Accounting for the Hidden Economy: Barriers to Legality and Legal Failures\textsuperscript{1}

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September 28, 2004

\textsuperscript{1}This paper has benefitted from the financial support of Égide at Universidade Nova and Fundação para a Ciência e Tecnologia (FCT-Portugal) and POCTI through FEDER, and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil). We thank Stephen L. Parente, Mário Páscoa, João Maurício Rosal, and Anne Villamil for useful conversation and suggestions. We are also indebted to seminar participants at the Universidade NOVA de Lisboa informal workshop, Universidad de Alicante, Banco de Portugal, VII Workshop on Dynamic Macroeconomics (Vigo) and LACEA for helpful discussions and comments. We are responsible for any remaining errors.

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Abstract

This paper studies whether or not regulation costs (barriers to legality) and contractual imperfections in financial markets (legal failures) account for the observed differences in the informal sector across countries. In order to investigate this question it constructs and solves numerically a general equilibrium model with credit constrained heterogeneous agents, occupational choices over formal and informal businesses, contractual imperfections and a government sector which imposes taxes and regulations on formal firms. The benefit from formalization is better access to outside finance. Differences in regulation costs and the degree of enforcement in financial contracts generate endogenously differences in the size of the informal sector and in total factor productivity (TFP). The numerical exercises suggest that: i) regulation costs and not financial market imperfections account for the difference in the size of the informal sector between United States and Mediterranean Europe; however, ii) this result is different when for countries with very weak enforcement systems, such as Peru. In this case, contractual imperfections have a stronger impact on informal sector size than regulation costs. Regarding differences in output per capita across countries, regulation costs and strength of enforcement explain roughly half of the difference in observed international incomes.

JEL Classification: E6; O11; O17

Keywords: Inequality; Credit constraints; Corruption; Informal sector
1 Introduction

A fundamental issue in economic development is to study the determinants of the informal (unregulated) sector. Why does the size of the informal sector vary so much across countries? Informal production accounts for roughly 10 percent of total production in the United States and this statistic is about 25 and 50 percent in Italy and Peru, respectively.\(^1\) De Soto (1989) emphasizes that informal production in developing countries is mainly driven by *entry barriers*, under the form of regulation and corruption, that entrepreneurs face to acquire legal status. Empirical studies have corroborated this hypothesis.\(^2\) In a cross-section of 85 countries, Djankov et al. (2002) show that stricter regulation of entry\(^3\) is associated with sharply higher levels of corruption, and a greater relative size of the unofficial economy.

Recently, De Soto (2000) suggests that these *barriers to legality* are not only important to explain differences in the size of the informal sector but also differences in per capita income across countries. His idea is that, without legal status, entrepreneurs cannot exercise full property rights over their assets, and, as a consequence, cannot use their wealth as collateral for a loan and cannot generate capital from their savings. Due to these *legal failures*, they, in general, under-invest and are locked into running low productivity technologies.

This paper investigates the role of corruption, regulation (*barriers to legality*) and credit market imperfections (*legal failures*) in accounting for differences in the size of the unofficial economy across countries. We put forth a general equilibrium model to address De Soto’s (2000) hypothesis analytically and quantitatively. Agents in our framework can choose to be either a worker or an (informal or formal) entrepreneur. In this respect, this paper is related to Lucas’ (1978) “span of control” model, which was later extended by Rauch (1991) in his study of informal production. Unlike these models, ours is built upon a dynamic framework and uses credit constraints in the analysis of occupational choice. Agents are dif-

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\(^1\)See Schneider and Enste (2000).

\(^2\)See Friedman, Johnson, Kaufman and Zoido-Lobaton (2000) and Djankov, La Porta, Lopez-de-Dilanes and Shleifer (2002).

\(^3\)They evaluate all formal procedures that an entrepreneur needs to carry out to begin legally operating a firm.
differentiated by entrepreneurial ability and initial wealth. They care about their own consumption and the initial wealth of their offspring. In order to open a formal/informal business, agents must buy capital in advance to finance their project. Capital markets are imperfect and therefore the best project will not necessarily be undertaken. This interaction between wealth distribution and capital market imperfection is based on Banerjee and Newman (1993).

The theoretical environment, therefore, considers three occupational choices (worker, and formal or informal entrepreneur), inequality in wealth and in entrepreneurial ideas, corruption and regulation, and limited enforcement. Occupational choices and the size of each project are determined endogenously. They depend on the agent’s “type” (wealth and project), start up costs, and credit market imperfections. Different levels of bureaucracy and limited enforcement generate not only differences in the occupational choice (size of the informal sector), but also differences in total factor productivity (TFP). Our model therefore provides a theory of differences in TFP (as required by Prescott (1998)). The main factors in this theory are corruption and credit market imperfection.

Our quantitative experiments based on the empirical evidence on the size of the informal sector, regulation costs and the degree of enforcement across countries suggest that: i) regulation costs, not financial market imperfections, account for the difference in the size of the informal sector between United States and Mediterranean Europe; however, ii) this result is somewhat different when we consider countries with very weak enforcement systems, such as Peru. In this case, contractual imperfections have a stronger impact on informal sector size than regulation costs. Regarding differences in output per capita across countries, limited enforcement and corruption costs explain roughly half of the difference in observed international incomes. Therefore, the model accounts for the difference in informal sector sizes across countries, but just part of the difference in output per capita.

Another important empirical fact is also consistent with our exercises: the

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4See also Lloyd-Ellis and Bernhardt (2000) for a close framework which studies the macroeconomic and distributional dynamics associated with the process of economic development. They develop important tools that we use to prove the long run dynamics of our model economy.

5Antunes and Cavalcanti (2003) and Erosa and Cabrillana (2004) also develop model economies where capital market imperfections and regulation costs endogenously generate differences in TFP.
proportion of small scale activities is negatively related with per capita income levels. Tybout (2000), for instance, shows that government regulations and taxes are enforced only among large firms. In order to avoid the costs associated with formal production, entrepreneurs scale down the size of their firms and operate outside the realm of government regulation.

There is already a tradition in economics which studies the hidden economy.6 One branch of the literature studies the effects of the informal sector on growth and government policies,7. Another branch, which is more related to our study, investigates the determinants of the hidden economy. Among these empirical studies it is important to highlight Friedman et al. (2000) and Johnson, Kaufmann and Zoido-Lobaton (1998). Their findings suggest that it is not tax rates8 per se that lead entrepreneurs to go underground, but bureaucracy, corruption, and a weak legal system. Their findings are consistent with our quantitative results. With respect to the theoretical studies9 Rauch’s (1991) model suggests that entrepreneurs go underground to avoid minimum wages. Consequently, in their model the formal equilibrium wage rate is higher than that in the informal sector.10 In Dessy and Pallage (2003) entrepreneurs become legal because they can use productive infrastructure (which enters in the production function), while in our model the benefit from formalization is better access to outside finance. Quintin (2002) has a similar premium from formalization. Our model is different from his because we add bureaucracy costs and bequest transfers to the analysis. Regulation costs are empirically and quantitatively important to determine the size of the informal

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6For a comprehensive overview see Schneider and Enste (2000).
7Easterly (1993) and Loayza (1996) show that growth is negatively related to informal production. Cavalcanti and Villamil (2003) shows how the optimal monetary policy and the welfare costs of inflation might be affected by the size of the informal sector (tax evasion).
8They argue that the burden of tax rates has two effects: (i) it drives agents into the informal sector to avoid official taxes; but (ii) it can also raise revenue to improve government institutions and enforcement, which leads to a lower informal sector. In general, these two effects offset each other, and the relationship between tax rates and the size of the informal sector across countries is not statistically significant.
9An interesting article is Azuma and Grossman (2002). They study a related but different question: why do governments impose or tolerate burdensome taxes, bribes and bureaucracy that lead many producers to operate in the informal sector?
10See Amaral and Quintin (2003) for a model where differences in worker’s characteristics and in the wage rate arise endogenously due to better investment opportunities in the formal sector. The question that they address is different from ours. They explain why informal production emphasizes low-skill work.
sector and the interaction between credit constraints and bequest inequality is key for occupational choice.

Finally, a literature on the two sector growth model and economic development is also related to our quantitative theoretical analysis. Parente, Rogerson and Wright (2000), for instance, introduces home production, while Restuccia (2004) considers a traditional and a modern sector in a Neoclassical growth model. Similar to our model, these economies can generate larger differences in “official” output levels across countries than standard models for a given policy differential. In our model technologies will be the same in each sector but differences in productivity will arise endogenously due to policies that affect differently each sector.\(^{11}\)

This paper is divided as follows: the next section describes the model economy. Section 3 describes the agents’ optimal behavior, defines the competitive equilibrium allocations, and presents some analytical results. Section 4 solves the model numerically and conducts policy experiments. The last section provides some concluding remarks and policy implications.

2 The model

2.1 Preferences, endowments and technology

2.1.1 Preferences

In each time period \((t = 0, 1, 2, \ldots)\), the economy consists of a continuum of individuals in the unit interval. Each agent lives and is productive for one period, then reproduces another individual so that the population is constant. Agents care about their own consumption and leave bequests to their offspring. Let \(c_i^t\) and \(b_{i+1}^t\) denote consumption and bequests, respectively, by agent \(i\) in period \(t\). Preferences are represented by

\[
U^i = (c_i^t)\gamma (b_{i+1}^t)^{1-\gamma}, \quad \gamma \in (0, 1).
\] (1)

This utility function implies that agents are risk-neutral with respect to income as the indirect utility function is linear in wealth. This implies that any additive

\(^{11}\text{Related to this result is Guner, Ventura and Yi (2004) who consider a model with occupational choice and investigate the effects of restrictions on size on productivity.}\)
punishment or reward in utility may be measured in terms of income. Notice that, for tractability, we assume that preferences are for the bequest and not the offspring’s utility (see Banerjee and Newman (1993) and Lloyd-Ellis and Bernhardt (2000) for a similar formulation).

2.1.2 Endowments

Each individual can be either a worker or an entrepreneur. Entrepreneurs create jobs and manage their labor force, n. As in Lucas (1978a) each individual is endowed with a talent for managing, xi, drawn from a continuous cumulative probability distribution function \( \Gamma(x) \) with finite support \([x, \bar{x}]\), where \( \bar{x} \geq 0 \). Therefore, in each period agents are distinguished by their initial wealth and ability as entrepreneurs, \((b_i, x_i)\). We assume that the agent’s talent for managing is not hereditary. For notational convenience, in the remainder of the paper we drop agent superscript \( i \).

2.1.3 Production technologies

Managers operate a technology that uses labor, n, and capital, k to produce a single consumption good, y, that is represented by

\[
y = xk^\alpha n^\beta, \quad \alpha, \beta > 0, \quad \text{and} \quad \alpha + \beta < 1.
\] (2)

Capital fully depreciates each period. Managers can operate only one project. Entrepreneurs can choose to declare their establishments (formal sector) or to work in the shadow economy (informal sector). In order to operate in the formal sector, entrepreneurs must pay a start up cost, \( \varsigma \), in the form of complying with regulations and corruption. This cost is assumed to be independent of firm output since it is an \textit{ex-ante} payment to the government. This is a barrier to become formal. De Soto (1989, 2000) has shown that this cost varies across countries and is especially high in developing countries. Firms that are legally declared also pay a uniform payroll tax, \( \tau \). Informal firms do not pay any start up costs or taxes, but pay fines in case they are detected by the tax authority. We assume that the expected punishment rate in the informal sector is a fraction of output, \( \eta y \), where
\( \eta \in [0, 1] \). This is consistent with observers (see Loayza (1996) and De Soto (2000)), who point out that large firms are more easily detected, and informal firms scale down their size to avoid detection.

### 2.2 The capital market

Agents can borrow capital from a financial intermediary with access to perfect outside capital markets, in which a risk-free bond earns a gross return of \( r \geq 1 \). Let \( l \) be the amount of funds that an agent borrows from the financial intermediary. In order to finance their projects, constrained agents must put up their initial wealth, \( b \), as collateral. Borrowers cannot commit *ex-ante* to their individual promises and can avoid the repayment obligation, \( rl \), by defaulting on their debt and loosing \( rb \). Those that renege on their debt loose the collateral and incur a cost proportional to what was produced, \( \phi y \). This is equivalent to an additive utility punishment. This cost reflects the strength of contract enforcement in the economy. Higher \( \phi \) means a better quality legal system. The point here is that, in contrast to Banerjee and Newman (1993), the quality of the project will be an important determinant of external debt. Since contracts are easily monitored in the formal sector, we assume for simplicity that \( \phi \) is zero in the informal sector. This is consistent with De Soto (2000), who points out that projects and assets in the informal sector are not adequately documented and therefore “cannot be turned into capital or cannot be used as a collateral for a loan.” Loans will be limited by the agents’ inheritance and the degree that contracts are enforced.

Notice that resource allocation involves the division of individuals among formal and informal managers and workers, and then the allocation of factors of production among managers. Occupational choices will be driven by the agent’s type, \((b, x)\), the efficiency of the capital market, \( \phi \), and government tax and regulation, \( \tau, \eta \) and \( \varsigma \).

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12 \( \eta y \) can also be seen as the costs associated with hiding information for operating in the shadow economy. De Soto (2000) argues that entrepreneurs in the informal sector pay roughly 10 to 15 percent of their annual income in bribes and commissions to authorities to avoid penalties.

13 Several studies have shown (see, in particular, Cohn and Coleman (2000)) that profitability of the firm is an important predictor of external debt, suggesting that lenders may use individual and business characteristics to evaluate projects.
3 Optimal behavior and equilibrium

3.1 Entrepreneurs

Those who have enough resources and managerial ability to become entrepreneurs choose the level of capital and the number of employees to maximize profit subject the technological constraint. Since capital markets are imperfect, let us describe the problem of an entrepreneur for a given level of capital $k$. Let $I_F$ ($I_I$) be an indicator function, which takes value one when $j = F$ ($j = I$), and zero otherwise. The problem of an entrepreneur with capital $k$ is

$$\pi_j(k, x; w) = \max_{n_j} (1 - \eta I_I) x k^\alpha n_j^\beta - (1 + \tau I_F) w n_j, \quad (3)$$

where $j \in \{F, I\}$. Equation (3) gives the labor demand of each entrepreneur in both sectors:

$$n_j(k, x; w) = \left( \frac{\beta (1 - \eta I_I) x k^\alpha}{w (1 + \tau I_F)} \right)^{\frac{1}{1 - \beta}}. \quad (4)$$

Substituting (4) into (3) yields the entrepreneur’s profit function for a given level of capital,

$$\pi_j(k, x; w) = [(1 - \beta)(1 - \eta I_I) x k^\alpha]^{\frac{1}{1 - \beta}} \left( \frac{\beta}{w (1 + \tau I_F)} \right)^{\frac{\beta}{1 - \beta}}. \quad (5)$$

In an environment with perfect enforcement, $\phi = 1$, managers will solve the following problem

$$\max_{k_j \geq 0} \pi_j(k_j, x; w) - r(k_j + \varsigma I_F). \quad (6)$$

This gives the optimal physical capital level:

$$k_j^*(x; w) = \left[ \left( \frac{\beta}{w (1 + \tau I_F)} \right)^{\beta} \left( \frac{\alpha}{r} \right)^{1 - \beta} (1 - \eta I_I) x \right]^{\frac{1}{1 - \alpha - \beta}}. \quad (7)$$

Since agents cannot commit to their promises, debt contracts must be self-enforcing. Let $a$ be the amount of capital that is self-financed (or used as collateral) and $l$ be the amount of funds raised in the outside capital market. The income from
running a project is

\[ V_j(b, x; w) = \max_{0 \leq a_j \leq b, \ l_j \geq 0} \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_{j=F}) \]  

subject to

\[ \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_{j=F}) \geq (1 - \phi \mathcal{I}_{j=F}) \pi_j(a_j + l_j, x; w) - r a_j. \]

This problem yields optimal policy functions \( a_j(b, x; w) \) and \( l_j(b, x; w) \), and we define the optimal policy function for capital as \( k_j(b, x; w) = a_j(b, x; w) + l_j(b, x; w) \). The last restriction is an incentive compatibility constraint, which guarantees that individual promises will be fulfilled (see Kehoe and Levine (1993)). We can rewrite this constraint such that

\[ l_j(b, x) \leq \left( \frac{\phi}{r} \pi_j(a_j(b, x; w) + l_j(b, x; w), x; w) - \varsigma \right) \mathcal{I}_{j=F}. \]

It can be shown that constrained entrepreneurs put their entire wealth in the project as long as \( b \leq k_j^*(x; w) \). This implies that the size of a project of an entrepreneur \((b, x)\) is

\[ k_j(b, x; w) \leq b + \left( \frac{\phi}{r} \pi_F(b + l(b, x; w), x; w) - \varsigma \right) \mathcal{I}_{j=F}. \]  

Therefore, projects are limited by the agents’ inheritance and the incompleteness of the capital market.

The following lemma summarizes the value of undertaking each project:

**Lemma 1** For any \( x \in [x, \bar{x}] \), and \( w > 0 \), the value function \( V_j(b, x; w) \), and the associated policy function \( l_j(b, x; w) \) for \( j \in \{F, I\} \) have the following properties:

1. \( V_j(b, x; w) \) is continuous and differentiable in \( x \) and \( w \). If \( x > 0 \), it is also strictly increasing in \( x \) and strictly decreasing in \( w \).

2. For \( b < k_j^*(x; w) \), \( V_j(b, x; w) \) is continuous, differentiable and strictly increasing in \( b \). For \( b > k_j^*(x; w) \), \( V_j(b, x; w) \) is constant in \( b \). Moreover, \( V_j(b, x; w) \)

\[ \text{See Appendix A.} \]
is continuous in $b = k_j^*(x; w)$.

3. For all $b$ and $x$, $l_I(b, x; w) = 0$. $l_F(b, x; w)$ is strictly increasing for $b < k_F^*(x; w)$ and $l_F(b, x; w) = 0$ for $b > k_F^*(x; w)$.

**Proof.** See Appendix B. ■

It is important to highlight the trade-offs to operate in each sector. In the informal sector entrepreneurs do not pay the payroll tax and the start up cost, but pay a cost to hide information from the tax authority, and projects are limited by the agents' initial wealth. In the formal sector, managers have access to the financial market, but have to pay taxes and costs associated with regulation and corruption.

### 3.2 Occupational choice

The occupational choice of each agent defines his lifetime income. For any $w > 0$, an agent $(b, x)$ will become an entrepreneur if $(b, x) \in E(w)$, where

$$E(w) = \{(b, x) \in [0, \infty) \times [x, \bar{x}] : \max\{V_F(b, x; w), V_I(b, x; w)\} \geq w\}. \quad (10)$$

Let $E^c(w)$ denote the complement set of $E(w)$. Obviously, if $(b, x) \in E^c(w)$, then agents are workers. Among those who are able to operate a business, they will become a formal entrepreneur if $(b, x) \in E_F(w) \subseteq E(w)$, where

$$E_F(w) = \{(b, x) \in E(w) : V_F(b, x; w) \geq V_I(b, x; w)\}. \quad (11)$$

They operate in the informal sector if $(b, x) \in E_I(w) \subseteq E(w)$, where

$$E_I(w) = \{(b, x) \in E(w) : V_I(b, x; w) \geq V_F(b, x; w)\}. \quad (12)$$

The following lemma characterizes the occupational choice for a given bequest and entrepreneurial ability.

**Lemma 2** Define $b_e(x; w)$ as the curve in the $(b, x)$ plane such that $(b, x) \in [0, \infty) \times [x, \bar{x}]$ and $\max\{V_F(b, x; w), V_I(b, x; w)\} = w$. Then there exists $x^*(w)$ such that $\frac{\partial b_e(x; w)}{\partial x} < 0$ for $x > x^*(w)$ and $\frac{\partial b_e(x; w)}{\partial x} = -\infty$ for $x = x^*(w)$. 

9
1. For all \( x \), if \( b < b_e(x; w) \), then \( (b, x) \in E^c(w) \).

2. For all \( x \), if \( b \geq b_e(x; w) \), then \( (b, x) \in E(w) \).

In addition, define \( b_s(x; w) \) as the curve in the \((b, x)\) plane such that \((b, x) \in [0, \infty) \times [x, \bar{x}] \) and \( V_F(b, x; w) = V_I(b, x; w) \).

3. For all \( x \), if \( b \geq b_e(x; w) \) and \( b > b_s(x; w) \), then \( (b, x) \in E_I(w) \).

4. For all \( x \), if \( b \geq b_e(x; w) \) and \( b \leq b_s(x; w) \), then \( (b, x) \in E_F(w) \).

**Proof.** See Appendix C. ■

Figure 1 illustrates this lemma. It shows the occupational choice in the \((b, x)\) space for the baseline economy (see parameters in section 4). Lemma 2 and figure 1 suggest that agents are workers when the quality of their project is low, i.e., \( x < x^*(w) \) (the lightest shaded area). For \( x \geq x^*(w) \), then agents might become entrepreneurs depending if they are credit constrained or not (notice that for very low bequests agents are workers even though their entrepreneurial ability is higher than \( x^*(w) \)). The negative association between \( b_e(x; w) \) and \( x \) suggests that managers with better projects need a lower level of initial wealth to run a project. This is rather intuitive since profits are increasing in the quality of the project. Given the low operational costs in the informal sector, unconstrained entrepreneurs will stay illegal. Constrained entrepreneurs will operate in the informal sector only if the premium from formalization (access to outside finance) is not high enough. Since this premium increases with the quality of the project, only high-productivity projects will operate in the formal sector (darkest shaded area). The area in between the darkest and lightest shaded areas corresponds to managers in the informal sector.

The size of the informal sector depends on the institutional and policy parameters \( \tau, \eta, \varsigma \) and \( \phi \), as well as on distribution \( \Gamma \). Two limiting cases identify the role of each parameter:

**Proposition 3** For each \( w > 0 \),

1. (Full Formalization) if \( \eta \geq 1 - \frac{w^{1-\alpha-\beta}}{(1+\tau)\eta(\bar{w}+\varsigma)^{1-\alpha}} \), then \( E_I(w) = \emptyset \);
Figure 1: Firm size distribution in the formal and informal sectors.

2. (Full Informalization) if \( \eta \leq 1 - \frac{w^{1-\alpha-\beta}}{(1+\tau)^{\beta}(w+r\varsigma)^{1-\alpha-\beta}} \) and \( \phi = 0 \), then \( E_F(b, x) = \emptyset \).

Proof. The proof follows directly from (8). For each case, just compare the net income \( V_j(b, x; w) \) in each sector.

Item 1 implies that if the cost of hiding information from the tax authority is too high and corruption is low, then agents have no incentive to hide their activities and there will be no informal activity. On the other hand, item 2 suggests that if the costs associated with the informal sector are low and debt contracts are not enforced, agents will not operate in the formal sector. If \( \eta \leq 1 - \frac{w^{1-\alpha-\beta}}{(1+\tau)^{\beta}(w+r\varsigma)^{1-\alpha-\beta}} \), but \( \phi \) is positive, then unconstrained entrepreneurs will prefer to operate in the informal sector. However, formal entrepreneurs might appear in equilibrium, since
in this sector it is possible to obtain outside finance.

3.3 Consumers

In period $t$, the lifetime wealth of an agent characterized by $(b_t, x_t)$ is given by

$$ Y_t = Y(b_t, x_t; w_t) = \max\{w_t, V_F(b_t, x_t; w_t), V_I(b_t, x_t; w_t)\} + r b_t. \quad (13) $$

Lifetime wealth is thus a function of agent-specific $b_t$ and $x_t$, and economy-wide $w_t$. Given lifetime wealth, (13), agents choose consumption and bequests to maximize preferences (1). This problem defines the optimal consumption, $c_t = c(Y_t)$, and bequest, $b_{t+1} = b(Y_t)$, policies. The functional form of (1) implies that agents leave a proportion $1 - \gamma$ of their lifetime wealth as a bequest. Notice that bequests cannot be negative because every agent is allowed to become a worker. Define $z_t = (b_t, x_t)$ and let $W_t$ be the bequest distribution at period $t$.\(^{15}\)

3.4 Competitive equilibrium

**Definition 4** Given $(\tau, \phi, \eta, \varsigma)$, $\Gamma$ and $W_t$, equilibrium at date $t$ is a list $w_t$, \(\{n_j(x; w_t)\}_{j \in \{F, I\}}, \{l_j(b, x; w_t)\}_{j \in \{F, I\}}, \{a_j(b, x; w_t)\}_{j \in \{F, I\}}, \{V_j(b, x; w_t)\}_{j \in \{F, I\}}, c_t = c(\cdot), b_{t+1} = b(\cdot), \) such that:

A. Given the wage rate and government policies, an agent of type $(b, x)$ chooses his occupation to maximize his lifetime wealth, (13).

B. $l_j(b, x; w_t)$ and $a_j(b, x; w_t)$ solve (8) for $j \in \{F, I\}$.

C. Given the lifetime wealth, (13), each agent maximizes utility, (1).

D. Given the wage rate, technology constraint, credit markets, and government policies, formal and informal entrepreneurs select their labor force to maximize profits, (3).

\(^{15}\)See the definition of $W_t$ in appendix D.
E. The Labor Market clears:

\[
\int\int_{z \in E(w)} n_F(x; w_t) W_t(db_t) \Gamma(dx_t) + \int\int_{z \in E_l(w_t)} n_I(x; w_t) W_t(db_t) \Gamma(dx_t) = (14)
\]

\[
\int\int_{z \in E^c(w_t)} W_t(db_t) \Gamma(dx_t).
\]

In the quantitative exercises it is important to evaluate policy experiments in “stable” economies, where, for instance, the real wage and income distribution are not changing significantly over time. Indeed, it is possible to show that when policies and institutions are stationary a unique steady-state equilibrium exists (i.e., an equilibrium with a constant real wage, \(w\), and invariant distribution, \(H = WT\)) and from any initial condition the economy converges to this equilibrium.

**Proposition 5** There exists a unique stationary equilibrium with \(0 < w < \infty\) and invariant distribution \(W\). In addition, for any initial bequest distribution \(W_0\) and stationary government policies and institutions \(\lambda\), the bequest distribution converges to \(W\).

**Proof.** See Appendix D. ■

In the calibration and quantitative experiments we will study the economy in this particular equilibrium and therefore we will consider the long run impact of changes in policies and institutions.

4 Quantitative results

Before we consider the quantitative results it is important to know the share of informal production relative to total output. Table 1 (column 1) shows the size of the informal economy relative to GDP for a selected set of countries.\(^{16}\) Notice that informal production is not only substantial for developing countries, but is also significant for some industrial countries, such as Italy, Portugal and Spain.

\(^{16}\)See Johnson et al. (1998), Schneider and Enste (2000), and Friedman et al. (2000) for additional data, and for an extensive discussion of the underground economy.
**Table 1**: Selected Statistics. Sources: Informal sector size is from Schneider and Enste (2000, Tables 2, 3 and 7), and Friedman et al. (2000, table 1, first column), and is total production in the informal sector as a share of GDP. Official GDP per capita is the Gross Domestic Product per capita in U.S. dollars in 1999 (World Bank, 2001). Unofficial GDP per capita is calculated using the first and the second columns above. For countries with a range of informal sector sizes, the upper limit was used. Regulation costs are from Djankov et al. (2002, table 3, column 8). They are direct costs as a fraction of GDP per capita that entrepreneurs face to meet government regulations. Efficiency of the judicial system, protection against expropriation, rule of law, and risk of contract repudiation are from La Porta et al. (1998, table 5, columns 1, 2, 4, and 5). The last column is the average from columns 5 to 8.

<table>
<thead>
<tr>
<th>Country</th>
<th>Informal sector size, % (1)</th>
<th>Official GDP per capita (2)</th>
<th>Unofficial GDP per capita (3)</th>
<th>Regulation costs (4)</th>
<th>Efficiency of judicial system (5)</th>
<th>Protection against expropriation (6)</th>
<th>Rule of law (7)</th>
<th>Risk of contract repudiation (8)</th>
<th>average enforc. (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>9.4</td>
<td>32,030</td>
<td>35,041</td>
<td>0.1000</td>
<td>10.00</td>
<td>9.67</td>
<td>10.00</td>
<td>9.31</td>
<td>9.74</td>
</tr>
<tr>
<td>Canada</td>
<td>14.8</td>
<td>19,320</td>
<td>22,179</td>
<td>0.0145</td>
<td>9.25</td>
<td>9.67</td>
<td>10.00</td>
<td>8.96</td>
<td>9.47</td>
</tr>
<tr>
<td>Germany</td>
<td>13.2</td>
<td>25,350</td>
<td>28,696</td>
<td>0.1569</td>
<td>9.00</td>
<td>9.90</td>
<td>9.23</td>
<td>9.77</td>
<td>9.47</td>
</tr>
<tr>
<td>France</td>
<td>13.8</td>
<td>23,480</td>
<td>26,720</td>
<td>0.1430</td>
<td>8.00</td>
<td>9.65</td>
<td>9.05</td>
<td>9.19</td>
<td>8.97</td>
</tr>
<tr>
<td>United States</td>
<td>10</td>
<td>30,600</td>
<td>33,660</td>
<td>0.0049</td>
<td>10.00</td>
<td>9.98</td>
<td>10.00</td>
<td>9.00</td>
<td>9.74</td>
</tr>
<tr>
<td>Belgium</td>
<td>15.3</td>
<td>24,510</td>
<td>28,260</td>
<td>0.0998</td>
<td>9.50</td>
<td>9.63</td>
<td>10.00</td>
<td>9.48</td>
<td>9.65</td>
</tr>
<tr>
<td>Portugal</td>
<td>22.1</td>
<td>10,600</td>
<td>12,942</td>
<td>0.1844</td>
<td>5.50</td>
<td>8.90</td>
<td>8.68</td>
<td>8.57</td>
<td>7.91</td>
</tr>
<tr>
<td>Spain</td>
<td>22.4</td>
<td>14,000</td>
<td>17,136</td>
<td>0.1730</td>
<td>6.25</td>
<td>9.52</td>
<td>7.80</td>
<td>8.40</td>
<td>7.99</td>
</tr>
<tr>
<td>Italy</td>
<td>26</td>
<td>19,710</td>
<td>24,834</td>
<td>0.2002</td>
<td>6.75</td>
<td>9.35</td>
<td>8.33</td>
<td>9.17</td>
<td>8.40</td>
</tr>
<tr>
<td>Argentina</td>
<td>21.8</td>
<td>7,600</td>
<td>9,257</td>
<td>0.1019</td>
<td>6.00</td>
<td>5.91</td>
<td>5.35</td>
<td>4.91</td>
<td>5.54</td>
</tr>
<tr>
<td>Brazil</td>
<td>25–35</td>
<td>4,420</td>
<td>5,967</td>
<td>0.2014</td>
<td>5.75</td>
<td>7.62</td>
<td>6.32</td>
<td>6.30</td>
<td>6.50</td>
</tr>
<tr>
<td>Peru</td>
<td>40–60</td>
<td>2,390</td>
<td>3,824</td>
<td>0.1986</td>
<td>6.75</td>
<td>5.54</td>
<td>2.50</td>
<td>4.68</td>
<td>4.87</td>
</tr>
<tr>
<td>Nigeria</td>
<td>68–76</td>
<td>310</td>
<td>546</td>
<td>2.5700</td>
<td>7.25</td>
<td>5.33</td>
<td>2.73</td>
<td>4.36</td>
<td>4.92</td>
</tr>
</tbody>
</table>
Table 2: Parameter values, baseline economy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.33</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.55</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.25</td>
</tr>
<tr>
<td>$r$</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.055</td>
</tr>
</tbody>
</table>

4.1 Parameterization

In order to solve the model numerically we have to choose a functional form for the ability distribution and assign values to the parameters of the model. We parameterized the model such that, in the stationary equilibrium, we could match some key empirical observations of the United States economy.

Table 2 summarizes the parameter values, which were determined as follows.\textsuperscript{17} We interpret the model period to be 35 years and we let $r = 2$, which implies a yearly real interest rate of roughly 2 percent. We set $\alpha$ and $\beta$ such that about 55 percent of income is paid to labor, 35 percent is paid to the remuneration of capital, and 10 percent are profits. We chose a payroll tax of $\tau = 0.33$, which is consistent with the literature ((Jones, Manuelli and Rossi, 1993)). The share of bequests in the instantaneous utility function, $1 - \gamma$, was taken to be 0.2, which is consistent with those estimated by Laitner and Juster (1996). The fraction of output that an entrepreneur can keep in case of default was set to $1 - \phi = 0.75$, which is similar to the one used by Cagetti and De Nardi (2002). Finally, we assumed that the entrepreneurial cumulative distribution function is $\Gamma(x) = Ax^{\frac{1}{\epsilon}}$ and we normalized the support of this distribution to the $[0,1]$ interval, so that $A = 1$. We chose parameters $\eta$, $\epsilon$ and $\varsigma$ such that the size of the informal sector was 10 percent, the percentage of entrepreneurs in the steady-state equilibrium was 9 percent, and regulation costs were around 0.5 percent\textsuperscript{18} of GDP per capita.

The baseline economy reproduces statistics consistent with those of the US economy, except for the income Gini coefficient (table 3). Since every worker receives the same wage in the model economy, the model income Gini coefficient

\textsuperscript{17}Appendix E contains some quantitative results using different parameterizations.
\textsuperscript{18}See table 1 column (4).
Table 3: Basic statistics, US and baseline economy. Sources: Schneider and Enste (2000), World Bank (2001) and Cagetti and De Nardi (2002); all figures in percentage.

<table>
<thead>
<tr>
<th></th>
<th>US economy</th>
<th>Baseline economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informal sector size</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Regulation and corruption costs</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Income Gini</td>
<td>40–44</td>
<td>34</td>
</tr>
<tr>
<td>% of entrepreneurs</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

should underestimate its real world counterpart. The income Gini coefficient when we consider only entrepreneurs’ income is higher in the data. For instance, data in Quadrini (1999) imply a value around 45 percent for the US economy. In our model the Gini index for entrepreneurs’ income is roughly 49 percent, which is close to the data. We therefore shall focus less on inequality and more on the other key statistics.

Figure 2 shows the distribution of capital allocated in the two sectors. It illustrates well the premium from formalization. The horizontal area (when ability is low) corresponds to those agents that are workers. As the entrepreneurial ability and bequests increase, the size of the project increases monotonically. The capital in the informal sector, however, is constrained by initial wealth, which implies that agents with different abilities end up having the same level of capital. On the other hand, in the formal sector the total debt increases with the quality of the project. This implies that, for a given level of bequest, the amount of capital employed in this sector is higher for those with better projects. As a consequence, formal entrepreneurs operate more productive technologies.

Next, we will run two quantitative experiments. In the first one (section 4.2.1) we will change corruption costs, \( \varsigma \), and the level of enforcement, \( \phi \), separately to provide their long run impact on the share of production generated outside the realm of government regulation, productivity, and percentage of entrepreneurs. In the other experiment (section 4.2.2) we will parameterize four economies based on data on corruption costs, enforcement level, and the informal sector size (see table 1) to assess how the degree of contract enforcement and corruption costs account for variations in the data.
4.2 Contract enforcement versus regulation costs

4.2.1 Experiment 1

In order to assess how changes in the degree of contract enforcement impact the model, we map linearly the last column of table 1 to $\phi$, assuming that the US case corresponds to $\phi = 0.25$ and a zero level of enforcement corresponds to $\phi = 0$. Mediterranean Europe (Italy, Portugal, Spain) has a value of $\phi$ of 0.209, while for Peru this figure is 0.13.

Table 4 shows that when enforcement decreases from the United States level to the Mediterranean Europe level, the size of the informal sector increases from 10% to 13.6%, total output falls by 6 percent, and measured output decreases by
Table 4: Policy Experiments.

<table>
<thead>
<tr>
<th>Policy Experiment</th>
<th>Informal sector size, % of baseline</th>
<th>Total output, % of baseline</th>
<th>Official output, % of baseline</th>
<th>% of entrepreneurs</th>
<th>Income Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline φ = 0.209</td>
<td>13.6</td>
<td>94</td>
<td>91</td>
<td>9.3</td>
<td>34</td>
</tr>
<tr>
<td>φ = 0.13</td>
<td>29</td>
<td>80</td>
<td>68</td>
<td>10.6</td>
<td>33</td>
</tr>
<tr>
<td>ζ = 0.011</td>
<td>26</td>
<td>94</td>
<td>82</td>
<td>9.1</td>
<td>32</td>
</tr>
<tr>
<td>ζ = 0.013</td>
<td>29</td>
<td>93</td>
<td>79</td>
<td>9.16</td>
<td>32</td>
</tr>
<tr>
<td>η = 0</td>
<td>28</td>
<td>99</td>
<td>85</td>
<td>9.4</td>
<td>31</td>
</tr>
</tbody>
</table>

9 percent. A further decrease of φ to 0.13 (Peru level) yields an increase in the size of the informal sector of 19 percentage points relative to the baseline economy, while total output falls in 20%.

Comparing this experiment to the data on table 1, we can conclude that the strength of financial contract enforcement alone cannot account for the differences in the size of the informal sector across countries. In addition, it accounts for just a small fraction of the difference in official output per capita.

From table 4 we can also observe that, as φ decreases, the fraction of entrepreneurs in the total population increases. This is a stylized fact in developing economies: the percentage of entrepreneurs over the total working population decreases with output per capita. Less developed countries tend to have more entrepreneurs but less productive entrepreneurs (Lucas, 1978b, Tybout, 2000). When, for instance, enforcement improves, the number of entrepreneurs decreases but the quality (size and productivity) of each project increases (see also Kumar, Rajan and Zingales (2004)).

We now verify whether or not variations in corruption costs, ζ, generate differences in the size of the informal sector observed in the data. We increased ζ from the baseline economy (United States) value (ζ = 0.0004), which corresponds to 0.5% of the output per capita, to 18% of this output (ζ = 0.011). This new value is the average regulation cost as a fraction of output per capita found in Mediterranean Europe. Notice that the informal sector size increases from 10% to 26%, while observed output per capita decreases by 18%. The variation in the
informal sector size is similar to that found in the data (see column (1) of table 1), but the variation in the official output per capita is smaller in the model than in the data. We further increase corruption costs to the level observed in Peru, which is roughly 20% of the output per capita($ = 0.013$). We observe that the informal sector size increases to 28% and the output per capita decreases to 79% of the United States level. For the case of Peru, corruption costs alone cannot account for the size of informal sector. Relative output per capita is again much higher in the model than in the data.

We can also verify that variations in $\eta$ can generate considerable differences in the size of the informal sector. These variations account for both changes in the probability of being caught in the informal sector and changes in the payroll tax rate.\textsuperscript{19} When its value goes from the baseline to zero, there are virtually no changes in total output, but a large impact on the size of the informal sector. When $\eta$ decreases, not only do the costs to operate in the informal sector decrease (this increases the informal sector, which has in general smaller projects), but the productivity in this sector increases. The two effects on productivity roughly offset each other.

\subsection*{4.2.2 Experiment 2}

From the statistical evidence presented in table 1, we can see that some countries (Argentina, Brazil, Nigeria, Peru) present a large informal sector (above 20 percent) coupled with a low degree of contract enforcement (below 6) and middle to low measured GDP per capita (below $10,000$); others (Italy, Portugal, Spain)\textsuperscript{20} also have strong shadow economies (above 20 percent), but show moderate levels of enforcement (above 6 and below 8.5) and high measured income (above $10,000$); and the remaining have small shadow economies (below approximately 15 percent) and high levels of income and contract enforcement. To further study the effects of financial constraints on output and the size of the shadow economy, we create four reference economies. One is the baseline case, corresponding to the US economy.

\textsuperscript{19}In the model, these two distortions are summarized by the quantity $\frac{1-\eta}{(1+\tau)^{\beta}}$. Therefore, for fixed $\beta$, $\tau$ can be ignored in the analysis by adjusting only $\eta$.

\textsuperscript{20}Greece could easily be added to the group, but some data are missing from the sources we used.
It is characterized by a very high level of income and a small informal sector, as well as the highest level of contract enforcement. The second is “Mediterranean Europe,” characterized by a high level of income, a moderate degree of contract enforcement, and a sizeable shadow economy. The third is “Western Europe,” which features a high level of income, a high level of contract enforcement, and a relatively small informal sector. Finally, we pick the Peruvian economy to characterize economies with a low level of income, a large informal sector, and a low degree of contract enforcement.

We again map linearly the last column of table 1 to \( \phi \), and assume the baseline economy, with \( \phi = 0.25 \), is the US. Given \( \phi \), which is directly calculated from the average level of contract enforcement, we assign values to parameters \( \varsigma \) and \( \eta \) so as to match the size of the informal sector and the corruption costs of the reference economies. We also compute the average GDP per capita for each reference economy.

Table 5 presents basic statistics for these reference economies, as well as results from model simulations. For Western Europe, \( \phi \) is 0.244, which is almost the US level. Mediterranean Europe has a value of 0.209, while for Peru the figure is 0.13. As for parameter \( \varsigma \), its value is lowest for the US and highest for Mediterranean Europe. In order to assess the impact of financial constraints due to the lack of contract enforceability on productivity vis à vis regulation costs, we pick one of the reference economies and simulate the model assuming that either the level of contract enforcement or corruption costs changed to the US level. We then compare the size of the informal sector and the productivity for these two hypothetical cases.

Let us first look at Mediterranean Europe. With contract enforcement equal to the US level, this economy would have an informal sector of around 17 percent, which is comparable to the 10 percent level in the US. This figure contrasts to 6 percent if the costs of corruption and regulation were set to the US level instead.

\[ 21 \text{While Portugal is strictly not a Mediterranean country, it shares many of the features of the other Mediterranean countries considered, including the size of the informal sector.} \]

\[ 22 \text{For Mediterranean Europe and Western Europe, these values are averages of the countries referred to above. We did not use Canada, but this country could have been included in the Western Europe group.} \]

\[ 23 \text{The numerical value for } \varsigma \text{ is higher for Mediterranean Europe than for Peru. However, as a fraction of output per capita the value is higher for Peru.} \]
<table>
<thead>
<tr>
<th>Region</th>
<th>$\phi$</th>
<th>$\varsigma$</th>
<th>Informal sector size, % of official output</th>
<th>Corruption, reg. costs, % of official output</th>
<th>Output per capita, % of US level</th>
<th>Official output; gap to US, perc. points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Data Official</td>
<td>Total</td>
</tr>
<tr>
<td>Baseline case</td>
<td>0.25</td>
<td>0.0004</td>
<td>10</td>
<td>0.5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Mediterranean Europe</td>
<td>0.209</td>
<td>0.0107</td>
<td>24</td>
<td>18.6</td>
<td>55</td>
<td>62</td>
</tr>
<tr>
<td>Financial constraint</td>
<td>0.25</td>
<td>0.0107</td>
<td>17</td>
<td>16.2</td>
<td>88</td>
<td>94</td>
</tr>
<tr>
<td>Corruption and regulation</td>
<td>0.209</td>
<td>0.0004</td>
<td>6</td>
<td>0.5</td>
<td>99</td>
<td>95</td>
</tr>
<tr>
<td>Western Europe</td>
<td>0.244</td>
<td>0.0086</td>
<td>13</td>
<td>12.5</td>
<td>81</td>
<td>83</td>
</tr>
<tr>
<td>Financial constraint</td>
<td>0.25</td>
<td>0.0086</td>
<td>12</td>
<td>12.3</td>
<td>94</td>
<td>96</td>
</tr>
<tr>
<td>Corruption and regulation</td>
<td>0.244</td>
<td>0.0004</td>
<td>4</td>
<td>0.5</td>
<td>106</td>
<td>100</td>
</tr>
<tr>
<td>Peru</td>
<td>0.13</td>
<td>0.0075</td>
<td>60</td>
<td>19.9</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Financial constraint</td>
<td>0.25</td>
<td>0.0075</td>
<td>18</td>
<td>11.2</td>
<td>89</td>
<td>96</td>
</tr>
<tr>
<td>Corruption and regulation</td>
<td>0.13</td>
<td>0.0004</td>
<td>24</td>
<td>0.8</td>
<td>71</td>
<td>81</td>
</tr>
</tbody>
</table>

Table 5: Empirical data and simulation results for reference economies. Mediterranean Europe comprises Italy, Portugal and Spain; Western Europe comprises Belgium, Denmark, France and Germany.
In terms of productivity, as measured by official GDP per capita, the unaltered model predicts 77 percent of the US level. Setting the financial barrier equal to its US value, measured output per capita would be 88 percent of the US level, whereas this value would be 99 percent if the regulation barriers were set to the US level. These results suggest that, for this group, the financial barrier, although important, seems to have a smaller impact on both productivity and the size of the informal sector than regulation and corruption costs. Changing regulation and corruption costs to the US level, however, tends to generate an informal sector size that is too small relative to the US. Results are quite similar for Western Europe.

For Peru, the picture is somewhat different: with a level of contract enforcement equal to the US level the model predicts: i) a reduction in the size of the informal sector from 60% to 18%; and ii) the relative official output per capita increases from 50% to 89% of the US level. When regulation costs are set to the US level, the informal sector decreases from 60% to 24%, while relative output per capita increases from 50% to 71%. The same conclusions are drawn for countries with a relatively low degree of enforcement. Differences in the financial constraint therefore seem to generate large differences in both productivity and the size of the informal sector for these economies than do regulation costs. This is consistent with De Soto’s (2000) argument: countries with poor contract enforcement and ineffective financial systems would benefit a lot if their entrepreneurs could effectively use their assets as collateral.

4.3 TFP differences

We now focus on the question of whether differences in contract enforcement and regulation costs between countries account for the differences in output per capita and productivity across countries.

Parente and Prescott (2000) show that the gap in output per capita among rich and poor countries is explained by differences in TFP and not by factor accumulation. Their theory, which is based on technology adoption, shows that inside groups with vested interests block the adoption of more advanced technologies
and explains the use of inferior production processes. In this model, differences in TFP across countries arise endogenously due to differences in regulation costs (barriers to legality) and the enforcement system (legal failures). Looking at the last two columns of table 5 we can assess the relevance of those two factors in explaining TFP differences across countries. For Mediterranean Europe, the gap between measured output per capita and the US level is 45 pp. The model accounts for a gap of 23 pp, which leaves another 22 pp to be explained by other factors that affect TFP, such as vested interest. For Western Europe, the gap to US measured output per capita is 19 pp, of which 8 is accounted for by the model. Finally, for Peru the 49 pp of the gap in output per capita is explained by corruption costs and contractual imperfections, while the remaining 43 pp are explained by other factors affecting TFP. In sum, the model explains roughly half of the TFP differences observed between the US measured output per capita and the other reference economies. These results are still valid if we consider other countries. Our results are thus complementary to those of Parente and Prescott (2000), since corruption costs and credit market imperfections can account for just part of the differences in output per capita across countries.

5 Concluding remarks and policy implications

This paper contributes to the literature by characterizing how government policies and institutions interact with the distribution of wealth and entrepreneurial ability in a general equilibrium model with formal and informal sectors, corruption, and contractual imperfections. Formal entrepreneurs have better access to outside finance. The quantitative exercises suggest that:

i. Regulation costs rather than limited enforcement in financial contracts account for a larger part of the difference in the size of the shadow economy between Mediterranean Europe and the United States.

ii. In countries with very weak enforcement, credit market imperfections rather

\[^{24}\text{This clearly could be extended to explain the presence of institutions that inhibit economic development. Improvements in the judicial system might be blocked by inside groups which benefit from a poor judicial system.}\]
than corruption explain a larger part of the difference in the size of the informal sector. Corruption costs, however, still explain an important part of this difference.

iii. The productivity gains from improving financial contract enforcement and from decreasing bureaucracy and corruption to the United States level are sizable. Output per capita in Mediterranean Europe and Peru would increase by 23% and 43%, respectively.

In order to investigate the policy implications of the model it is important to understand the real counterpart of parameters $\varsigma$ and $\phi$. Parameter $\varsigma$ measures the barriers to legality. It corresponds to costs and procedures to comply with government requirements to start a business. These costs deter entrepreneurs from becoming formal. Djankov et al. (2002), for instance, show that in Italy entrepreneurs on average need to follow 16 procedures, pay US$ 3946 in fees, and wait roughly 62 business days to acquire legal status. In Canada, however, the process needs only 2 procedures, takes two days and costs around US$280 in fees. Parameter $\phi$, on the other hand, is a proxy for legal failures in financial markets. When financial markets operate poorly, capital is misallocated and lucrative investment opportunities are forgone. According to De Soto (2000), capital markets fail in developing countries for two main reasons: i) the majority of residents in developing countries do not have the title of their property and therefore cannot use their asset as collateral for a loan to convert it into capital - notice that this is related to parameter $\varsigma$; and ii) even when their assets are formally registered their properties and financial contracts are not well enforced.

Policies should therefore simplify the entry process, decrease bureaucracy and provide titles to informal properties. Besides this, a set of complementary institutional reforms to secure properties and debt contracts are needed, such as: reforms of bankruptcy laws, banking regulation, and judiciary and enforcement systems. Our quantitative experiments suggest that policies that decrease regulation and improve the functioning of credit markets might have important effects on the size of the informal sector and productivity. These policies, however, are not easy to implement since they will change the status quo of politicians and bureaucrats who are the main beneficiaries from high regulation costs (see Djankov et al. (2002))
and a weak legal system.

Our model could also be used to investigate related questions. One of them is the long run effects of tax reforms on tax evasion (size of the informal sector) and productivity. Intuitively, for instance, a tax on capital income might not only decrease capital accumulation, but it can also have an effect on the occupational choice of the agents, preventing agents from becoming formal entrepreneurs. We leave this and other questions for future research.

References


A  Kuhn-Tucker conditions for problem (8)

The Lagrangean associated with problem (8) is

\[ L_j = \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma) + \lambda_j[l_j + \varsigma] + \chi_j[b - a_j]. \]

The Kuhn-Tucker conditions are:

\[ \frac{\partial L_j}{\partial l_j} = \pi_j(a_j + l_j, x; w) - r + \lambda_j[l_j + \varsigma] \leq 0, \quad (15) \]
\[ \frac{\partial L_j}{\partial a_j} = \pi_j(a_j + l_j, x; w) - r + \lambda_j[l_j + \varsigma] - \chi_j \leq 0, \quad (16) \]
\[ \lambda_j[l_j + \varsigma] = 0, \quad (17) \]
\[ \chi_j[b - a_j] = 0, \quad (18) \]
\[ l_j \geq 0, \quad \frac{\partial L_j}{\partial l_j} l_j = 0, \quad a_j \geq 0, \quad \frac{\partial L_j}{\partial a_j} a_j = 0, \quad \lambda_j \geq 0, \quad \chi_j \geq 0, \]

along with the incentive compatible constraint and the upper limit on \( a_j \). If the entrepreneur is credit constrained, \( \lambda_j > 0 \), that is, he would be better off if the credit constraint were eased. Notice first that, from (17), \( l_j = 0 \). We know that \( \pi_{F1}(k_F^*(x; w), x; w) = r \) and \( \pi_{F1}(a_F + l_F, x; w) \) is decreasing with \( l_F \). Notice that \( a_F + l_F \leq k_F^*(x; w) \), since \( k_F^*(x; w) \) is the unconstrained optimal level of capital. Then, equation (16) implies \( \chi_F > 0 \), which implies by (18) that \( a_F = b \).
B Proof of Lemma 1

Continuity of $V_j(b, x; w)$ follows from the Maximum Theorem and differentiability from Theorem 4.11 of Stokey and Lucas (1989). From the envelope theorem it is easily seen that, provided $x > 0$,

$$V_{j2}(b, x; w) = \pi_j(b + l_j(b, x; w), x; w)(1 + \lambda_j \phi I_j = F) > 0,$$

$$V_{j3}(b, x; w) = \pi_j(b + l_j(b, x; w), x; w)(1 + \lambda_j \phi I_j = F) < 0,$$

If $b \leq k^*(x; w)$, then

$$V_{j1}(b, x; w) = \pi_j(b + l_j(b, x; w), x; w)(1 + \lambda_j \phi I_j = F) > 0.$$

When $b > k^*_F(x; w)$, then by definition of $k^*_F(x; w)$ the net income from entrepreneurship cannot increase and $V_{j1}(b, x; w) = 0$. $l_F(b, x; w) = 0$ since there is no borrowing in the informal sector. For $b > k^*_F(x; w)$ it is also obvious that $l_F(b, x; w) + a_F(b, x; w) = k^*_F(x; w)$. When agents are credit constrained, the incentive compatible constraint holds with equality and

$$\phi_F(b + l_F(b, x; w), x; w) = r(l_F(b, x; w) + \varsigma).$$

Thus,

$$\frac{\partial l_F(b, x; w)}{\partial b} = \phi_F(k_F, x; w) - \frac{\phi_F(k_F, x; w)}{r - \phi_F(k_F, x; w)}.$$

By condition (15), we have that $r - \phi_F(k_F, x; w) = \frac{\pi_F(k_F, x; w) - r}{\lambda_F}$. Since this is for constrained agents, $\lambda_F > 0$ and, as we have seen previously, $\pi_F(k_F, x; w)$ is greater than $r$. Therefore,

$$\frac{\partial l_F(b, x; w)}{\partial b} = \lambda_F \frac{\phi_F(k_F, x; w)}{\pi_F(k_F, x; w) - r} > 0.$$
C Proof of Lemma 2

If agents have sufficiently high $b$ and

$$\max_{j \in \{I,F\}} \{V_j(b, x; w)\} \geq w,$$

there is $x^*(w)$ such that for $x < x^*(w)$ agents prefer to be workers rather than managers:

$$x^*(w) = \min_{j \in \{I,F\}} \left\{ \left( \frac{r}{\alpha} \right) \alpha \left( \frac{w(1 + \tau I_{j=F})}{\beta} \right)^\beta \left( \frac{w + r I_{j=F}}{1 - \alpha - \beta} \right)^{1-\alpha-\beta} \frac{1}{1 - \eta I_{j=I}} \right\}.$$ 

$x^*(w)$ is independent of $b$. For constrained agents with $x \geq x^*(w)$, we have that $\max_{j \in \{I,F\}} \{V_j(b, x; w)\} = w$ defines $b_e(x; w)$, such that

$$\frac{\partial b_e(x; w)}{\partial x} = -\frac{V_{F2}(b, x; w)}{V_{F1}(b, x; w)} - \frac{V_{I2}(b, x; w)}{V_{I1}(b, x; w)},$$

where $j = \arg \max_{j \in \{I,F\}} \{V_j(b, x; w)\}$, in all points where $b_e(x; w)$ is differentiable. This is negative from Lemma 1.

Define

$$G(b, x; w) = V_F(b, x; w) - V_I(b, x; w).$$

Provided $G_1(b, x; w) \neq 0$, by the implicit function theorem $G(b, x; w) = 0$ defines $b_s(x; w)$, where

$$\frac{\partial b_s(x; w)}{\partial x} = -\frac{V_{F2}(b, x; w) - V_{I2}(b, x; w)}{V_{F1}(b, x; w) - V_{I1}(b, x; w)}.$$ 

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We have
\[
V_{F2}(b, x; w) - V_{I2}(b, x; w) = x^{\frac{1}{1-\beta}} \left( \frac{\beta}{w} \right)^{\frac{\beta}{1-\beta}} \times \\
\left[ \left( \frac{(b + l)^{\alpha}}{(1 + \tau)^{\beta}} \right)^{\frac{1}{1-\beta}} (1 + \lambda_F \phi) - (b^\alpha(1 - \eta))^{\frac{1}{1-\beta}} \right]
\]
\[
V_{F1}(b, x; w) - V_{I1}(b, x; w) = \alpha x^{\frac{1}{1-\beta}} \left( \frac{\beta}{w} \right)^{\frac{\beta}{1-\beta}} \times \\
\left[ \left( \frac{(b + l)^{\alpha}}{(1 + \tau)^{\beta}} \right)^{\frac{1}{1-\beta}} \frac{(1 + \lambda_F \phi)}{b + l} - \frac{(b^\alpha(1 - \eta))^{\frac{1}{1-\beta}}}{b} \right].
\]

Notice that \( V_F(b, x; w) \geq V_I(b, x; w) \) implies that
\[
x^{\frac{1}{1-\beta}} \left( \frac{\beta}{w} \right)^{\frac{\beta}{1-\beta}} \left[ \left( \frac{(b + l)^{\alpha}}{(1 + \tau)^{\beta}} \right)^{\frac{1}{1-\beta}} - (b^\alpha(1 - \eta))^{\frac{1}{1-\beta}} \right] \geq \frac{r(l + s)}{1 - \beta} > 0.
\]

Since \( \lambda_F \geq 0 \), this implies that the numerator of \( \frac{\partial b_s(x; w)}{\partial x} \) is always positive for \( V_F(b, x; w) \geq V_I(b, x; w) \). For a given \( b, l \) increases and \( \lambda \) decreases with \( x \), which implies that for sufficiently high \( x \) the denominator is negative and \( \frac{\partial b_s(x; w)}{\partial x} \) is positive (see Figure 1).

\section{D Proof of Proposition 5}

First we need to show that, for every bequest distribution, there exists a finite equilibrium wage rate that clears the labor market. Given the bequest and ability distributions, \( W \) and \( \Gamma \), define the excess demand function \( ED(w) \) by
\[
ED(w) = \int \int_{z \in E_F(w)} n_F(x; w)W(db)\Gamma(dx) \quad (19) \\
+ \int \int_{z \in E_I(w)} n_I(x; w)W(db)\Gamma(dx) - \int \int_{z \in E^c(w)} W(db)\Gamma(dx).
\]

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The excess demand $ED(w)$ is continuous since both $n_j(x; w)$ and $V_j(b, x; w)$ are continuous in $w$ (see equation (4) and Lemma 1). In addition, $n_j(x; w)$ and $V_j(b, x; w)$ are also strictly decreasing in $w$. Notice that as $w$ goes to zero, no agent wants to become a worker, $V_j(b, x; w)$ is unbounded and $ED(w) > 0$. Analogously, when $w$ increases, then $ED(w) < 0$. Therefore, by continuity of $ED(w)$ there must be some $w^*$ such that $ED(w^*) = 0$.

It remains to show that $w^* \in [w, \overline{w}]$, where $w > 0$ and $\overline{w} < \infty$. Let us consider an initial bequest distribution that assigns zero bequest to all agents. Set $E_F(w)$ is the measure of all agents for sufficiently small $w$ as long as $\phi > 0$. In this case, the equilibrium wage rate, $w$, is positive and finite, as stated in the previous paragraph. Since the wage rate is positive, next periods’ bequests will all be positive. Therefore, the set of possible occupational choices cannot shrink, and might even expand. This implies that for the previous wage rate $w$, the excess demand is nonnegative, $ED(w) \geq 0$, which in turn means that for this new bequest distribution the wage rate that clears the labor market is $w' \geq w$. Consequently, $w > 0$ is the lowest equilibrium wage rate for any initial distribution.

Now suppose an initial bequest distribution that assigns $\overline{b}_0$ to all agents such that $\overline{b}_0 \geq k^*_j(x; w)$. By the first argument in this proof, there exists a positive and finite equilibrium wage rate, $\overline{w} < \infty$. In this case, no agent is credit constrained. Either the smallest bequest, $(1 - \gamma)(\overline{w} + r\overline{b}_0)$, is higher than $\overline{b}_0$, in which case the next periods’ equilibrium wage rate will be the same; or it is smaller than $\overline{b}_0$ and the set of occupation choices might shrink. Therefore, for this new wealth distribution $ED(\overline{w}) \leq 0$. In this case the new equilibrium wage rate is $w' \leq \overline{w}$.

We can thus conclude that $w_t \in [w, \overline{w}]$ for all $t$. The maximum possible bequest is thus $\overline{b}$ such that

$$\overline{b} = (1 - \gamma)(\max_{j=1}^{k^*_j(x; w)} + r\overline{b}), \quad (20)$$

where we assume that $(1 - \gamma)r < 1$. On the other hand the minimum bequest is

$$\underline{b} = (1 - \gamma)(w + r\underline{b}) \quad (21)$$

Define $Z = [\underline{b}, \overline{b}]$ and $z_t = (b_t, x_t)$. $Z$ is compact. Define the measurable
space \((Z, \mathcal{B})\), where \(\mathcal{B}\) is the Borel algebra for the set. Define \(\Lambda(Z, \mathcal{B})\) as the set of all possible probability measures defined on the measurable space \((Z, \mathcal{B})\). For instance, \(W_t\), which specifies the probability of each event in \(\mathcal{B}\) at time \(t\), belongs to \(\Lambda(Z, \mathcal{B})\). Measure \(W_t\) defines a non-stationary transition probability function, 

\[
P_t(b_t, A) = \Pr\{b_{t+1} \in A | b_t\},
\]

for any \((b_t, A)\) in \((Z, \mathcal{B})\). Function \(P_t\) assigns a probability to event \(A\) for the descendant of an agent that has bequest \(b_t\) but does not know yet \(x_t\). We want to show that the operator \(T^* : \Lambda(Z, \mathcal{B}) \rightarrow \Lambda(Z, \mathcal{B})\) defined as

\[
(T^*W_t)(A) = \int P_t(b_t, A)W_t(db_t),
\]

where \(P_t\) is the transition function defined above, has a unique fixed point \(T^*W = W\) for any Borel subset \(A \in \mathcal{B}\), given the initial bequest distribution \(W_0\). \((T^*W_t)(A)\) can be interpreted as the probability that the next period’s state lies in \(A\) according to the present period’s distribution. Of course, \(T^*W_t = W_{t+1}\). Notice first that \(w_t\) is well defined for every distribution \(W_t\), as we argued previously. Second, we know that \(b_{t+1} = h(z_t; w_t)\), where \(h(z_t; w_t) = (1-\gamma)Y(z_t; w_t)\) (see equation 13), is increasing in \(z_t\) for any \(w_t\), and \(Z\) is compact. Operator \((Tf)(b_t) = \int f(b_{t+1})P_t(b_t, db_{t+1})\), defined for any bounded function \(f : B(Z) \rightarrow B(Z)\), where \(B(Z)\) is the set of real-valued bounded functions defined on \(Z\), is the conditional expectation of function \(f\) at \(t+1\) given that the state at \(t\) is \(b_t\). Since, for any wage rate \(w_t \in [w, \bar{w}]\), \(h(z_t, u_t)\) is bounded and increasing in \(b_t\), and \(x_{t+1}\) is independent of \(b_t\), the conditional expectation of \(f(b_{t+1})\) on \(b_t\) is also increasing and bounded provided that \(f\) is increasing. Intuitively, this means that, given the equilibrium wage rate \(w_t\), an agent’s descendant would never be worse off in terms of the expected value of \(b_{t+1}\) if, for any \(\varepsilon > 0\), the agent’s state were \(b_t + \varepsilon\) instead of \(b_t\). As function \(Tf\) is increasing, \(T^*\) is increasing and \(P_t\) is a monotonic transition function.\(^{25}\) By Corollary 2 of Hopenhayn and Prescott (1992), there is a fixed point for map \(T^*\).

It remains to show that \(P_t\) satisfies the Monotone Mixing Condition (MMC). First, define \(P_{t+n}(b_t, A) = \Pr\{(b_{t+n}) \in A | b_t\}\). This is the \(n\)-step transition function

beginning at \( t \). We must show that the transition function \( P_{t+n} \) satisfies, for all \( t \),

\[
P_{t+N}(\bar{b}, [b_a, \bar{b}]) > \epsilon \quad \text{and} \quad P_{t+N}(\bar{b}, [\bar{b}, b_a]) > \epsilon
\]

for some \( b_a \in Z \), \( \epsilon > 0 \), and \( N \in \mathbb{N} \). Let us, for simplicity and without loss of generality, omit subscript \( t \). Let \( w \) be the wage rate associated with the fixed point of map \( T^* \), \( W \). Define the minimum stationary bequest \( b_l \) such that \( b_l = (1-\gamma)(w + rb_l) \). Let \( b_a = (1-\gamma)(w + rb_l) + \varrho \) for some small \( \varrho > 0 \). We now show that there is a positive probability that the \( N^{th} \) descendent of an agent with \( b = \bar{b} \) receives a bequest above \( b_a \). Notice first that the agent’s descendants will have bequest in the vicinity of \( b_l \) in finite time because they will all be workers. Since the measure of sets \( E(w) \) and \( E^c(w) \) is non-zero and constant (as the labor market clears with wage in \( [w, \bar{w}] \)), and ability is independent across generations, there is a positive probability that a worker becomes entrepreneur and vice-versa. Suppose that agents with ability in the vicinity of \( \bar{x} \) and bequest in the vicinity of \( b_l \) cannot have descendants that become entrepreneurs. Since all agents’ descendants face a positive probability of having bequest in the vicinity of \( b_l \) in finite time (as they can have successive low \( x \)’s), this implies that the measure of agents (workers) in the vicinity of \( b_l \) is 1, a contradiction to the fact that \( E(w) \) has non-zero measure. Therefore, agents with ability in the vicinity of \( \bar{x} \) and bequest in the vicinity of \( b_l \) have descendants that become entrepreneurs. Moreover, they can become so in the following generation. This implies that they can also have bequest higher than \( b_a > b_l \) as long as they have a sufficiently high \( x \), in which case they have high credit limits. Starting from \( b = \bar{b} \) is easier: a succession of low \( x \)’s leaves the agent’s descendants with bequest lower than \( b_a \), as they will become workers and remain so until one of them gets a sufficiently high \( x \). Therefore, by Theorem 2 of Hopenhayn and Prescott (1992), there exists a unique time invariant distribution \( W \) and associated equilibrium wage \( w \), such that from any initial distribution \( W_0 \), the operator \( T^*W_t \) converges to \( W \).
Table 6: Basic statistics, changes in parameters relative to the baseline.

<table>
<thead>
<tr>
<th>Baseline</th>
<th>Informal sector size, % output</th>
<th>Total output per capita, % of baseline</th>
<th>Official output per capita, % of baseline</th>
<th>% of entrep.</th>
<th>Income Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>10</td>
<td>100</td>
<td>100</td>
<td>9</td>
<td>34</td>
</tr>
<tr>
<td>Lifespan = 45</td>
<td>21</td>
<td>91</td>
<td>84</td>
<td>9.1</td>
<td>31</td>
</tr>
<tr>
<td>$\gamma = 1$</td>
<td>0</td>
<td>97</td>
<td>107</td>
<td>6.0</td>
<td>40</td>
</tr>
<tr>
<td>$\gamma = 0.7$</td>
<td>46</td>
<td>106</td>
<td>80</td>
<td>8.8</td>
<td>27</td>
</tr>
<tr>
<td>$\phi = 0$</td>
<td>-</td>
<td>65</td>
<td>0</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>$\phi = 0.3$</td>
<td>7.4</td>
<td>106</td>
<td>109</td>
<td>8.1</td>
<td>33</td>
</tr>
</tbody>
</table>

E Sensitivity analysis

This model does not display wage inequality. This could be introduced by means of a “working ability” that would differentiate among workers. This inequality, however, would simply reflect the randomness of working productivity, which is not important to our purposes. We could have increased inequality by increasing the curvature of the ability distribution (i.e., parameter $\epsilon$). The quantitative exercises are roughly the same for a parameterization with higher inequality (income Gini in range 0.40–0.44).\(^{26}\)

Table 6 shows some quantitative results when we change other parameters of the model. The second row shows the impact of increasing the lifespan to 45 years. The number of entrepreneurs does not change, whereas, as expected, output falls 9 percent.

The third row displays the case where agents are not altruistic. The effect is large on all variables except output. The informal sector is null because there is no bequest in this economy and every entrepreneur needs outside resources to undertake a project. Notice, however, that with $\gamma = 1$ and $\phi = 0$ the economy would collapse because everybody would be credit-constrained. In this case, financial constraints could explain any difference in the size of the informal sector and on output across countries. But this is a rather extreme case. The fourth row shows the results for a higher propensity to leave bequest. Output is higher

\(^{26}\)For the sake of space, we omit these results but we can provide them upon request.
because agents are less credit constrained and as a consequence productivity increases. Notice that the existence of equilibrium requires that $\gamma > 1 - 1/r$. The model therefore displays some sensitivity to parameter $\gamma$, but there is no reason to assume that the altruism degree varies across countries.

The last two rows of table E consider two extreme cases: one with an enforcement such that the entrepreneur keeps only 70 percent of the assets in case of default (against 75 percent in the baseline), and another with no enforcement of debt contracts. The informal sector size varies from 7.4 percent of measured output to full informalization, while output per capita varies from 106 to 65 percent of the baseline economy. This confirms our previous findings that credit market policies can account for the differences in the size of the informal sector, especially for countries with low enforcement, but just part of the differences in output per capita.\footnote{When $\phi$ goes from 0 to 0.13, total output per capita increases 79 pp, while when it goes from 0.13 to 0.25 total output increases by only 21 pp.}