not prescribe a pure floating exchange rate regime. Domestic inflation targeting is optimal only under a particular parameterization, where the only relevant distortion in the economy is price stickiness. When inefficient steady state output and trade imbalances are present, exchange rate targeting arises as part of the optimal monetary plan.

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1 Introduction

Should monetary policy target the exchange rate? Numerous papers have analyzed the choice of monetary policy objectives in closed and open economies. In the former, the debate has focused on whether inflation should be the unique policy target. In open economies, the characterization of optimal policy extends beyond policy makers’ decision of whether to concentrate on the minimization of domestic price distortions. That is, one needs to evaluate the role of other relevant relative prices, more specifically, the role of the exchange rate on monetary policy formulation. The framework presented in this paper addresses these particular issues in a small open economy setting.


Price stability has been advocated in many works employing closed economies models: Woodford (2002), Clarida, Gali and Gertler (2000), Goodfriend and King (1997), among others. In a small open economy setting, Gali and Monacelli (2002) find that, with a specific degree of monopolistic competition and no trade imbalances, the small open economy problem is isomorphic to a closed economy. Several authors have questioned the advantage of complete exchange rate flexibility and of a pure inward looking monetary policy (i.e., a policy that aims to stabilize domestic variables). Corsetti and Pesenti (2000) first emphasize that a country might benefit from influencing its terms of trade. Benigno and Benigno (2003) illustrate the potential gains from cooperation of monetary policy between countries by analyzing the incentives of individual countries to affect the exchange rate. Corsetti and Pesenti (2001b) show that with incomplete pass through, optimal monetary policy is not purely inward looking. The same conclusion is drawing Tille (2003) in the presence of sector specific shocks.

This paper contributes to this rich literature by formalizing a general analytical solution for welfare in a small open economy. Moreover, we obtain a tractable representation of the
optimal policy problem and solution (in the form of a targeting rule). We show that the utility-based loss function is a quadratic expression on domestic inflation, the output gap and the real exchange rate. Therefore, the exchange rate is part of the stabilization goals of monetary policy which induce a more efficient allocation of resources and minimize the distortions present in the economy.

The paper characterizes a small open economy model as a limiting case of a two country dynamic general equilibrium framework featuring monopolistic competition and price stickiness. Moreover, the framework assumes no trade frictions (i.e. the law of one price holds) and perfect capital markets (i.e. asset markets are complete). This benchmark specification allows us to focus on the implications for monetary policy of the interaction of the following factors: (a) Calvo type of staggered price setting; (b) monopolistic competition in goods’ production and the resulting inefficient level of output; (c) trade imbalances and (d) deviations from purchasing power parity that arise from the home bias (in consumption) specification.

The small open economy representation severs the link between domestic policy and world prices, permitting us to abstract from policy interaction between countries. Our focus lies on studying how the monetary authority should react to fluctuations on external conditions, when this reaction has no feedback effects. We further analyze how the optimal response changes with the degree of openness of the economy.

Following the method developed by Benigno and Woodford (2003) and Sutherland (2002), we derive a quadratic loss function for our small open economy. The analysis presented here encompasses as special cases the closed economy framework (Benigno and Woodford, 2003) and the small open economy case with a specific degree of monopolistic competition and no trade imbalances. As previously mentioned, our loss function is quadratic in domestic producer inflation, the output gap measure and the real exchange rate. The weight given to each of these variables depends on structural parameters of the model. In addition, the policy targets depend on the source of the disturbance affecting the economy, which includes a real external shock.

The analytical representation of welfare presented here allows for a precise qualitative analysis of monetary policy in a small open economy. The results obtained show that domestic inflation targeting is optimal only under a particular parameterization, where the only relevant
distortion in the economy is price stickiness. When non efficient steady state output and trade imbalances are present, exchange rate targeting emerges as part of the optimal monetary plan. Therefore, the policy prescription in a small open economy is not isomorphic to a closed economy and does not prescribe a pure floating exchange rate regime.

The intuition behind this result can be obtained from the different features of the small open economy. When production is characterized by monopolistic competition and the elasticity of substitution between domestic and imported goods is high, changes in the real exchange rate might generate costly movements in production. Moreover, in a world where purchasing power parity does not hold, real exchange rate changes generate nominal wealth variations and therefore, create fluctuations in households’ spending. According to agents’ risk aversion, these movements in domestic consumption create welfare losses.

The remainder of the paper is structured as follows. Section 2 introduces the model. Section 3 derives the small open economy dynamics. Section 4 is dedicated to the derivation of welfare and the quadratic loss function. Section 5 analyses the optimal plan and the performance of standard policy rule. Section 6 concludes.

2 The Model

The framework consists of a two-country dynamic general equilibrium model with complete asset markets. Deviations from purchasing power parity arise from the existence of home bias in consumption. The dimension of this bias depends on the degree of openness and the relative size of the economy. This specification allows us to characterize the small open economy by taking the limit of the home size to zero. The two countries Home and Foreign represent the small open economy and the rest of the world.

Monopolistic competition and sticky prices are introduced in the small open economy in order to address issues of monetary policy. We further assume that home price setting follows a Calvo-type contract, which introduces richer dynamic effects of monetary policy than a setup where prices are set one period in advance. Moreover, we abstract from monetary frictions by considering a cashless economy as in Woodford (2003, chapter 2).
2.1 Preferences

We consider two countries, \( H \) (Home) and \( F \) (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment \([0, n)\) belong to country \( H \) and the population in the segment \((n, 1]\) belong to country \( F \). The utility function of the representative consumer \( j \) in country \( H \) is given by:

\[
U_j^t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C^j_s) - \frac{1}{n} \int_0^n V(y^j_s, \varepsilon_{Y,s}) \right],
\]

(1)

Households obtain utility from consumption \( U(C^j) \) and contribute to the production of all the goods \( y^1 \) attaining disutility \( \frac{1}{n} \int_0^n V(y^1_s, \varepsilon_{Y,s})^2 \). Productivity shocks are denoted by \( \varepsilon_{Y,s} \). Moreover, \( C \) is a Dixit-Stiglitz aggregator of home and foreign goods as

\[
C = \left[ \frac{v^{1/\sigma}}{\theta} C_H^{\sigma-1} + (1 - v)^{1/\sigma} C_F^{\sigma-1} \right]^{\sigma/\theta} \quad \text{(2)}
\]

where \( \theta > 0 \) is the intratemporal elasticity of substitution and \( C_H \) and \( C_F \) are the two consumption sub-indices that refer, respectively, to the consumption of home-produced and foreign-produced goods. The parameter determining home consumers’ preference for foreign goods, \( (1 - v) \), is a function of the relative size of the home economy, \( 1 - n \), and of the degree of openness, \( \lambda : (1 - v) = (1 - n)\lambda \).

Similar preferences are specified for the rest of the world:

\[
C = \left[ \frac{v^*^{1/\sigma}}{\theta} C_H^{\sigma-1} + (1 - v^*)^{1/\sigma} C_F^{\sigma-1} \right]^{\sigma/\theta}
\]

(3)

with \( v^* = n\lambda \), that is, foreign consumers’s preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that the specification of \( v \) and \( v^* \) generates a home bias in consumption, as in Sutherland (2002).

The indexes \( C_H \) (\( C_H^x \)) and \( C_F \) (\( C_F^x \)) are Home (Foreign) consumption of the differentiated

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1 For clearness on the welfare derivation, we will assume the following functional forms: \( U(C_t) = \frac{C_t^{1+\rho}}{1+\rho} \) and \( V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^{1+n}}{1+n} \). Where \( \rho \) is the coefficient of relative risk aversion and \( \eta \) is the inverse of the elasticity of goods production.

2 This specification would be equivalent to one in with firms are modelled separately. These firms employ workers who have disutility of supplying labour and this disutility is separable from the consumption utility.
products produced in countries $H$ and $F$. These are defined as follows:

$$ C_H = \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{1}{\sigma}} $$

$$ C_F = \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{1}{\sigma}} $$

where $\sigma > 1$ is the elasticity of substitution across the differentiated products. The consumption-based index for the small open economy that corresponds to the above specifications of preferences is given by:

$$ P = \left[ vP_H^{1-\theta} + (1 - v)(P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \ \theta > 0 $$

$$ P = \left[ v^*P_H^{1-\theta} + (1 - v^*)(P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \ \theta > 0 $$

where $P_H$ ($P_H^*$) is the price sub-index for home-produced goods expressed in the domestic (foreign) currency and $P_F$ ($P_F^*$) is the price sub-index for foreign produced goods expressed in the domestic (foreign) currency.

$$ P_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} P(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \ P_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} P(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} $$

$$ P_H^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} P^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \ P_F^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} P^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} $$

The law of one price holds: $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$, where $S$ is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Therefore, equations (5) and (6), imply that $P_H = SP_H^*$ and $P_F = SP_F^*$. However, equations (7) and (8) show that the home bias specification leads to purchasing power parity deviations, that is, $P \neq SP^*$. For this reason we define the real exchange rate as $RS = \frac{SP^*}{P}$. From consumers’ preferences, we can derive the total demands of the generic good $h$, produced in country $H$, and of the good $f$, produced in country $F$:

$$ y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\theta} \left[ \frac{P_H}{P} \right]^{-\theta} \left[ vC + v^*(1-n) \left( \frac{1}{RS} \right)^{-\theta} C^* \right] + G $$

$$ y^d(f) = \left[ \frac{p(f)}{P_F} \right]^{-\theta} \left[ \frac{P_F}{P} \right]^{-\theta} \left[ (1-v) \frac{n}{1-n} C + (1-v^*) \left( \frac{1}{RS} \right)^{-\theta} C^* \right] + G^* $$
where $G$ and $G^*$ are country-specific government purchase shocks. To portray our small open economy we use the definition of $v$ and $v^*$ and take the limit for $n \to 0$ so that

$$y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[ \frac{P_H}{P} \right]^{-\theta} \left[ (1 - \lambda)C + \lambda \left( \frac{1}{RS} \right)^{-\theta} C^* \right] + G_t \right\}$$

(10)

$$y^d(f) = \left[ \frac{p^*(f)}{P^*_F} \right]^{-\sigma} \left\{ \left[ \frac{P^*_F}{P^*} \right]^{-\theta} C^* + G_t \right\}$$

(11)

From equations (10) and (11) we can see that external changes in consumption will affect our small open economy, but the reverse is not true. Moreover, movements in the real exchange rate will not modify the rest of the world’s demand.

### 2.2 The asset market structure

We assume that, as in Chari et al. (2002), markets are complete domestically and internationally. All household have access to state contingent nominal claims that deliver a unit of Home currency in each state of the world. In this setting there is perfect risk sharing and, consequently, the rate marginal utility of consumption (in nominal terms) is equalized across countries at all times and states of nature.

$$\frac{U_C(C^*_{t+1})}{U_C(C^*_t)} \frac{P^*_t}{P^*_{t+1}} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}}$$

(12)

This specification for the asset market imply that the risk of movements in agent’s nominal wealth is shared with the rest of the world. However, deviations from purchasing power parity imply that changes in agents nominal wealth may occur even with a complete set of nominal contingent bonds. Therefore, real exchange rate movements lead to differences in agents nominal income and in the evolution of consumption across borders.

### 2.3 Price-setting Mechanism

Producers of differentiated goods know the form of their individual demand functions (given by equations (11) and (10)), and maximize profits taking the overall market prices and products, $P_t, P^*_t, Y_t$ and $Y_t^*$, as given. Prices follow a partial adjustment rule a la Calvo (1983). In each period a fraction $\alpha \in [0, 1)$ of randomly picked producers is not allowed to change the

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3Chari et al (2002) represent the asset market structure by having complete, contingent, one period nominal bonds denominated in Home currency. Including bonds denominated in foreign currency would be redundant.
nominal price of the good it produces. The remaining fraction of firms \((1 - \alpha)\) chooses prices optimally by maximizing the expected discounted value of profits\(^4\). Therefore, the optimal choice of producers that can set their price \(\hat{p}_t(h)\) at time \(T\) is:

\[
E_t \left\{ \sum (\alpha \beta)^{T-t} U_c(C_T) \left( \frac{\hat{p}_t(z)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[ \frac{\hat{p}_t(z) P_{H,T}}{P_T} \frac{\mu_t V_y(\hat{y}_T(z), Y_t)}{U_c(C_T)} \right] \right\} = 0 \quad (13)
\]

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by \(\mu_t = \frac{\sigma}{(1 - \tau_t)(\sigma - 1)}\). We allow for fluctuations on this wedge by assuming a proportional tax \(\tau_t\). Hereafter we refer to \(\mu_t\) as a mark up shock.

Given the Calvo-type setup, the price index evolves according to the following law of motion:

\[
(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) (\hat{p}_t(h))^{1-\sigma} \quad (14)
\]

The rest of the world has an analogous price setting mechanism.

### 3 A log-linear representation of the model

We approximate the model around a steady state in which the exogenous variables \((\varepsilon_{yt}, G_t, mc_t)\) assume the values \(\varepsilon_y \geq 0, \bar{G} = 0, \mu \geq 1\) and producer price inflation is set as \(\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1} = 1\). In addition, in this steady-state \(\bar{RS} = 1, \bar{C} = \bar{C}^*, \bar{Y} = \bar{Y}^*,\) and \(\bar{U}_C(\bar{C}) = \mu \bar{V}_y(\bar{Y}, 0)\). Therefore, unless \(\mu = 1\), the steady-state output is inefficiently low (see appendix for a full characterization of the steady state).

In this section we lay out a log-linear version of the structural equilibrium conditions for our small open economy and the rest of the world. We denote with a hat log deviation from the steady state. The system of equilibrium conditions in the small open economy can be expressed by the following six equations:

The Phillips curve, derived from the pricing equation (13) can be written as follows

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\(^4\) All households within a country that can modify their prices at a certain time face the same discounted value of the streams of current and future marginal costs (under the assumption that the new price is maintained). Thus, they will set the same price.

\(^5\) Note that in the case of no proportional taxation and infinitely elastic demand, \(\mu = 1\), characterizing the monopolistic competition case.

\(^6\) We consider that \(\tau_t\) is exogenous and, a priori, uncorrelated with other disturbances, by assuming that government spending is solely financed by lumpsum transfers.

\(^7\) As shown in the appendix, this imply a specific level of initial distribution of wealth across countries.
\begin{equation}
\hat{\pi}_t^H = k \left( \rho \hat{C}_t + \eta \hat{Y}_t - \hat{p}_H + \hat{\mu}_t - \eta \hat{\epsilon}_Y \right) + \beta E_t \hat{\pi}_{t+1}^H \tag{15}
\end{equation}

where \( k = (1 - \alpha \beta)(1 - \alpha)/\alpha(1 + \sigma \eta) \), \( p_{H,t} \) represents the relative price \( P_{H,t}/P_t \) and \( \pi_t^H \) denotes the producer price inflation. This is the usual Open Economy New Keynesian Phillips Curve and it describes the supply side relationship between relative prices, output and consumption. Fluctuations on this condition are led by productivity and mark up shocks.

The log linear version of equation (11) is

\begin{equation}
\dot{Y}_t = -\theta \hat{p}_H + (1 - \lambda) \hat{C} + \lambda \hat{C}^* + \theta \lambda R S_t + \hat{g}_t \tag{16}
\end{equation}

where the fiscal shock \( \hat{g}_t \) is defined as \( \frac{G_t - \bar{G}}{\bar{G}} \). Equation (16) summarizes the aggregate demand condition in the small open economy, and is affected by external and fiscal disturbances.

Assuming a symmetric steady state consumption across borders, equation (12) can be written as

\begin{equation}
\hat{C}_t = \hat{C}_t^* + \frac{1 - \lambda}{\rho} \hat{R} S_t \tag{17}
\end{equation}

Therefore real exchange rate movements lead to differences in consumption across borders. This is due to the fact that deviations from purchasing power parity imply that changes in agents nominal wealth may occur even with a complete set of nominal contingent bonds.

From the price index specification, we can write the following relationship between \( p_{H,t} \) and the real exchange rate

\begin{equation}
(1 - \lambda) \hat{p}_H + \lambda \hat{R} S = 0 \tag{18}
\end{equation}

Moreover, households’ intertemporal choices are represented by the Euler equation

\begin{equation}
\rho E_t \hat{C}_{t+1} = \rho \hat{C}_t + \hat{\mu}_t - (1 - \lambda) E_t \hat{\pi}_{t+1}^H - \lambda (E_t \Delta \hat{S}_{t+1} + E_t \hat{\pi}_{t+1}^*) \tag{19}
\end{equation}

where \( \hat{\pi}_t^* \) is foreign domestic inflation\(^8\). Finally, from the definition of the real exchange rate, we have

\(^8\)Given foreign preferences, in the limiting case where \( \eta \to 0 \), foreign CPI inflation coincide with its domestic - or PPI - inflation.
\[ \Delta RS_t = (1 - \lambda)(\Delta S_t + \pi^*_t - \pi^H_t) \]  

(20)

The small open economy system of equilibrium conditions is closed by specifying the monetary policy rule. Given the domestic exogenous variables \( \hat{y}_{y,t}, \hat{y}_{tt}, \hat{\mu}_t \) and the external real and nominal external shocks \( \hat{C}^*_t, \hat{\bar{p}}^*_t \), we can determine the equilibrium dynamics of \( \hat{Y}_t, \hat{RS}_t, \hat{C}_t, \hat{\pi}^H_t, \hat{p}_{H,t}, \hat{S}_t \) and \( \hat{\mu}_t \).

Foreign dynamics can be represented by the following 3 equations. The foreign Phillips curve;

\[ \hat{\pi}^*_t = k \left( \rho \hat{C}^*_t + \eta \hat{Y}^*_t + \hat{\mu}^*_t - \eta \hat{\varepsilon}^*_t \right) + \beta E_t \hat{\pi}^*_{t+1} \]

The foreign demand equation

\[ \hat{Y}^*_t = \hat{C}^* + \hat{\mu}^*_t \]  

(21)

And the Euler equation

\[ \rho E_t \hat{C}^*_{t+1} = \rho \hat{C}^*_t + \hat{\mu}^*_t - E_t \hat{\pi}^*_{t+1} \]  

(22)

The system in the rest of the world is also completed by specifying a monetary policy rule in the foreign economy. Note that the dynamics of the rest of the world is not affected by the small open economy. Therefore, the variables of interest for the small open economy, \( C^*_t \) and \( \hat{\pi}^*_t \), can be treated as exogenous. Moreover, \( C^*_t \) and \( \hat{\pi}^*_t \) are functions of the foreign structural disturbances and the policy choice of the rest of the world modifies the way these structural shocks affect \( C^*_t \) and \( \hat{\pi}^*_t \).

4 Welfare

The policy objective for the small open economy is given by the expected utility of its agents:

\[ W = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U(C_t) - V(y_t, h, \varepsilon_{Y,t})] \right\} \]  

(23)

In the appendix we derive a second order approximation of the policy objective. In order to eliminate the discounted linear terms that appear in the Taylor expansion, we follow the method of Benigno and Woodford (2003) and Sutherland (2002) and use a second order approximation.
to some of the structural equilibrium conditions. The result is a complete second order solution for the evolution of the endogenous variables of interest. We also assume that policy makers respect past promises following a timeless perspective commitment (as in Woodford, ch 7). The final expression of the loss function for our small open economy can be written as a quadratic function of $\hat{Y}_t, \hat{RS}_t,$ and $\hat{\pi}_t^H$:

$$L_{t=0} = U_c CE_{t=0} \sum \beta^t \left[ \frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\hat{RS}_t - \hat{RS}_t^T)^2 + \frac{1}{2} \Phi_\pi (\hat{\pi}_t^H)^2 \right] + t.i.p \quad (24)$$

where $\hat{Y}_t^T$ and $\hat{RS}_t^T$ are the policy target variables $\Phi_\pi, \Phi_Y$ and $\Phi_{RS}$ are the weights of inflation, output and real exchange rate gaps in welfare losses. The targets are functions of the various shocks (apart from the nominal external disturbance $\pi_t^{09}$) and the weights depend on the structural parameters of the model. The determinants of welfare losses in the small open economy can be analyzed by inspection of the weights $\Phi_Y, \Phi_{RS},$ and $\Phi_\pi$ and the targets $\hat{Y}_t^T$ and $\hat{RS}_t^T,$ which are specified in the appendix.

In order to understand the factors driving welfare losses (i.e., the terms in equation (24)) we should explore the different characteristics of our small open economy. As in a closed economy, the presence of staggered prices brings in gains from minimizing relative price fluctuations. Moreover, monopolistic competition in production introduces a wedge between the marginal utility of consumption and the marginal disutility of production. The existence of market power can lead to gains in deviating from price stability. In order to demonstrate the above argument we can characterize a closed economy by setting $\lambda = 0$. In this case, the policy trade of can be illustrated by: the relative weight of inflation with respect to output $\Phi_\pi/\Phi_Y$ and by the fact that $\hat{Y}_t^T \neq \hat{Y}_t^{Flex},$ where $\hat{Y}_t^{Flex}$ represents the flexible price allocation for output. In a closed economy we can write those as

$$\frac{\Phi_\pi}{\Phi_Y} = \frac{\sigma}{k(\eta + \rho)}$$

and

$$\hat{Y}_t^T - \hat{Y}_t^{Flex} = \frac{(\mu - 1)}{(\eta + \rho)(\mu(\eta + \rho + (\mu - 1))} \{-\eta \mu_t - \rho g_t\}$$

Foreign inflation just affects the nominal interest rate and nominal exchange rate, but not the policy targets. Technically, the system is recursive and $\pi_t^*, S_t$ and $\hat{i}_t$ do not impact the evolution of $\hat{Y}_t, \hat{RS}_t, \hat{C}_t, \hat{\pi}_t^H$ and $\hat{p}_{H,t}$. 


Therefore, the policy trade off is influenced essentially by the parameters related to the monopolistic competitive production, i.e., the degree of market power and level of mark up, $\sigma$ and $\mu$, and the price rigidity parameter, given by the share of households that can not adjust prices in each period $\alpha$.

As shown in equation (24), in a small open economy, apart from domestic relative prices and output, the real exchange rate also have an effect on welfare. The intuition for this result is related to the different features of the small open economy. With complete markets, agent’s nominal wealth is insured against any risk. However, when purchasing power parity does not hold, changes in the real exchange rate lead to changes in real wealth across borders. Therefore, the real exchange rate affects the evolution of consumption (this can be seen by inspection of equation (17)). When the coefficient of relative risk aversion is not unitary, these movements in consumption have a direct effect on welfare.

Moreover, equation (18) and (16) show how the real exchange rate affects the relative price of Home produced goods and modifies the small open economy’s demand. The impact of the real exchange rate on output and consumption distresses the wedge between marginal utility of consumption and marginal disutility of production. And fluctuations in this gap affect the small open economy’s welfare.

The value of the intertemporal and intratemporal elasticity of substitution, $1/\rho$ and $\theta$, determine the real exchange rate effect on consumption and output through the risk sharing and demand channels explained above. Therefore, the real exchange rate weight in the loss function, $\Phi_{RS}$, depends crucially on these parameters.

We now turn to the constraints of the policy problem. The first constraint the policy maker faces is given by the Phillips Curve:

\[
\hat{\pi}_t^H = k \left( \eta(\hat{Y}_t - \hat{Y}_t^T) + (1 - \lambda)^{-1}(\hat{R}S_t - \hat{RS}_t^T) + u_t \right) + \beta E_t \hat{\pi}_{t+1}^H \tag{25}
\]

where $u_t$ is a linear combinations of the shocks defined in the appendix. The policy problem is further constrained by the small open economy aggregate demand equation (16) and the risk sharing condition (12). Combining these two conditions, the following relationship between output and the real exchange rate arises:

\[
(\hat{Y}_t - \hat{Y}_t^T) = (\hat{R}S_t - \hat{RS}_t^T) \frac{(1 + l)}{\rho(1 - \lambda)} + \chi u_t \tag{26}
\]
where \( l = (\rho \theta - 1) \lambda (2 - \lambda) \) and \( \chi \) is a vector with coefficients depending on the structural parameters shown in the appendix. From equation (25) we can see that the policy targets \( \hat{Y}_t^T \) and \( \hat{RS}_t^T \) are not necessarily the flexible price allocations of output and the real exchange rate. Moreover, equation (26) shows that closing the "output gap" does not eliminate the "real exchange rate gap". This is the case only under a special parametrization, where the structural shocks composing \( u_t \) - referred in the literature as cost-push shocks - are eliminated from equations (25) and (26). These cases will be explored later in the text when we analyze the optimality of producer price inflation target.

5 Optimal Monetary Policy

We will proceed by characterizing the optimal plan under the assumption that policy makers can commit to maximizing the economy’s welfare. The policy problem consists of choosing the path of \( \{\hat{\pi}_t^H, \hat{Y}_t, \hat{RS}_t\} \) in order to minimize (24), subject to the constraints (25) and (26) and given that the initial condition \( \hat{\pi}_{t_0} \) and \( \hat{Y}_{t_0} \) equal the precommitted values\(^{10}\). The multipliers associated with (25) and (26) are respectively \( \varphi_1 \) and \( \varphi_2 \). Therefore, the first order conditions with respect to \( \hat{\pi}_t^H, \hat{Y}_t \) and \( \hat{RS}_t \) are given by:

\[
(\varphi_{1,t} - \varphi_{1,t-1}) = \phi \Phi_{\pi} \hat{\pi}_t^H
\]

\[
\varphi_{2,t} - \eta \varphi_{1,t} = \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)
\]

\[
-\varphi_{2,t} - \frac{\rho}{(1 + l)} \varphi_{1,t} = \frac{\rho (1 - \lambda)}{(1 + l)} \Phi_{RS} (\hat{RS}_t - \hat{RS}_t^T)
\]

In order to obtain a targeting rule for the small open economy, we combine equations (27), (28), and (29):

\[
(1 + l) \Phi_Y \Delta (\hat{Y}_t - \hat{Y}_t^T) + \rho (1 - \lambda) \Phi_{RS} \Delta (\hat{RS}_t - \hat{RS}_t^T) + (\rho + \eta (1 + l)) \Phi_{\pi} (\hat{\pi}_t^H) = 0
\]

where \( \Delta \) denotes the first difference of the variable. The above expression is the optimal policy rule for the small open economy: it specifies the objectives of monetary policy, which

\(^{10}\)This should be consistent with the 'timeless perspective' equilibrium. For a discussion on the timeless perspective of optimal rule see Woodford, 2003.
are given by the targets $\bar{Y}_t^T$ and $\bar{RS}_t^T$. Equation (30) shows that the optimal rule prescribes targeting inflation, output and real exchange rate\textsuperscript{11}. Targeting rules have been described in the literature as flexible inflation targets (see Woodford and Giannoni (2003)). With this form of policy rules, the central bank may allow some variability in inflation in order to respond to harmful movements in other variables. Equation (30) specifies the policy maker’s behavior that minimizes such costly fluctuations. The targeting rule determine the policy objectives that can then be achieved by using the nominal interest rates as a policy instrument\textsuperscript{12}.

We now proceed to analyze the optimal plan and some of its special cases. In the next sections, we explore how certain economic characteristics influence the optimal plan.

### 5.1 Producer Price Inflation Target

Under certain circumstances, our approximation of the loss function leads to clear cut results in term of optimal policy. In this section we analyze when the optimal policy consists of production inflation target, i.e., it prescribes complete domestic price stability. The conditions under which this is true involve assumptions on the parameter values of the model and on the source of the shock present in the economy.

Two assumptions are necessary in order to have inflation target as the optimal plan independent of the source of the disturbance present in the economy. Those are: (1) Home and Foreign goods are perfect substitutes and there are no trade imbalances and (2) there are no pure mark-up shocks.

The above assumptions are necessary and sufficient conditions if the economy just experiences productivity and foreign shocks. More specifically, in this case, the conditions that eliminate the term $u_t$ from the equations (25) and (26) are: (1) $\rho = \theta = 1$; and (2) $\mu_t = 0$ and $g_t = 0$ in every state.

In this case, the weights on the loss function are:

$$\frac{\Phi_Y}{1 - \lambda} = (\eta + 1)$$ (31)

\textsuperscript{11}Even if we express equation (30) as a function of Consumer Price Index inflation instead of producer price inflation $\hat{\pi}_t^p$, the targeting rule still includes the term $\Delta(\bar{RS}_t - \bar{RS}_t^T)$.

\[ \Phi_{RS} = 0 \quad (32) \]

\[ \frac{\Phi_x}{(1 - \lambda)} = \frac{\sigma}{k} \quad (33) \]

And the target for output is:

\[ \hat{Y}^T_t = \hat{Y}^{\text{Flex}}_t = (\eta + 1)^{-1} \{ \eta \xi_{Y,t} \} \quad (34) \]

As in the closed economy case, studied in Benigno and Woodford (2004), in order to have inflation target as the optimal plan in an economy subject to fiscal shocks \((g_t \neq 0)\) we need to impose a restriction related to the steady state level of mark up. In the closed economy case, here fully characterized by \(\lambda = 0\), the economic distortion introduced by market power can be eliminated by subsidizing production\(^{13}\). That is, \(\tau\) is set such that \(\mu = 1\). In an open economy, the "optimal subsidy" induces a level of market power in the steady state of \(\mu = (1 - \lambda)^{-1}\). Under this parametrization, the marginal effect of the real exchange rate on consumption utility and labour disutility offset each other and no stabilization procedure is needed.

The target output in this case is:

\[ \hat{Y}^T_t = \hat{Y}^{\text{Flex}}_t = (\eta + 1)^{-1} \{ \eta \xi_{Y,t} + g_t \} \quad (35) \]

The relative weights specified in equations (31) and (33) are analogous to those in the closed economy and the policy target coincides with the flexible price allocation. Moreover, the optimal plan does not respond to external shocks. In what follows, under this specification, the optimal monetary policy in a small open economy is isomorphic to a closed economy (as in Gali and Monacelli (2002)).

### 5.2 Violation of Purchasing Power Parity

A new economic characteristic that can be explored in this framework is the violation of purchasing power parity (PPP hereafter). The presence of deviations from PPP imply differences

\(^{13}\)See Woodford (2000) and Benigno and Woodford (2004) for a full discussion of optimal policy in closed economies.
in the marginal utility of consumption across borders. To simplify the analysis and access how this specific factor changes the policy trade-off, we maintain the subsidy as in the previous section (i.e. \( \tau \) is set such that \( \mu = (1 - \lambda)^{-1} \)) and rule out mark-up shocks. In addition, we impose a less restrictive assumption on the intertemporal and intratemporal elasticities of substitution \( \rho \theta = 1 \).

With the above specification but in the absence of home bias (and therefore, in an environment where PPP holds) Benigno, G and Benigno, P (2003) show that individual countries have no incentives to influence the terms of trade. However, in our small open economy, real exchange rate movements generates uninsured variability in agent’s nominal wealth. Therefore, violation of purchasing power parity \textit{per se} introduces a policy incentive to manipulate the exchange rate. For this reason, the loss function includes a non zero weight on the variability of the real exchange rate.

More accurately, with this parametrization, the weights and the targets on the loss function are:

\[
\begin{align*}
\Phi_Y (1 - \lambda) & = \eta + \rho \\
\Phi_{RS} (1 - \lambda) & = (\rho - 1)\lambda(2 - \lambda) \left[ \frac{\lambda}{\rho(1 - \lambda)} \right]^2 \\
\Phi_x (1 - \lambda) & = \frac{\sigma}{k} \\
\hat{Y}_t T = \hat{y}_t^{Flex}, \hat{R}S_t T = 0
\end{align*}
\]

we can write the economy loss function as

\[
L_{lo} = U_t \tilde{C} E_{t_0} \sum \beta^t \left[ \frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{y}_t^{Flex})^2 + \frac{1}{2} \Phi_{RS} \hat{R}S_t^2 + \frac{1}{2} \Phi_x (\hat{x}_t^H)^2 \right] + t.i.p \quad (37)
\]

This particular specification is the closest to the one in subsection (5.1) where deviations from \textit{inward looking} policy arise in the optimal rule. That is, the optimal plan does not focus uniquely on domestic variables. Even though stabilizing inflation eliminates the output gap, the variability of the real exchange rate still affects welfare. The assumption \( \rho \theta = 1 \)
implies that consumption of Home and Foreign goods are proportional across borders and this relationship is independent of the exchange rate behavior. However, in the presence of home bias, movements in the real exchange rate lead to differences in aggregate consumption at Home relative to the rest of the world.

5.3 The General Optimal Plan: Quantitative results

In this section we present some numerical analysis of the optimal monetary policy. In our benchmark calibration we assume a unitary elasticity of intertemporal substitution, i.e. $\rho = 1$. Following Rotemberg and Woodford (1997) we assume $\eta = 0.47$. The elasticity of substitution between home and foreign goods, $\theta$, is assumed to be 3\(^{14}\) (Obstfeld and Rogoff (1998) argue that it should be between 3 and 6). The degree of openness $\lambda$ is assumed to be 0.4, implying a 40% import share of the GDP. In addition, we consider the case of a "optimal subsidy" policy, where $\tau$ is set such that $\mu = 1/(1 - \lambda)$. Moreover, the elasticity of substitution between differentiated goods is assumed to be 10 as in Benigno and Benigno (2003). To characterize an average length of price contract of 3 quarters, we assume $\alpha = 0.66$. We further consider that shocks follow an AR(1) process, with persistence coefficient of 0.3. Finally, we assume $\beta = 0.99$.

Starting from this benchmark specification, we analyze how optimal monetary policy varies when the economic environment changes.

The small open economy characterization allows us to study the optimal policy reaction to external shocks. The zero measure specification of the Home economy enables us to study how the monetary authority should react to fluctuations on external conditions, when this response has no feedback effects. Therefore, we start by describing the dynamic effect of an external shock on output, inflation and the real exchange rate. Note that in this exercise we consider a change in $C_t^*$ maintaining $\pi_t^* = 0$ in every state. This is the case in which the Foreign authority is adopting a policy of price stability. However, relaxing this assumption does not alter the evolution of the variables relevant to the monetary authority (see footnote 9).

Figure 1 shows the impulse responses of domestic inflation - or producer price index, home output and the real exchange rate to an innovation in $C_t^*$. A higher export demand increases the level of domestic output, appreciates the real exchange rate and increases domestic inflation. We can see that optimal policy prescribes a higher stabilization of the real exchange rate when

\(^{14}\)This to a specification where Home and Foreign goods are substitutes in the utility, given that $\rho \theta > 1$. 

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the economy is more open. Comparing the variables’ reaction to the shock when \( \lambda = 0.4 \) and \( \lambda = 0.2 \) we see a smaller movement of domestic variables in the later case. Therefore, the extent to which policy should be "inward looking" depends crucially on the openness dimension.

A similar exercise can be made with the elasticity of substitution between Home and Foreign goods. In Figure 2 we display the impulse responses of domestic inflation, output and the real exchange rate following a foreign shock. This is done for different specifications of \( \theta \). In the case where goods are complements, \( \theta = 0.5 \), the foreign shock generates a significant appreciation of the exchange rate and a small drop in inflation. Moreover, the increase in output is higher the larger the elasticity of substitution. A trade off between stabilizing domestic variables and the real exchange rate is also affected by the elasticity of substitution between goods.

But is this a feature of the optimal plan? Figure 3 shows the same exercise under an inflation target regime. In this case, the variability of the real exchange rate does not vary significantly with the elasticity of substitution. On the other hand, when the central bank is behaving optimally, it stabilizes movements in the real exchange rate when those are more costly. That is, when the elasticity of substitution between goods is high, movements in the exchange rate have bigger effects on the marginal disutility of production and generate harmful fluctuations in the "monopolistic competition gap". In these circumstances, there are larger gains form managing the exchange rate.

Exercises as the one shown above allow us to judge under which environment (represented by the structural parameters of the model) standard policy rules, such as domestic inflation target or exchange rate peg are more appropriate. Figures 1 and 2 indicate that low values of \( \theta \) and \( \lambda \) imply a bigger exchange rate flexibility under the optimal plan. Therefore, the optimal targeting rule moves further away from an exchange rate peg when the elasticity of substitution between home and foreign goods is low and the country has a small degree of openness.

However, the source of the shock affecting the economy is an important determinant of the performance of policy rules. In the optimal plan this is captured by the composition of the target variables \( \bar{Y}_i^T \) and \( \bar{RS}_i^T \), which stipulates how optimal policy should respond to the different shocks. Figure 4 displays the impulse response function of domestic output gap, inflation and the real exchange rate gap following a productivity shock. This is done for the case in which the policy maker is following the optimal plan and when there is a policy of
domestic price stability. In both cases Home output and real exchange rate jump on impact returning gradually to their initial value. As in Benigno and Benigno (2003), even though qualitatively the optimal plan does not consist of complete price stability, quantitatively they coincide.

This exercise shows that domestic inflation target (i.e. producer price index inflation target), is the appropriate policy rule if the economy experiences productivity shocks. However, this cannot be generalized for other types of disturbances. Figure 5 displays the impulse response function of domestic output gap, inflation and the real exchange rate gap following an external shock. The responses are computed for the case in which the monetary authority is following the optimal plan and when there is a policy of price stability. Figure 6 compare the optimal responses with the ones implied buy an exchange rate peg regime (also following a foreign shock). Under our benchmark calibration, a fixed exchange rate regime resembles closer the optimal plan when compared to PPI inflation target. Therefore, it is important to notice that the performance of standard policy rules depends both on the economy’s characteristics (that is, variations in the structural parameters) and the source of shock affecting the economic environment.

6 Conclusion

This paper has formalized a small open economy model as a limiting case of a two country general equilibrium framework and has characterized its utility-based loss function. The optimal monetary plan was derived and represented in the form of a targeting rule. The setup developed in this work encompasses as special cases the closed economy framework and the small open economy case with efficient levels of steady state output. As a result, the examination of monetary policy in such environments is nested in our analysis.

The utility-based loss function for a small open economy is a quadratic expression on domestic inflation, the output gap and the real exchange rate. This paper demonstrates that a small open economy, completely integrated with the rest of the world, should be concerned about exchange rate variability. Therefore, the optimal policy in a small open economy is not isomorphic to a closed economy and does not prescribe a pure floating exchange rate regime. Price stability (or domestic inflation target) is optimal only under specific parameterization.
When the economy experiences only productivity and foreign shocks, domestic inflation target is optimal if preferences are specified as to rule out trade imbalances. If demand (or fiscal) disturbances are also present, price stability as the optimal plan further requires a production subsidy. In this case the only relevant distortion in the economy is price stickiness.

The simple violation of purchasing power parity brings in a role for targeting the real exchange rate. Similarly, when the assumptions on the steady state level output and trade balances are relaxed, deviations from inward looking policies arise in the optimal plan. The intuition behind these results can be obtained from the different features of the small open economy. When production is characterized by monopolistic competition and the elasticity of substitution between domestic and imported goods is high, changes in the real exchange rate might generate costly movements in production. Moreover, in a world where purchasing power parity does not hold, real exchange rate changes generate nominal wealth variations and therefore, create fluctuations in households’ spending. According to agents’ risk aversion, these movements in domestic consumption create welfare losses.

The sole determinants of monetary policy are the economic environment and the source of the disturbance distressing the economy. These two factors are represented in our analysis by the weights in the loss function - which depend on the structural parameters; and the target variable - that are function of the various disturbances. The quantitative exercise in the last section of the paper explores these factors. The results indicate that low values of $\theta$ and $\lambda$ imply bigger exchange rate flexibility under the optimal plan. Therefore, the optimal targeting rule moves further away from an exchange rate peg when the elasticity of substitution between home and foreign goods is low and the country has a small degree of openness. Moreover, domestic inflation target is shown to be the appropriate policy rule if the economy only experiences productivity shocks. In the case of foreign shocks, under our benchmark calibration, a fixed exchange rate regime resembles closer the optimal plan when compared to PPI inflation target.

The tools developed in this paper can be applied to different economic environments. It is important to notice that the model presented here assumes that there are complete asset markets. Relaxing such assumption would lead to a more realistic representation of the model. Moreover, the introduction of asset market imperfections and their welfare consequences would enrich the optimal monetary policy analysis.

Another interesting extension would involve analyzing fiscal policy by allowing the pro-
portional taxation to be an endogenous variable. This would enable the investigation of the interaction between fiscal and monetary authorities and the optimal policy mix. The small open economy representation allow for the assessment of interesting issues such as the implication of different government bonds denomination for fiscal policy. Moreover, one could analyze optimal fiscal arrangements under a fixed exchange rate regime.

References


7 Appendix A

In this appendix we derive the steady state conditions and define some parameters that depend on these conditions. All variables in steady state are denoted with a bar.

From the demand equation at Home, we have:

\[ y^d(h) = \left[ \frac{P_H}{P} \right]^{-\sigma} \left[ \frac{P_H}{P} \right]^{-\theta} \left[ vC + \frac{v^*(1-n)}{n} \left( \frac{1}{RS} \right)^{-\theta} C^* \right] \]

(38)

\[ y^d(f) = \left[ \frac{P_F}{P} \right]^{-\sigma} \left[ \frac{P_F}{P} \right]^{-\theta} \left[ (1-v)nC + (1-v^*) \left( \frac{1}{RS} \right)^{-\theta} C^* \right] \]

Normalizing \( P_H = P_F \), we have:

\[ Y = vC + \frac{v^*(1-n)}{n}C^* + \bar{G} \]

(39)

If we specify the proportion of foreign-produced goods in home consumption as \( 1 - v = (1 - n)\lambda \) and the proportion of home-produced goods in foreign consumption is \( v^* = n\lambda \), and take the limiting case where \( n = 0 \), we have.

\[ Y = (1 - \lambda)C + \lambda C^* + \bar{G} \]

(41)

And from the Foreign demand we have

\[ Y^* = C^* \]

(42)

For further reference, let’s define some steady state dependent constants:

\[ d_s = \frac{C}{Y} \]
\[ d_b = (1 - \lambda) \frac{C}{Y} \]

Moreover, using equation (41), we can notice that:

\[ \frac{\lambda C^\pi_v}{Y} = 1 - d_b - d_g \]

**The Symmetric Steady State:**

From the complete asset market assumption we have:

\[ RS_t = \kappa_0 \left( \frac{C_t}{C^\pi_v} \right) \tag{43} \]

where

\[ \kappa_0 = RS_0 \left( \frac{C_0}{C^\pi_v} \right) \tag{44} \]

So if we assume an initial level of wealth such that \( \kappa_0 = 1 \), the steady state version of (43) imply \( C = C^\pi_v \).

If, moreover we assume \( \bar{G} = 0 \) we have:

\[ d_g = 0 \]

\[ d_b = (1 - \lambda) \]

Applying our normalization to the price setting equations we have:

\[ U_C(C) = \mu V_g \left( \lambda C^\pi_v + (1 - \lambda)C \right) \tag{45} \]

\[ U_C(C^\pi_v) = \mu^* V_g \left( C^\pi_v^* \right) \tag{46} \]

Where

\[ \mu = \frac{\sigma}{(1 - \tau)(\sigma - 1)} \]

we are also going to use the following notation throughout the appendix

xxv
\( (1 - \phi) = \frac{1}{\mu} \)

\( \phi = 1 - \frac{(1 - \bar{r})(\sigma - 1)}{\sigma} \)

\( 0 \leq \phi < 1; \mu > 1 \)

8 Appendix B

In this appendix, we derive the 1st and 2nd order approximation of the equilibrium conditions of the model. Moreover, we show the second order approximation of the utility function in order to address welfare analysis. To simplify and clarify the algebra, we use the following isoelastic functional forms:

\[
U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}
\]

\[
V(y_t(h), \varepsilon_{Y,T}) = \frac{\varepsilon^{-\eta} y_t(h)^{\eta+1}}{\eta + 1}
\]

9 Demand

As shown in the text, home demand equation is:

\[
Y_{H,t} = \left[ \frac{P_{H,t}}{P_t} \right]^{-\theta} \left[ (1 - \lambda)C_t + \lambda \left( \frac{1}{RS_t} \right)^{-\theta} C_t^* \right] + g_t \tag{47}
\]

Using the "steady state depend" parameters defined in Appendix A, and with home relative price defined as \( P_{H,t}/P_t = p_{H,t} \), we obtain the following first order approximation to demand in the small open economy:

\[
\hat{Y}_H = -\theta(1 - \hat{g}_t)\hat{p}_H + d_b \hat{C} + (1 - d_b - d_g)\hat{C}^* + \theta(1 - d_b - d_g)\hat{R}S + \hat{g} \tag{48}
\]

Note that fiscal shock \( \hat{g}_t \) is defined as \( \frac{G_t - \bar{G}}{\bar{G}} \), allowing for the analysis of this shock even when the zero steady state government consumption is zero. In the symmetric steady state, where \( d_b = 1 - \lambda \) and \( d_g = 0 \), the equation (48) becomes:
\[ \hat{Y}_H = -\theta \hat{p}_H + (1 - \lambda) \hat{C} + \lambda \hat{C}^* + \theta \lambda \hat{R} \hat{S} + \hat{g} \]  

(49)

And the second order approximation to the demand function is:

\[ \sum \beta^t \begin{bmatrix} d'_y y_t + \frac{1}{2} y'_t y_t + y'_t D_c e_t \end{bmatrix} + s.o.t.i.p = 0 \]

where

\[ y_t = \begin{bmatrix} \hat{Y}_t & \hat{C}_t & \hat{p}_{Ht} & \hat{R}S_t \end{bmatrix} \]

\[ e_t = \begin{bmatrix} \hat{\varepsilon}_y t & \hat{\mu}_t & \hat{g}_t & \hat{C}_t \end{bmatrix} \]

\[ d'_y = \begin{bmatrix} -1 & d_b & -\theta(1 - d_g) & \theta(1 - d_b - d_g) \end{bmatrix} \]

\[ D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1 - d_b)d_b & -\theta d_b d_g & -\theta(1 - d_b - d_g)d_b \\ 0 & -\theta d_b d_g & \theta^2(1 - d_b)d_g & -\theta^2 d_g(1 - d_b - d_g) \end{bmatrix} \]

\[ D'_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -d_b & -(1 - d_b - d_g)d_b \\ 0 & 0 & \theta(1 - d_g) & -\theta d_g(1 - d_b - d_g) \end{bmatrix} \]

Moreover, in the symmetric equilibrium:

\[ d'_y = \begin{bmatrix} -1 & 1 - \lambda & -\theta & \theta \lambda(1 - \lambda)^2 \end{bmatrix} \]

\[ D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda(1 - \lambda) & 0 & -\theta \lambda(1 - \lambda) \\ 0 & 0 & 0 & 0 \\ 0 & -\theta \lambda(1 - \lambda) & 0 & \theta^2 \lambda(1 - \lambda) \end{bmatrix} \]
\[
D_e' = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1 - \lambda) & -\lambda(1 - \lambda) \\
0 & 0 & \theta & 0 \\
0 & 0 & -\theta\lambda(1 - \lambda) & \theta\lambda(1 - \lambda)
\end{bmatrix}
\]

10 Risk Sharing Equation

In a perfectly integrated capital market, the value of the intertemporal marginal rate of substitution is equated across borders:

\[
\frac{U_C(C^*_{t+1})}{U_C(C^*_t)} \frac{P^*_t}{P^*_{t+1}} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}} \tag{50}
\]

Assuming the symmetric steady state equilibrium, the log linear approximation to the above condition is:

\[
\dot{C}^*_t = \dot{C}_t + \frac{1}{\rho} \dot{R}S_t \tag{51}
\]

Given our utility function specification, equation (50) gives rise to a exact log liner expression and therefore the first and second order approximation are identical.

In matrix notation, we have:

\[
\sum E_t\beta^t \left[ c'_y y_t + \frac{1}{2} y'_t C_y y_t + y'_t C_e e_t \right] + \text{s.o.t.i.p.} = 0
\]

\[
c'_y = \begin{bmatrix}
0 & -1 & 0 & \frac{1}{\rho}
\end{bmatrix}
\]

\[
c'_y = \begin{bmatrix}
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
C'_y = 0
\]

\[
C'_e = 0
\]
11 The Real Exchange Rate

Given our preference specification in the small open economy, and knowing that in the rest of the world $P_F = SP^*$, we can write the price level in the following form:

$$\left( \frac{P}{P_H} \right)^{1-\theta} = (1 - \lambda) + \lambda \left( \frac{RS}{P_H} \right)^{1-\theta}$$

Therefore, the first order approximation to the above expression is:

$$\tilde{p}_H = -\frac{\lambda RS}{1 - \lambda}$$

Moreover, the second order approximation to equation (52) is:

$$\sum E_t\beta_t \left[ f'_y y_t + \frac{1}{2} y_t F_y y_t + y_t F_e e_t \right] + s.o.t.i.p. = 0$$

$$f'_y = \begin{bmatrix} 0 & 0 & -(1 - \lambda) & -\lambda \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & (1 - \lambda(1 - \lambda)) \end{bmatrix}$$

$$F'_y = \lambda(\theta - 1)$$

12 Price Setting

The first and second-order approximation to the price setting equation follow Benigno and Benigno (2001) and Benigno and Benigno (2003). They are derived from the following first order condition of sellers that can reset their prices:

$$E_t \left\{ \sum (\alpha \beta)^{T-t} U_c(C_T) \left( \frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[ \frac{\tilde{p}_t(h) P_{H,T}}{P_T} - \mu V_y (\tilde{y}_{h,T} (h), e_{Y,T}) \right] \right\} = 0$$

where

$$\tilde{y}_t (h) = \left( \frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,t}$$

For a detail derivation of the first-order approximation to the price setting see technical appendix in Benigno and Benigno (2001). Benigno and Benigno (2003) have the details on the second-order approximation.
and

\[(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) (\hat{p}_t(h))^{1-\sigma} \quad (56)\]

With mark up shocks \(\mu_t\) defined as \((\sigma-\sigma)/(\sigma-\tau_H)\), the the first order approximation to the price setting equation can be written in the following way:

\[
\hat{\pi}_t^H = k \left( \rho \hat{C}_t + \eta \hat{Y}_t - \hat{p}_t + \hat{\mu}_t - \eta \hat{\varepsilon}_{Y,t} \right) + \beta E_t \hat{\pi}_{t+1}^H \quad (57)
\]

where \(k = (1 - \alpha_\beta)(1 - \alpha)/\alpha(1 + \sigma\eta)\).

The second order approximation to the price setting can be written as follows:

\[
Q_{to} = \phi \sum E_t \beta^t \left[ a_y' y_t + \frac{1}{2} y_t' A_y y_t + y_t' A_e e_t \right] + s.o.t.i.p. \quad (58)
\]

\[
a_y' = \begin{bmatrix} \eta & \rho & -1 & 0 \end{bmatrix}
\]

\[
A_y' = \begin{bmatrix}
\eta(2 + \eta) & \rho & -1 & 0 \\
\rho & -\rho^2 & \rho & 0 \\
-1 & \rho & -1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
A_e' = \begin{bmatrix}
-\eta(1 + \eta) & 1 + \eta & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

For further reference, we can use the other log linear structural relationships of the model and write equation (57) with variables in deviation from the flexible price allocation:

\[
\hat{\pi}_t^H = k \left( \eta (\hat{Y}_t - \hat{Y}_{t}^{F,lex}) + \frac{(RS_t - \hat{R}_{S_t}^{F,lex})}{(1 - \lambda)} \right) + \beta E_t \hat{\pi}_{t+1}^H \quad (59)
\]

where:

\[
\hat{Y}_t^{F,lex} = \left( (\eta + \rho) + \eta \lambda \right)^{-1} \left\{ \eta (1 + l) \hat{\varepsilon}_{Y,t} - (1 + l) \hat{\mu}_t + \rho \hat{\mu}_t + \rho l \hat{\mu} \right\} \quad (60)
\]
\[
\tilde{RS}_t^{\text{Flex}} = \frac{(\eta + \rho + \eta \lambda)^{-1}}{(1 - \lambda)} \left( \eta \tilde{\varepsilon}_{y,t} - \tilde{\mu}_t - \eta \tilde{y}_t + (\eta + \rho) \tilde{C}_t^* \right)
\]  

(61)

### 13 Welfare

Following Benigno and Benigno (2003), the second order approximation to the utility function can be written as:

\[
U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s^j, \varepsilon_{Y,s})]
\]

(62)

\[
W_{to} = U_c \tilde{C}_t e_t \sum \beta^t \left[ w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w'_n \pi^2_t \right] + \text{s.o.t.i.p}
\]

(63)

\[
w'_n = \frac{\sigma}{\mu k}
\]

\[
w'_y = \begin{bmatrix}
-1/\mu & 1 & 0 & 0
\end{bmatrix}
\]

\[
W'_y = \begin{bmatrix}
\mu(1 + \eta) & 0 & 0 & 0 \\
0 & -(1 - \rho) & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
W'_e = \begin{bmatrix}
\frac{\eta}{\rho} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Using the second order approximation of the equilibrium condition, we can eliminate the term \( w'_y y_t \). Do do so, we will derive the vector \( Lx \), such that:

\[
\begin{bmatrix}
a_x & d_x & f_x & b_x
\end{bmatrix} Lx = w_y
\]

Giving the values of \( a_y, b_y, c_y, d_y \), defined in this appendix, we have:

\[
Lx_1 = \frac{1}{(\rho + \eta) + \eta} \left[ \frac{\eta \mu^{-1}}{1 + \lambda} - \mu^{-1} \right]
\]

(64)
\[ Lx_2 = \frac{1}{(\rho + \eta) + l\eta} \left[ \rho(\mu^{-1} - (1 - \lambda)) + (1 - \lambda)(\eta + \rho) \right] \]  
\[ Lx_3 = \frac{1}{(\rho + \eta) + l\eta} \left[ (\rho\theta - 1)(1 - \lambda)\mu^{-1} - (\eta\theta + 1) \right] \]

where \( l = (\rho\theta - 1)\lambda(2 - \lambda) \)

And the loss function \( L_{to} \) will have the following form:

\[ L_{to} = U_c\bar{C}E_{t_0} \sum \beta^t \left[ \frac{1}{2} y_t' y_t + y_t' L_e e_t + \frac{1}{2} l^2 \pi^2 \right] + s.o.t.i.p \]

where:

\[ L_y = W_y + Lx_1 A_y + Lx_2 D_y + Lx_3 F_y \]

\[ L_e = W_e + Lx_1 A_e + Lx_2 D_e \]

\[ L_\pi = w_\pi + Lx_1 a_\pi \]

To write the model just in terms of the output, real exchange rate and inflation, we define the matrixes \( N \) and \( N_e \) mapping all endogenous variables into \([Y_t, T_t]\) and the errors in the following way:

\[ y_t' = N [Y_t, T_t] + N_e e_t \]

\[
N = \begin{bmatrix}
1 & 0 \\
1 & -\frac{1+\lambda}{\rho(1-\lambda)} \\
0 & -\frac{\lambda}{(1-\lambda)} \\
0 & 1 \\
\end{bmatrix}
\]

\[
N_e = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Equation (67) can therefore be expressed as:

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\[ L_{t_0} = U_c C E_{t_0} \sum \beta^t \left[ \frac{1}{2} \left[ \hat{Y}_t, \tilde{R}S_t \right]' L_y' \left[ \hat{Y}_t, \tilde{R}S_t \right] + \left[ \hat{Y}_t, \tilde{R}S_t \right]' L_e' e_t + \frac{1}{2} l_y \pi_t^2 \right] + t.i.p \quad (69) \]

where:

\[ L_y' = N' L_y N \]

\[ L_e' = N' L_y N_e + N' L_e \]

Finally, we rewrite the previous equation as deviations from the target variables:

\[ L_{t_0}' = U_c C E_{t_0} \sum \beta^t \left[ \frac{1}{2} \Phi_Y (\hat{Y}_t - \tilde{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\tilde{R}S_t - \tilde{R}S_t^T)^2 + \frac{1}{2} \Phi_\pi (\hat{\pi}_t)^2 \right] + s.o.t.i.p \quad (70) \]

where:

\[ \Phi_Y = (\eta + \rho)(1 - \phi) + \frac{(\rho - 1)[-l(1 - \phi) - (\lambda - \phi)]}{(1 + l)} + L_x \left[ (\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho - 1)}{(1 + l)} \right] - \frac{L_x (1 - \lambda)^2 \lambda(\rho \theta - 1)}{(1 + l)} \]

\[ \Phi_{RS} = -\frac{(\lambda + l)(\rho - 1)}{(1 - \lambda)^2 \rho} \]

\[ \quad + \frac{L_x l(\rho - 1 - l)}{(1 - \lambda)^2 \rho} \]

\[ \quad + \frac{L_x 2(\rho \theta - 1)[\rho \theta (1 - \lambda) + \lambda + l]}{\rho^2} \]

\[ \quad + \frac{L_x 3 \left[ 1 + \lambda^2 (2 - \lambda) \right] \lambda(\theta - 1)}{1 - \lambda} \]

\[ \Phi_\pi = \frac{\sigma}{\mu k} + (1 + \eta) \frac{\sigma}{k} L_x \]

and

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\[ \dot{Y}_t^T = q_y e_t \]

with
\[
q_y = \frac{1}{\Phi_Y} \begin{bmatrix}
\frac{\eta}{\rho} + Lx_1(1 + \eta) & -Lx_1(1 + \eta) & (\rho^{-1}(1-\lambda) + Lx_2) \\
\end{bmatrix}
\]

\[ \dot{R}_S_t^T = q_{rs} e_t \]

with
\[
q_{rs} = \frac{1}{\Phi_{RS}} \begin{bmatrix}
0 & 0 & (\rho^{-1}-1)Lx_1(c_{RS}^T - c_{RS}) + \frac{Lx_2(\lambda(1-\lambda) + 1)(\rho^\theta - 1)}{\rho(1-\lambda)} & \frac{-Lx_2(\lambda(1-\lambda) + 1)(\rho^\theta - 1)}{\rho}
\end{bmatrix}
\]

We can write the constraints of the maximization problem as:

\[ \dot{\pi}_t^H = k \left( \eta(\dot{Y}_t - \dot{Y}_t^T) + (1 - \lambda)^{-1}(\dot{R}_S_t - \dot{R}_S_t^T) + u_t \right) + \beta E_t \dot{\pi}_t+1 \] (71)

\[ (\dot{Y}_t - \dot{Y}_t^T) = (\dot{R}_S_t - \dot{R}_S_t^T) \frac{(1 + l)}{\rho(1 - \lambda)} + \chi u_t \] (72)

where

\[ u_t = \left[ \eta, \frac{1}{1 - \lambda} \left( (\dot{Y}_t^T - \dot{Y}_t^{Flex}), (\dot{R}_S_t^T - \dot{R}_S_t^{Flex}) \right)' \right] \]

\[ \chi = \left[ \frac{1}{\eta}, \frac{(1 + l)}{\rho} \right] \]

14 Special Cases

In this case we show the special cases described in the main text

Special Case 1:

The assumptions made in this special case are:

1. \( \rho = \theta = 1 \)

2. No mark-up shocks.
therefore, we have:

\[ l = 0; Lx_1 = 0; Lx_2 = (1 - \lambda); Lx_3 = -1 \]

\[
\frac{\Phi_Y}{(1 - \lambda)} = \eta + 1
\]

\[
\Phi_{RS} = 0
\]

\[
\frac{\Phi_x}{(1 - \lambda)} = \frac{\sigma}{\phi}
\]

Moreover:

\[
\hat{Y}_t^T = q_y e_t = \hat{Y}_t^{Flex} = [\eta + 1]^{-1} \{ \eta \xi_{Y,t} + g_t \}
\]

**Special Case 2:**

In this case we assume:

1. \( \rho \theta = 1 \)

2. \( \mu = 1/(1 - \lambda) \)

3. No mark-up shocks.

\[ l = 0; Lx_1 = 0; Lx_2 = (1 - \lambda); Lx_3 = -\theta \]

\[
\frac{\Phi_Y}{(1 - \lambda)} = \eta + \rho
\]

\[
\frac{\Phi_{RS}}{(1 - \lambda)} = \frac{(\rho - 1)\lambda(2 - \lambda)}{\rho^2} \left[ \frac{\lambda^2}{(1 - \lambda)} \right]^2
\]

\[
\frac{\Phi_x}{(1 - \lambda)} = \frac{\sigma}{\phi}
\]

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