Abstract.
This paper proposes a model of economy with weakly non-separable preferences for both work effort and consumption. Households who derive utility from consumption of single commodity and leisure take into account the habitual dependency of their utility on both labour supply and consumption in the past. As a result, the model provides analysis of the effects of labour income and consumption taxes increase on the asset holdings, consumption and labour supply of households. We show that with non-distortionary taxation of either labour income or consumption, habits will have different effects on the macroeconomic variables in presence of composite habits, than in the case of consumption habits alone. Three regimes are distinguished and discussed depending on the underlying parameters of the model. In the presence of comprehensive habits, we show that the equivalence between consumption tax and income tax no longer holds in contrast with the benchmark model. In addition, unlike traditional habits models, ours is capable of capturing the time-persistence qualities of both consumption and leisure.

Key Words: Habits, Consumption, Leisure, Income Tax

JEL Classification: E10, E21, E62
Introduction.

In recent years, we have seen a resurgence of interest in the issue of time-inseparability of consumption. Following empirical research, many macroeconomists have used habit formation as a tool for developing models of consumption and savings behaviour that provided a ‘better fit’ for the data. In particular, habits were shown to be useful in resolving several major puzzles in macroeconomics. Alongside the work of Abel (1990), Constantinides (1990) uses habitual inseparability in consumption to resolve the equity premium puzzle in asset price models. Similarly, Gali (1994) resorts to habits in consumption to explain several puzzles with portfolio choice and asset pricing models. Carroll et al (2000) developed a habit formation model to explain the empirically documented reversal of causality between savings and growth relative to the traditional models of economic growth. Earlier, Muellbauer (1988) and Dynan (1992) have shown that habits in consumption are successful in explaining the hump-shaped delayed response of consumption to changes in income. Fuhrer (1999) addresses the role of habits in explaining the excess smoothness puzzles in consumption, inflation and spending. Last, but not least, Carroll (1999) shows that contrary to the traditional permanent income hypothesis, empirically low marginal propensity to consume out of income shocks can be captured by the habits in consumption.

Across the habits literature, the main idea of habit formation remains the same. The habitual consumption stock accumulated over the past history of consumption by a representative agent has a direct effect on consumers’ current utility. This may involve either the inward- or the outward-looking habits, as discussed in depth in Carroll et al (2000). The agents may care about the aggregate economy-wide stock of habits (as in case of outward-looking agents) or only concern themselves with the ratio of their own consumption to their own habits (as in
case with the inward-looking habits mechanism). Depending on their type, consumers wish to smooth ether both the level and the growth rate in consumption over time or only the level of their consumption. Hence, in response to an exogenous income shock, consumption will adjust gradually to a new steady state.

The robustness of the theoretical predictions of habit formation models in consumption to the utility function and the choice of laws of motion specifications is a feature well established in the literature. Mansoorian (1996) shows that under a set of general assumptions on the utility function given by Ryder and Heal (1973), a general model of habit formation in consumption will exhibit saddle path stability in steady state solutions. Similar results are shown in Gurdgiev (2003) paper that introduces inward-looking habits in leisure.

The central feature of the majority of habit formation models in consumption is the ad-hoc nature of labour supply. Following the tradition established by Boyer (1978), the majority of papers on habits in consumption are based on the assumption of inelastic labour supply. As a result, changes in economic policy or other exogenous shocks cannot be discussed in the context of labour/leisure trade-off.

On the other hand, a few papers such as Faria (2001) and Graham (2003) discuss the implications of including elastic labour supply considerations into the model with habitual consumption. Faria (2001) looks at the inward-looking habits mechanism in consumption and its effects on leisure in the standard neoclassical growth model. Graham (2003) shows that in dynamic simulations the traditional habit formation models extended to include the endogenous labour supply perform equally poorly with respect to both capturing the persistence of leisure and consumption as the standard RBC models. This result, albeit not
discussed analytically, indicates that a new approach to the way we interpret habits is warranted in order to capture the dynamics of endogenous leisure.

To the best of our knowledge no literature currently comprehensively discusses the role of leisure in determining the macroeconomic behaviour of the agents in presence of habits in consumption and leisure taken either separately or jointly. As argued in Gurdgiev (2003), labour supply exhibits strong degree of persistence in response to economic shocks. It is a commonly known fact (see for example discussion in Ehrenberg and Smith (1982), and McConnell and Brue (1995)) that empirical adjustments in labour supply to exogenous income and wage shocks do not follow jump discontinuity dynamics. Instead leisure demand and labour supply both adjust gradually in response to exogenous shocks. This warrants the approach of the present paper to incorporate directly habituality of leisure into the household optimisation problem. In broader terms, as pointed out by Becker (1965) and others, leisure may be at least partially inseparable from consumption. Hence, if past consumption patterns are important to the determination of future consumption plans, past leisure choices may also have a direct impact on the responses in leisure to changes in the economic environment.

The following paper aims to close several important gaps in the habit formation literature. To begin with, we propose a straightforward extension of the habitual consumption model to include explicit consideration of the elastic labour supply decisions by the consumers. Assuming, at first, that leisure enters utility function independent of habits we develop our first model. As expected, time separability of leisure implies that while consumption exhibits gradual adjustment to income tax change, leisure demand and thus labour supply exhibit jump discontinuity in their adjustment paths. This part of the study, therefore supplies closed end solution result that confirms Graham (2003) simulated solutions. We treat this model as a benchmark.
In addition to incorporating explicitly labour supply decisions into the standard model of habit formation in consumption, the first model presented below contributes to the literature by providing explicit solutions for the real variables responses to changes in labour income and consumption tax policies. To the best of our knowledge, this discussion is new to the literature on habits in general.

The benchmark model, as shown hereinafter, fails to account for the gradual adjustment in labour supply. Hence we develop a model with habits arising from both consumption and leisure histories of the household. We show that in this model dynamic responses in both leisure and consumption are linked to each other and to the exogenous economic environment described by the parameters of the model. Implications of these links are discussed in the context of the households’ asset holdings, consumption, leisure and habits stock dynamics. The main results of the model with comprehensive habits are further enhanced by the discussion of the effects of tax policy changes on the real variables. Once again, both the model of comprehensive itself and the subsequent discussion of the tax effects in the model are entirely new to the literature.

In addition to motivate our emphasis on the combined (comprehensive) habits in leisure and consumption we consider two major reasons that, in general, make time-inseparability of consumption and leisure relevant to the issue of taxation. First, time additivity of either one of the components of household choice implies constant rate of time preferences. As shown in Sen and Turnovsky (1989) this makes the steady state analysis of tax policies extremely sensitive to the initial conditions. One way of solving it is through the introduction of endogenous time preferences, as done, for example, in Shi (1994). Another approach is to introduce habit dependency. Carroll (2000) shows the benefits of the latter approach which implies that along the adjustment path to the new steady state households’ time preferences
are not constant but are dependent on the habit formation parameters. Second, in time-separable models, wealth and the substitution effects of the income tax changes cannot be separated. This point is raised in Barro and King (1984). As the result of this, the relative changes in consumption and leisure due to changes in after-tax wage must be equal to the relative response caused by wealth change.

In recent work, Becker, Murphy and Werning (2002) discuss the importance of status, as a separate variable in household optimisation. Our model of ‘comprehensive’ habitual good closely follows their reasoning with respect to the preferences specification. In the Becker et al (2002) model, status is separate from a consumption good. As such there is a trade-off across consumption and status in household decisions. This implies that status raises the marginal utility of consumption and of income. In our model, marginal utility of consumption is increasing in habits. Traditional habits in consumption literature explains this by observing that for two agents with identical preferences who differ in their habits stock levels, the agent with the highest habitual reference stock will enjoy greater marginal utility of consumption. In addition to this traditional effect, our model suggests another link between habits and the marginal utility of consumption. By making habits stock dependable on leisure, we establish a complimentarity link between leisure and the marginal utility of consumption that acts in a similar fashion to the idea of status goods.

Another aspect of Becker et al (2002), is that the status good is characterised by a relatively fixed-supply nature. Traditional habit in consumption models do not have a similar feature with regards to habits stock. In our model, habits stock is determined at any period of time by both consumption and leisure. As consumption and leisure may vary in the opposite directions, comprehensive habits stock may not be as flexible as consumption itself. Thus our model of comprehensive habits allows for a closer relationship between habits role as either a
reference good or a status good. This is inherent in the distinction between internal and external habits, yet is not directly addressed by consumption habits alone.

This paper is organised as follows. Part 1 develops a model of habits in consumption in the presence of elastic labour supply. This provides a basis for future analysis of the implications of incorporating leisure into the mainstream habit formation models. Part 2 builds on the preceding model to develop a comprehensive model of adjacent complementarity in consumption and habits combined with the weak separability of consumption and leisure in preferences. Part 3 concludes the discussion of the two models by comparing various tax effects on the steady state levels of real variables, and by examining the effects of the habit formation parameters on the models predictions.

**Part 1. A Model of Leisure Demand in the Presence of Habits in Consumption.**

Following the intuition presented in the Introduction, we develop a representative agent model of habits in consumption in the presence of elastic labour supply. This provides a basis for future analysis of the implications of incorporating leisure into the mainstream habit formation models. As mentioned above, traditional habits in consumption literature assumes inelastic labour supply. As the result of this assumption, along the adjustment path, changes in consumption are associated with no labour-leisure trade-off. Model 1 below introduces explicit analysis of the effects of habits in consumption on dynamics of labour supply and discusses the interactions between labour supply, consumption and foreign asset holdings of the households in response to the exogenous shocks to income (tax policy changes).
1.1. General Solution.

Consider an economy with a single consumption good, \( C_t \). In labour markets, infinitely lived households face a fixed real wage rate \( w \) and supply \( 1-l \) units of labour. Labour income is taxed at the rate \( \tau_l > 0 \) while consumption is taxed at rate \( \tau_c > 0 \). Tax proceeds are rebated in a lump-sum fashion, with tax rebate denoted by \( T_t \). Let \( (1-\tau)w(1-l) \) be a given level of labour income after the income tax. We can express the tax revenue rebated by the government as

\[
T_t = \tau w(1-l) + \tau c C_t
\]  

(1)

As usual, households take \( T_t \) as exogenously given in the optimisation problem.

Agents maximise their lifetime utility from leisure and consumption accounting for consumption habits stock:

\[
U = \int_0^\infty U \left(C_t, h_t, l_t \right) e^{-\delta t} dt
\]  

(2)

subject to the budget constraint relating agents’ holdings of foreign bonds, \( B_t \), tax payments and transfer receipts (\( T_t \)):

\[
\dot{B}_t = rB_t + w(1-\tau_l)(1-l) - (1+\tau_c)C_t + T_t
\]  

(3)

In equation (2) we separate consumption and habits stock terms \( \left(C_t, h_t \right) \) from the demand for leisure term in order to capture the idea that in Model 1 consumption and leisure are weakly separable in the utility and that habits effects apply to consumption component of the utility function alone. As mentioned in the introduction above, this model will serve as a benchmark model for future analysis.
Following Ryder and Heal (1973), we assume that:

\[ U_C > 0 \text{ and } U_{CC} < 0 \]
\[ U_l > 0, \text{ and } U_{ll} < 0 \]
\[ U_h < 0 \text{ and } U_{hh} < 0 \]
\[ U_{hh} = U_{cl} = 0 \]
\[ U_{hh}U_{CC} - U_{Ch}^2 \geq 0 \text{ (concavity)} \]
\[ U_C + U_h > 0 \]
\[ U_{Ch} > 0 \]  

(4)

The assumptions presented in (4) are important in the context of comparing the benchmark model against the following extension. First, these assumptions support the analysis presented in Ryder and Heal (1973) with respect to the existence of a stable steady state. Second, these assumptions with respect to the separability of leisure and consumption parallel Hansen and Wright (1992) and Faria (2001). Finally, the last assumption reflects the arguments presented in both Mansoorian (1993) with respect to the traditional role of habits and in Becker, Murphy and Werning (2000) with respect to the status-like nature of a habitual standard of living. The latter aspect of these assumptions is important to the understanding of the following extension of the benchmark model to Model 2 below, the intuition for which is discussed in the Introduction.

Since labour tax payments are rebated in a lump-sum fashion, agents take \( T_r \) as given. This, in conjunction with the fact that marginal rate of substitution in consumption and leisure in period \( t \) are independent of their levels at time \( j \neq t \), implies that \( U(\ldots) \) is homothetic.

Furthermore, the assumption that the utility function is weakly separable in habits and leisure implies that, in our model, habits only serve as a reference point for the utility of consumption and do not distort the utility of leisure. This is standard for models with habit
formation in consumption. It also implies that preferences specified under (4) exhibit the adjacent complementarity property so that a change in the current consumption has the same direction effect on the future marginal utility discounted forward from today.

The assumed additivity of leisure and consumption in preferences can be justified on the following grounds.

First, using past choices of consumption as a reference point alone without distorting the marginal utility of leisure implies that the strength of habits will affect leisure demand only in so far as labour supply can be used by households to maintain a habitual standard of living in consumption. Thus stronger habits are now needed in order to generate the desired sensitivity of consumption than in the model with exogenous labour supply. This is the point made in the numerical results obtained by Graham (2003).

Second, in so far as there are no empirical tests of the persistence in leisure at the aggregate and individual levels, separation of consumption habits and demand for leisure eliminates the need for resorting to the arbitrary parameterisation of the model in its leisure component. The problem here is that while there exists extensive literature on acceptable parameterisation of the habits-in-consumption models, little or no empirical guidance currently can be found on parameterisation of the endogenous leisure components. This problem is especially pronounced in the context of the comprehensive habits model, i.e. model 2 below. In this model, neither the strength of habits-in-leisure in utility parameter, nor parameters on the speed of habits in leisure convergence and the relative weight of leisure in the law of motion for comprehensive habits can be justified empirically.
Third, the current specification, once extended to cover leisure in addition to consumption, as shown in model 2, allows us to reconcile habitual leisure with habits-like behaviour of aggregate consumption and the possibility for lower persistence in individual consumption vis-à-vis leisure.

Prior to considering the dynamic program based on equation (3), we must impose the following transversality conditions. The first condition rules out the possibility of unbounded borrowing by the economy at large. As standard,

$$A_t = \frac{w_t}{r} + B_t \geq 0$$

Since the right-hand-side (hereinafter, RHS) of inequality (5) involves both bonds and labour supply, $A_t$ is fully endogenous to the decision making by households. Hence, in the following optimisation program we can alternatively consider the budget constraint to be a function of the aggregate resources available to the household, namely $A_t$. Mansoorian (1993) takes this approach. However, we restrict our attention to households currently participating in the labour force, so that $l_t > 0$. Under this restriction, optimisation with respect to $A_t$ is equivalent to optimising with respect to $B_t$ and $l_t$. Subsequently we define the current value Hamiltonian in terms of these two variables.

Furthermore, we shall impose terminal conditions that rule out corner solutions for consumption and leisure:

$$\lim_{c \to a0} U_c (C_t, h_t; l_t) = \lim_{l \to a0} U_l (C_t, h_t; l_t) = \infty$$

$$\lim_{c \to a0} \left[ U_c (C, C; l_t) + U_b (C, C; l_t) \right] = \infty$$
In equation (3), the utility function depends on both leisure expenditure and stock of habits in consumption \((h_t)\). We assume that agents are endowed with some initial stock of habits \((h_0)\) and that habits stock evolves according to

\[
\dot{h}_t = \lambda (C_t - h_t)
\]  

(6)

In so far as \(\lambda\) captures the speed of adjustment of habits stock to consumption, setting \(\lambda = 0\) implies that in our model habits do not matter (their stock remains static relative to consumption demand). As \(\lambda \to 0\), consumption demand decisions of agents are less and less influenced by their initial endowment of habits \((h_0)\). In this case, households will be able to maintain different levels of habits stock and consumption even in the steady state. On the other hand, if \(\lambda \to 1\), agents reach their steady state level of consumption demand nearly instantaneously and their stock of habits has the weakest effect on their consumption demand. Once again, as standard, the ratio of consumption to habits along the steady state path will evolve so that \(c_t / h_t \geq 1\). Thus in the steady state, \((c/h)_{ss} = 1\).

The current value Hamiltonian specification for agents is given by:

\[
H_{CV} = U(C_t, h_t; l_t) + \mu_t \left[ rB_t + w(1 - \tau_t)(1 - l_t) - (1 + \tau_t)C_t + T_t \right] + \xi_t \lambda (C_t - h_t)
\]  

(7)

Note that equation (7) implies that habits are internal to the household decision making. An alternative to this assumption is to represent habits by an exogenous reference variable outside the choice variables of the agents. The two alternative specifications are discussed in depth in Carroll, Overland, Weil (1994), as well as in Gurdgiev (2003). However, from equation (7) it is clear that in absence of habits in the optimisation set, the model will be reduced to a single differential equation for asset holdings and the exogenously determined law of motion for aggregate habits stock. In this case, the dynamic multiplier on habits component of the current value Hamiltonian will be always zero. This in turn will mean that
the model solutions below will correspond to the case of a stable equilibrium that is qualitatively similar to the results of the model provided hereinafter.

Furthermore, to keep both models 1 and 2 tractable we omit consideration of the tax on income from the foreign assets held by the households. In part, analysis of this additional source of tax policy can be viewed in the context of interpreting the real interest rate, \( r \), as the net-of-tax rate of return. While interesting in general, tax on income arising from assets is most pertinent in the context of the wealthier households, while consumption and labour income taxes are by far more general in the breadth of their incidence.

Solutions to this model will follow closely along the lines of the methods presented in Mansoorian (1993), with two major exceptions. Mansoorian’s (1993) and later models do not include consideration of either income or consumption taxes presented below, nor do they include an endogenous determination of labour supply. In addition, in contrast with Mansoorian (1992, 1993, 1996), we solve the model in terms of direct utility function optimisation instead of the two-stage optimisation of an indirect utility function. The technical details of the derivations of the results presented below are relegated to the Appendix 1 for model 1, and to the Appendix 2 for model 2.

From (7), the first order conditions for the optimum are:

\[
U_c = \mu_r (1 + \tau_c) - \xi \lambda
\]  
(8)

\[-U_h + \xi (\lambda + \delta) = \bar{\xi},\]  
(9)

\[U_i = \mu_r w (1 - \tau_i)\]  
(10)

\[\mu_i = \mu_r (\delta - r)\]  
(11)
Clearly, in the steady state, $\dot{\mu} = 0$, as long as we assume:

$$\delta = r \quad (12)$$

In addition to the terminal conditions (5) we impose the following transversality conditions expressed as a function of the model multipliers. To ensure that the economy is on the steady state path, as consistent with (5), let

$$\lim_{t \to \infty} e^{-\delta t} \delta \zeta = 0$$
$$\lim_{t \to \infty} e^{-\delta t} \delta \mu = 0$$

Linearising first order condition (10) around the steady state and using equation (12) and assumptions (4) we have:

$$\left( l - T \right) = 0 \quad (13)$$

By equation (13) as predicted by the standard consumption habits model, gradual adjustment in consumption implies that leisure and thus labour supply act as jump-discontinuous variables that fully adjusts to the exogenous shocks at the impact. This is the effect that gives the same results as in Graham (2003). Following negative real after-tax income shock, according to Graham (2003) we can anticipate an increase in labour supply that fully offsets the impact of the changes in income. In absence of financial assets, households will fully adjust their leisure to smooth consumption. Thus future changes in consumption should be financed by changes in the asset positions of the households.

As shown in Appendix 1 below, first order conditions (8)-(12) imply that

$$\begin{bmatrix}
\dot{h} \\
\dot{e}
\end{bmatrix} = A \begin{bmatrix}
h_t - \bar{h} \\
e_t - \bar{e}
\end{bmatrix} \quad (14)$$
where
\[
a_{11} = -\lambda \frac{U_{cc} + U_{ch}}{U_{cc}} < 0 \quad \quad \quad \quad \quad a_{12} = -\frac{\lambda^2}{U_{cc}} > 0
\]
\[
a_{21} = \frac{U_{hb}^2 - U_{ba} U_{cc}}{U_{cc}} > 0 \quad \quad \quad \quad \quad a_{22} = r - a_{11} > 0
\]

(15)

Hence, saddle-path stability, which requires that \( \text{det} A < 0 \), is assured.

To derive solutions to the system given in (14), let \( \phi < 0 \) be a negative eigenvalue of \( A \), so that
\[
\phi = \frac{1}{2} \left[ r - \sqrt{r^2 - 4 \text{det} A} \right]
\]

(16)

By definition (14) and (15), and assumptions (4), \( \phi < 0 \) exists. Furthermore, by assumptions (4) and as shown in the Appendix 1:
\[
-\phi < \lambda \quad \iff \quad \lambda > -r \left[ \frac{U_{hb} + 2U_{ch}}{U_{ch}} \right]
\]

(17)

We express condition (17) in terms of the absolute magnitude of the negative eigenvalue \( \phi \) relative to the positive value of the speed of habits stock adjustment to the steady state. Such exposition is consistent with Mansoorian (1993).

Technically inequalities in (17) imply that there exist two different regimes with respect to the variables of choice response to changes in the rates of taxation. Whenever the marginal disutility of habits is strongly responsive to changes in the habitual stock relative to the speed at which habits stock adjusts towards the steady state level, the negative eigenvalue of the process in system (14) dominates the speed of growth in the habits stock. The converse applies when the marginal disutility of habits is weakly responsive to changes in habits stock along the steady state path.
Note that by equations (15) and (16), since there are no cross effects of habits on leisure, 
\( \phi < 0 \) can be interpreted as a rate of growth in the marginal utility of income due to the speed of adjustment in habits. When \( U_{Ch} \) is relatively high, so that habits have a strong effect on the marginal utility of consumption, marginal utility of income is strongly influenced by habits as well. In this case, \( -\phi < \lambda \), so that the speed at which consumption catches up with habits is above the speed at which marginal utility of income falls with the increase in the habits stock.

Standard form solutions for (14) are then given by the stable arm equations:

\[
\begin{align*}
(h_t - \bar{h}) &= (h_0 - \bar{h}) e^{\phi} \\
(\xi_t - \bar{\xi}) &= -(h_0 - \bar{h}) e^{\phi} \left( \frac{\phi U_{CC} + \lambda (U_{CC} + U_{Ch})}{\lambda^2} \right)
\end{align*}
\]  
(18), (19)

Using equations (18) and (19) we can solve for the asset holdings of households. Linearise equation (3) around the steady state, using equation (13):

\[
\dot{B}_t = r(B_t - \bar{B}) + \Omega (h_0 - \bar{h}) e^{\phi} \\
\Omega = -\frac{\phi + \lambda}{\lambda} \left(>0 \quad \Leftrightarrow \quad -\phi \gg \lambda \right)
\]  
(20)

In the above, inequality on \( \Omega \) arises from the assumptions (4) and inequalities (17).

Equation (20) provides a solution for the steady state deviation in asset holdings by households and the steady state differences in the habits stock:

\[
\bar{B} - B_0 = \frac{\Omega}{\phi - r} (\bar{h} - h_0) \left(>0 \quad \Leftrightarrow \quad -\phi \gg \lambda \right)
\]  
(21)
Note that by equations (20) and (21) households’ asset positions depend on the level of income taxation. This provides the analysis shown below. In addition, part 3 of the paper, and the Appendix 3 below discuss the effects of the model parameters in the context of consumption, leisure and foreign asset holdings adjustments.

In so far as our benchmark model predicts, we retain herein the main features of the traditional habit formation in consumption models. When consumption in the recent past becomes more important in reference to future consumption, $\lambda$ rises and the optimal consumption for the future periods decreases. At the same time, as the importance of habits in the utility function increases (so that $U_{ch}$ is positive as assumed in (4) above), the optimal consumption increases as well in order to maintain habitual standard of living.

Also, by inequalities (21), household bond holdings do respond to changes in wages and consumption habits parameters. Outside the steady state, increase in the habits endowment or decrease in the habits stock steady state value (so that $\left(h_0 - \bar{h}\right) \uparrow$) will lead to a faster growth rate in assets, as $B_t$ falls. This occurs whenever the speed of habits convergence is low compared to the effect of habits on the marginal disutility of habits relative to their effect on the marginal utility of consumption. Hence, agents with a greater initial habits stock in consumption will tend to have slower growing asset positions, and subsequently a higher volume of bond holdings whenever the adjustment of habits stock to the steady state is slower. Since the steady state level of habits converges to that of consumption by (6), we have the link between the initial level of consumption and the asset accumulation behaviour of the households.
This link arises due to the negative effect of the habits stock on intertemporal utility of consumption. Agents care about their income in two ways. First, income yields utility via consumption, second via demand for leisure. The second component is linked to the source of income. Income arising from savings (bonds) is not ‘taxed’ in terms of habits disutility and neither in this model is the labour income subject to habitual inertia. Hence, agents with stronger habits in consumption will fully substitute away from leisure in favour of savings (substitution effect). Secondly, *ceteris paribus*, agents with higher initial stock of habits will tend to allow lower variation in bond holdings away from the steady state (depth effect).

The impact of the speed at which habits stock catches up with consumption ($\lambda$) and the strength of habit formation effect in preferences ($U_{ch}$) on asset holdings by households and leisure will be further discussed in the following sub-section and in Part 3 below.

### 1.2. Effects of Tax Changes: General Solutions.

To consider the effects of changes in income and consumption taxes, we differentiate equations (21), (6), (3), (1), (8), (9) and (10) around the steady state which, with minor manipulations, yield:

\[
\frac{d\bar{B}}{\phi - r} = \frac{\Omega}{\phi - r} \bar{d} \bar{h}
\]

(22)

\[
d\bar{B} = \frac{1}{r} \left[ wd\bar{T} + d\bar{h} \right]
\]

(23)

\[
d\bar{h} = \frac{r + \lambda}{U_{hh} + U_{hc}} d\bar{T}
\]

(24)

\[
d\bar{T} = -\frac{\mu w \bar{T} + d\bar{\tau}}{U_{il}}
\]

(25)
\[ dh = \frac{d \tau_c \bar{\mu} - \lambda d \xi}{U_{cc} + U_{hc}} \]  

(26)

Observe that in equation (22): \[ \frac{\Omega}{\phi - r} >, < 0 \iff -\phi <, > \lambda \iff \Omega <, > 0 \]

This system of equations can be solved for the effects of changes in steady state level of habits stock in consumption as shown in Appendix 1 to yield:

\[ \frac{d \bar{h}}{d \tau_c} = \frac{\bar{\mu} d \tau_c}{U_{cc} + \frac{2 \lambda + r}{\lambda} U_{ch} + U_{hb}} \]  

(27)

Finally from the first order condition (10) we have:

\[ \bar{\mu} = \frac{U_i}{w(1 - \tau_f)} \]  

(28)

Equations (22)-(28) allow us to solve for the effects of changing the rate of taxation on the variables of choice.

1.3. Effects of Consumption Tax Change.

Starting as standard from the initial setting where both tax rates are originally at zero, suppose a government imposes a permanent change in the consumption tax, so that \( d \tau_c > 0 = d \tau_f \). Using equations (22)-(28) we can now solve for the resulting effects of consumption tax increase on the variables of choice. As shown in the Appendix 1:

\[ \frac{d \bar{h}}{d \tau_c} = \frac{d \bar{C}}{d \tau_c} = \frac{U_i}{w(1 - \tau_f)} \left[ U_{cc} + \frac{2 \lambda + r}{\lambda} U_{ch} + U_{hb} \right] < 0 \]  

(29)

\[ \frac{d \bar{T}}{d \tau_c} = -\phi (r + \lambda) \frac{d \bar{h}}{d \tau_c} > 0 \]  

(30)
\[
\frac{dB}{d\tau_c} = \frac{\Omega}{\phi - r} \frac{d\tilde{h}}{d\tau_c} >, < 0 \iff -\phi >, < \lambda
\]  

(31)

In the case of habitual consumption alone, we can (by using equation (31)) distinguish two possible environments. In the first case, whenever habits move to the steady state level at a sufficiently high speed, an increase in the consumption tax implies disposing of the household financial wealth along the transition path. However, when habits are slow to adjust, households accumulate financial wealth along the transition path. Intuitively, the costs of higher taxation net of leisure adjustments can be borne by either lower consumption today or lower consumption in the future. Since consumption is sluggish due to habits, while leisure adjusts instantaneously, the adjustments in consumption needed to compensate the household for higher taxation burden will be borne by the financial assets. When habits are fast to adjust to the new steady state, as is standard in consumption habits literature, consumption adjustments are insufficiently strong in the short run as consumers are less willing to postpone current consumption in favour of future consumption. As a result over time, consumers continue to lower their consumption and thus generate the need for extra smoothing through lowering of their asset holdings along the way. The converse applies to the case of slowly adjusting habits.

We thus distinguish two cases:

Case A: \(-\phi > \lambda\)

Case B: \(-\phi < \lambda\)

Figure 1 below illustrates the adjustment processes for leisure, consumption and asset holdings of the households facing an increase in consumption tax in Case A. The details of solutions for dynamics are given in the Appendix 1.
Leisure and labour supply act as jump-discontinuous variables in response to changes in the income tax. Under the assumptions above, an increase in consumption tax rate will have an effect of raising leisure at the impact level to the higher steady state level. The resulting decrease in labour supply by the households allows for downward overshooting at the impact in consumption. As labour income of the households falls due to dual effects of real after-tax wage decline and the contraction in labour supply, household asset holdings remain intact at the impact. Consumption of households initially falls below the new lower steady state level and then proceeds to gradually adjust upward toward the steady state. Asset holdings of the households are complemented by savings from overshooting the new steady state level of consumption. Hence, lower consumption demand fully absorbs the shock to income resulting from a higher consumption tax. As consumption approaches the new steady state gradually while labour supply instantaneously falls to the new steady state level, the surplus savings generated by the wedge between the post-impact consumption levels and the new steady state are absorbed by the households into higher holdings of assets. The economy continues to accumulate assets as it moves towards the new steady state.

Figure 2 below illustrates the case B. When habits adjust to a new steady state level at a relatively high speed, the negative impact of the more distant consumption decisions on the marginal utility of consumption is diminished. Agents, therefore, are less willing to adjust their consumption at the impact. The reason for this is that at the impact, the households are more inclined to maintain their habitual standard of living determined before the changes in consumption tax rate. This implies that households are willing to draw down their savings in order to finance incomplete adjustment of consumption toward the lower steady state. Such
partial reduction of consumption at the impact finances, in part, the immediate upward jump 
adjustment in leisure. Over time, consumption continues to decline toward the new steady 
state. The reduction in the asset position of households augments this decline to fully finance 
greater demand for leisure in the new steady state.

Figure 2. ABOUT HERE.

With the exception of explicitly incorporating labour supply decisions into the household 
optimisation problem, the results presented above are consistent with the general literature on 
habit formation in consumption (see, for example Mansoorian, (1993)). In simple terms, as 
with any exogenous shock to real income, an increase in the consumption tax acts to amplify 
the importance of habits in consumption in the overall decision of the agents. This results in a 
slower adjustment in consumption to the new steady state. The substitution effect in leisure 
continues to operate in the model, but is now magnified by the habits effects of rising 
consumption along the adjustment path to a new lower steady state. Hence, changes in labour 
supply and leisure are more pronounced in the model with habits in consumption than in the 
model where consumption is allowed to adjust instantaneously. This is counterintuitive, given 
that in general labour supply adjustments are slower in response to changes in income than 
the adjustments in consumption. Gurdgiev (2003) presents a summary of some evidence in 
favour of an argument that leisure adjusts slower than consumption. In this case, in the 
context of our model, the asset holdings of households act as the main shock absorber.

As shown in the Appendix 1, the effects of an income tax change on the variables of choice 
in this model are identical in direction to the effects of consumption tax changes discussed 
above. This result confirms theoretical RBC models that show the equivalence of a 
consumption and labour income taxes in the standard models with no habit formation (e.g.
see Judd (1985), Chamley (1986), and Milesi-Ferretti and Roubini (1998)). In general, consumption tax introduces a singularly important effect in household decision making: the trade-off between labour supply and leisure is tilted by a rise in the consumption tax rate in the direction of the greater demand for leisure. Outside the production decisions involving productive capital accumulation by the agents, the income tax has the same effect.

Yet, the endogenous growth literature (see Jones, Manuelli and Rossi (1993) for an excellent discussion) disputes this qualitative equivalence principle. So does the majority of the empirical studies regarding asset positions and investment decisions of households. These show that investment (in our model – asset positions) is commonly found to be negatively correlated with labour income tax and positively correlated with consumption tax. For example Mendoza, Milesi-Freretti and Asea (1997) show the regularity for a panel of nineteen OECD economies.

Hence, due to the counterintuitive results of the benchmark model mentioned above, it is warranted to explore other possible applications of the habit formation mechanism. The objective of such exploration is to account for both persistence of consumption and even stronger persistence of labour supply in the household responses to the exogenous income shocks given by the changes in tax rates.

To address the criticism supplied in the conclusions drawn from the solutions to Model 1 above, we examine a model of preferences that are history-dependent in all components of choice. The reason for this is that we want a model that would be able to account for sluggish dynamic adjustments in both consumption and leisure.

By analogy with Model 1 above, restricting leisure to be a habitual good, while allowing consumption to change without any reference to past choices will simply yield a model in which consumption is jump-discontinuous. In this regard, whenever habits stock arises from a single choice variable (either consumption or leisure, but not both simultaneously), the habitual variable will exhibit gradual adjustment. The variable that is independent of habits will act as a jump-discontinuous shock absorbing variable with complete adjustment to the new steady state at the impact. This can be seen from the first order conditions (10) and (11) in case when habits are formed in consumption alone.

Clearly, in order to achieve the desired dual persistence in both choice variables (consumption and leisure), it is important to introduce history-dependence in both of these variables simultaneously. With this in mind, we shall extend the earlier model of preferences to include the adjacent complementarity in both consumption and leisure. This link between consumption and leisure in habits stock determination generates the main difference between Models 1 and 2.
2.1. General Solution.

For simplicity, assume that consumption and leisure contribute to the stock of comprehensive habits. We assume that the law of motion for comprehensive habits stock follows the same process as in the earlier model, so that

\[ \dot{h}_i = \lambda_c \left[ \eta C_i + (1 - \eta) w l_i - h_i \right] \]  

(32)

By equation (32), the habits stock is a weighted average of the complete history of choice variables. Note that the model predictions do not change qualitatively if we replace the real expenditure with the level of leisure in equation (32). At the same time, given our assumption that the price of consumption goods is set at 1 we can interpret the stock of habits described by the law of motion (32) as referenced to both types of expenditure instead of levels of consumption and leisure. Thus overall, relative scaling of variables employed in equation (32) is of little analytical importance in the model. We can, therefore, interpret equation (32) as a law of motion for habitual stock determined by the total expenditure by the households.

In this context, $h_i$ is household income, net of foreign asset holdings.

Treating habits as a function of real expenditure, rather than levels of consumption and leisure allows us to solve the problem of different scaling of the two components of utility function. Since we restrict time endowment at unity while the price of consumption is assumed to be 1, specifying habits in terms of real expenditure on both consumption and leisure as measured in terms of consumption goods allows us to directly capture the possibility for the existence of a partial complementarity of leisure and consumption. When households trade away from consumption in favour of leisure they may choose to do so either by demanding more leisure hours or by switching away from pure consumption goods in favour of the leisure-complimentary goods. In the latter case, total leisure expenditure of the
household will rise even when the leisure hours demanded may stay fixed. The complementarity of consumption and leisure can, in our model, be captured without explicit separation of consumption goods and leisure-complementing goods by considering the habits in the real expenditure on consumption and leisure, rather than in the levels of consumption and leisure.

At the same time, this presents an interesting case for future analysis vis-à-vis the effects of price changes of leisure and consumption. For example, suppose that consumption and leisure expenditures form a signal of status of the agent. Then both the habitual standard of living and status standing of individuals will be altered in response to changes in prices and wages. As a result of this, in the models with explicit analysis of changing prices, higher price of consumption implies that as households are interested in smoothing both the habit-forming consumption and leisure, a lower weight will be placed by the households on keeping the leisure component of habit stock smoothed. This in turn means that agents will be more willing to adjust their leisure spending in response to the exogenous income shocks. On the other hand, higher wage rates will be associated with a greater weight of leisure in determination of the overall habitual stock, so that leisure adjustments will be more sluggish. Some evidence shows that in the environment of high price inflation, agents are willing to incur greater costs in obtaining consumption to maintain their standard of living, at the expense of supplying more labour. Thus, for example, in 1920’s Germany, women would commonly abandon both their labour and leisure activities in order to convert their husbands’ earnings into consumption goods.

These effects will remain unexplored in the present paper, offering an interesting avenue for inquiry in the future work. However, the link between the leisure expenditure and the consumption expenditure in our model can be partially investigated by the consideration of
the effect of parameter $\eta$ that captures the effect of consumption expenditure relative to leisure expenditure on the adjustments of consumption, leisure and foreign assets position, as discussed in Part 3 below and in the Appendix 3.

In general, the adjacent complementarity of leisure and consumption in the law of motion for habits, as well as the comprehensive habits specification linear dependence on both choice variables, are similar to the Ryder and Heal (1973) specification. In the Ryder and Heal model, the habitual standard of living is a weighted average of past choices of money balances and consumption, while utility function is weakly inseparable across real balances and consumption. In our model, the spirit of their specification is preserved with respect to leisure and consumption acting as joint determinants of the habitual reference stock.

To specify preferences allowing to account for the combined effects of consumption and leisure on habits stock, assume as in Model 1, that utility function is separable across consumption and leisure:

$$U(C_t, l_t ; h_t) = \nu^1(C_t, h_t) + \nu^2(l_t) + \nu^3(h_t)$$

where $\nu^3(h_t)$ captures the direct effects of habits on the utility of leisure.

The corresponding Ryder-Heal assumptions in addition to those listed in (4) are:

$$U_{l_t} > 0, \text{ and } U_{ll} < 0$$
$$U_{lh}^2 - U_{ll}U_{hh} \leq 0$$
$$U_{l_t} + U_{h_t} > 0$$

Finally the budget constraint (3) and the rebate identity (1) continue to hold as before. Once again, our assumptions allow for a broad range of utility functions, including those covered in Faria (2001). The strong assumption that preferences are separable in terms of habit
component in leisure, so that $U_{lh} = 0$, is maintained here for the reasons of analytical simplicity. Since the cross effect of the habits on leisure should be positive, as argued for the same effect in consumption, it would simply reinforce the effect of the consumption-habits link in the results below.

The current value Hamiltonian for the household optimisation problem is now given by:

$$
H_{CV} = U(C_t, l_t; h_t) + \mu_t \left[ rB_t + w(1 - \tau_i)(1 - l_t) - (1 + \tau_c)C_t + T_t \right] + \\
+ \xi_t \lambda_t \left( \eta C_t + (1 - \eta)wl_t - l_t \right)
$$

(34)

The first order conditions for optimisation are:

$$
U_C = (1 + \tau_c) \mu_t - \xi_t \lambda_t \eta
$$

(35)

$$
-U_h + \xi_t (\lambda_t + \delta) = \xi_t
$$

(36)

$$
U_l = \mu_t w(1 - \tau_i) - \xi_t \lambda_t w(1 - \eta)
$$

(37)

$$
\mu_t = \mu_t (\delta - r)
$$

(38)

As in Model 1 earlier, assumption (12) must be satisfied. In addition we impose the following transversality conditions:

$$
\lim_{t \to \infty} e^{-\delta t} \xi_t h_t = 0
$$

$$
\lim_{t \to \infty} e^{-\delta t} \mu_t A_t = 0
$$

where $A_t$ is defined in equation (5).

Solving the model as shown in the Appendix 2 we get:
\[
\begin{bmatrix}
\dot{h} \\
\dot{\xi}
\end{bmatrix}
= A_c \begin{bmatrix}
h_t - \bar{h} \\
\xi_t - \bar{\xi}
\end{bmatrix}
\]  \hspace{1cm} (39)

where

\[a_{11} = -\lambda_c \eta U_{ch} + U_{cc} < 0 \quad \quad a_{12} = -\frac{\lambda_c^2}{U_{cc} U_{ll}} \left( (1-\eta)^2 w^2 U_{cc} + \eta^2 U_{ll} \right) > 0\]

\[a_{21} = \frac{U_{ch}^2 - U_{cc} U_{hh}}{U_{cc}} > 0 \quad \quad a_{22} = r - a_{11} > 0\]  \hspace{1cm} (40)

Note that by equations (40), the square of trace of \( A \) is greater than \( 4 \det A_c \), satisfying saddle path stability conditions as stated in Chiang (1984). Furthermore, \( \mod \left( \det A_c \right) > 1 \) under assumptions (4) and (33) so that \( \mod \left( \phi_c \right) \neq 1 \). This implies that the basin attraction for the saddle point is large, as stipulated in Kozlowski, et al (2001)

Hence, with \( \det A_c < 0 \), the saddle-point stability is automatically assured, while the solutions to the system are given as follows. Consider the system of two differential equations given by (39) and (40). The negative eigenvalue of this system is given by:

\[\phi_c = \frac{1}{2} \left[ r - \sqrt{r^2 - 4 \det A_c} \right]\]  \hspace{1cm} (41)

Then

\[\lambda_c, < - \phi \quad \Leftrightarrow \quad \lambda_c, < r \frac{\eta U_{ch} U_{ll}}{w^2 (1-\eta)^2 \left( U_{ch}^2 - U_{cc} U_{hh} \right) - \eta^2 U_{ll} U_{hh} - 2 \eta U_{ch} U_{ll}}\]  \hspace{1cm} (42)

Inequality (42) implies that in our specification preferences in under conditions (33) exhibit the adjacent complementarity property defined by Ryder and Heal (1973) in terms of the composite choice of leisure and consumption.

Linearising the budget constraint:
\[
\dot{B}_t = r \left( B_t - \bar{B} \right) + \Omega_c \left( h_0 - \bar{h} \right) e^{\phi t}
\]  
(43)

where:

\[
\frac{w^2 \lambda (1-\eta)^2 U_{ch} - \eta U_{il} (\phi + \lambda)}{\lambda_c \left[ \eta^2 U_{il} + w^2 (1-\eta)^2 U_{cc} \right]} = \Omega_c <, > 0 \quad \iff \quad
\begin{cases}
Case A: -\phi < \lambda \\
Case B: -\phi > \lambda, \text{ and } 0 < \frac{\eta \phi U_{il}}{w^2 (1-\eta)^2 U_{ch} - \eta U_{il}} < \lambda \\
Case C: -\phi > \lambda, \text{ and } 0 < \lambda < \frac{\eta \phi U_{il}}{w^2 (1-\eta)^2 U_{ch} - \eta U_{il}}
\end{cases}
\]  
(44)

Note that case B corresponds to the situation where the marginal utility of consumption is strongly affected by the changes in habits stock, while case C corresponds to the case where \( U_{ch} \) is relatively low in comparison with the speed of habits stock adjustment to the steady state. This is similar to the discussion following equation (17) in Model 1.

Setting \( \eta = 1 \) gives consumption all weight in habits stock. In this case, we attain the same results as shown in Model 1. Alternatively, setting \( \eta = 0 \) yields results for a model where leisure is the only habitual good. In the latter case, \( \Omega_c < 0 \) unambiguously. As a result, by equation (35), consumption acts as a jump variable. Under the assumption of zero first order effects of habits on the marginal utility of leisure, only consumption and habits are linked through a second order effect. Thus asset holdings of the household will always countermove with leisure habits whenever no habitual consumption is built into the model. The short run effects of tax policy changes will be absorbed by consumption, while the long run effects will be at least partially checked by drawing down the household’s financial assets.
2.2. Effects of Tax Changes: General Solutions.

As in the earlier model, the following equations determine the effects of income tax change on the variables of choice:

\[
d\bar{B} = \frac{\Omega_c}{\phi - r} \, d\bar{h}
\]

\[
d\bar{h} = \frac{M_2}{M_1^+} \, d\tau_c + \frac{M_3}{M_1^-} \, d\tau_i
\]

where

\[
M_1 = \frac{[U_{cc} + \eta U_{ch}][(1-\eta)^2 w^2 \lambda U_{ch} - \eta^2 (\lambda + r) U_{ii} ] - \lambda w^2 (1-\eta)^2 U_{cc} - \lambda \eta^2 U_{ii} }{(1-\eta)^2 w^2 \lambda U_{ch} - \eta^2 (\lambda + r) U_{ii} } > 0
\]

\[
M_2 = \tilde{\beta} = \frac{U_c}{1+\tau_c} - \frac{\lambda \eta U_h}{(\lambda + r)(1+\tau_c)} = \frac{U_i}{1-\tau_i} + \frac{\lambda w(1-\eta) U_h}{(\lambda + r)(1-\tau_i)} > 0
\]

\[
M_3 = \frac{wM_2}{1-\eta} \left[ \frac{w^2 (1-\eta)^2 \lambda U_{ch} + (\lambda + r) \eta^2 U_{cc}(1 + w^2 (1-\eta)^2) + U_{ii} }{w^2 (1-\eta)^2 \lambda U_{ch} - (\lambda + r) \eta^2 U_{cc} } \right] < 0
\]

Once again, setting \( \eta = 1 \) gives consumption full weight in the habits determination process and yields the results of model 1.
2.3. Effects of Labour Income Tax Change.

We now consider the effects of a permanent increase in the labour income tax.

Assume \( d\tau > 0 = d\tau_c \). As shown in Appendix 2, the following equations determine the response of the choice variables. By (45)-(49):

\[
\frac{dh}{d\tau} < 0
\]

\[
\frac{dB}{d\tau} = \frac{\Omega}{\phi - r} \frac{dh}{d\tau} > 0 \quad \Leftrightarrow \quad \text{case C; cases A, B} \quad (50)
\]

While for leisure:

\[
\frac{dI}{d\tau} = \frac{w\lambda(1-\eta)[\eta U_{hh} + U_{ch}]}{(1-\eta)^2 w^2 \lambda U_{ch} - (\lambda + r) U_{ll}} \frac{dh}{d\tau} + \frac{\eta(\lambda + r)w\beta}{(1-\eta)^2 w^2 \lambda U_{ch} - (\lambda + r) U_{ll}} > 0 \quad \Leftrightarrow \quad \text{case A; cases B, C} \quad (51)
\]

Specifically for the assumptions of the model, (51) implies that

\[
\frac{dI}{d\tau} > 0 \quad \Leftrightarrow \quad \eta < \frac{U_{ch}}{U_{hh}}
\]

which confirms the intuitive argument concerning the second order effects of habits on consumption alone made in the preceding subsection.

Finally for consumption:

\[
\frac{dC}{d\tau} = \frac{1}{\eta} \frac{dh}{d\tau} - \frac{w(1-\eta)}{\eta} \frac{dI}{d\tau} > 0 \quad \Leftrightarrow \quad \text{case C; cases A and B}
\]

Next we proceed to discuss these three possible environments.
2.3.1. Case A: $-\phi < \lambda$

The adjustment dynamics in case A are shown in Figure 3.

Figure 3. ABOUT HERE

In the case of high speed of adjustment in habits stock as can be seen from Figure 3 and Appendix 3, at the moment of impact, stock of habits is below the original steady state level, having fallen only part of the way to the new steady state. This implies that consumption adjusts incompletely to the new lower steady state. Since the impact effect on leisure is to increase leisure, labour supply falls at the impact as well. Impact level of leisure undershoots the new steady state since incomplete adjustment in consumption prevents full adjustment of leisure in the absence of changes in asset holdings.

The reason for this is that in the environment of the high speed of habits adjustment to the steady state (case A), households attempt to maintain a habitual level of consumption and leisure simultaneously. Overall, in case A, $U_{ch}$ is relatively large as required to make $\phi$ small in absolute value, thus the costs of habits in terms of substitution of present consumption for future consumption will be dominated by the benefits of adjusting leisure today relative to the future. Agents will substitute in favour of future consumption and present leisure in relative terms (so that consumption adjustment at impact is incomplete and less deep than adjustment in leisure).

Over time, as consumption falls further while leisure rises, part of the leisure increase will be financed from lower asset holdings. Agents therefore will substitute away from ongoing
consumption and in favour of leisure, allowing for savings from falling consumption to partially compensate for leisure increase.

Note that as in Model 1, there are no overshooting results in Model 2. In Model 1, leisure is a non-habitual good, so that an instantaneous adjustment of leisure absorbs the shock allowing for the impact change in consumption. Financial assets provide added smoothing to consumption. In model 2, although leisure is smoothed alongside of consumption, fast moving habits stock implies that both adjustments at the impact will be incomplete. Hence the similarity arises due to the stronger effect on habits speed adjustment since this effect dominates the effect of high $U_{ch}$.

2.3.2. Case B: $-\phi > \lambda$ and $0 < \frac{\eta \phi U_{n}}{w^2 (1-\eta)^2 U_{ch} - \eta U_{n}} < \lambda$.

Case B corresponds to the environment of moderately fast moving habits. As shown in Appendix 2, habits continue to undershoot the new steady state level at the impact. Once again, fast adjustment of habits stock implies that costs of accumulating habitual standard of living accrue over a shorter period of time. However, at the impact, consumption falls below the new lower steady state level. This overshooting result is due to the fact that in case B, disutility of habits in second order effects dominates the effect of habits on the marginal utility of consumption. Thus the consumption component of habits can be sacrificed by the household to a greater degree than the leisure component. Leisure becomes a relatively more important component of habit stock. Following impact leisure falls only part of the way to the new lower steady state.
Overtime, as the asset position deteriorates, leisure continues to fall, allowing for financing of the rising consumption. These dynamics are shown in Figure 4 below.

Figure 4. ABOUT HERE

An interesting aspect of this scenario is that, in case A, taxes on labour income generate a positive co-movement between labour supply and consumption. In case B this relationship is reversed at the impact and preserved in the long run. Hence at the impact case B confirms Barro and King (1984) model predictions. In this context, due to habits in leisure and separability of leisure from habits stock in preferences, leisure acts as the main utility stabilising component. This requires the downward adjustment in the asset position of the household and a strong reduction in consumption expenditure. In case A, with the speed of habits adjustment being relatively high, leisure is relatively less important as a utility stabilising component, so that both leisure and consumption are smoothed simultaneously.

2.3.3. Case C: \(-\phi > \lambda\) and \(0 < \lambda < \frac{\eta \phi U_{ll}}{w^2 (1-\eta)^2 U_{ch} - \eta U_{ll}}\).

Finally in case C, the habits stock slowly moves to the new lower steady state. As a result of this at impact, consumption overshoots its new higher steady state level. Leisure falls down part of the way towards the new lower steady state. Once again, recall that these effects are driven by the relative importance of habits in terms of the marginal utility of consumption effect. Since habits are slow to move, while the marginal utility of consumption is weakly responsive to habits change (as required by the inequality above), agents have a greater incentive to use leisure as a smoothing variable. As a result, consumption overshoots the long run target.
Over time, as asset positions are altered, consumption slowly falls toward a new steady state from its overshooting position, while leisure continues to fall as well. The savings generated in the process are absorbed into the higher asset holdings. Figure 5 below provides the dynamics for consumption, leisure and asset holdings of the households.

Figure 5. ABOUT HERE

2.4. Effects of Consumption Tax Change.

Finally we consider the effects of a one time permanent increase in consumption tax, so that $d\tau_c > 0 = d\tau_l$. From equations (45)-(49) we can see that

$$\frac{d\bar{h}}{d\tau_c} > 0$$

$$\frac{dB}{d\tau_c} >; < 0 \quad \Leftrightarrow \quad cases \; A, B; \; case \; C$$

$$\frac{d\bar{T}}{d\tau_c} >; < 0 \quad \Leftrightarrow \quad \eta <; > \frac{U_{ch}}{U_{hh}} \quad \Leftrightarrow \quad case \; A; \; cases \; B \; and \; C$$

$$\frac{d\bar{C}}{d\tau_c} >; < 0 \quad \Leftrightarrow \quad cases \; B \; and \; C; \; case \; A$$

2.4.1. Case A: $-\phi < \lambda$

At the moment of policy change, habits stock adjusts incompletely to the new higher steady state. At the same time, consumption falls part of the way toward the new steady state, while leisure rises towards the higher long-run level. Since the relative price of consumption rises,
the opportunity cost of leisure falls making leisure more attractive. At the same time, habits adjust rapidly to the new steady state so that households place greater emphasis on maintaining their habitual standard of living and are unwilling to adjust either consumption or leisure dramatically. There are no overshooting results, as predicted by the traditional habit formation literature (see Model 1).

Since leisure dominates the smoothing motive for the households (consumption weight in habits stock formation is low relatively to the effect of habits on marginal utility of consumption, which is necessary in order to make \( \eta < - \frac{U_{ch}}{U_{hh}} \)), leisure adjusts more in terms of overall expenditure than consumption at the impact. Overtime, this is reversed, so that the net savings from a more significant decline in consumption expenditure, are absorbed into accumulation of the household financial wealth. Asset holdings continue to rise to the higher steady state level. The adjustment dynamics in case A are shown in Figure 6.

**Figure 6. ABOUT HERE**

Relative to the comparable case A in the exercise involving changes in the labour income tax, the model predicts the equivalence of two tax policies vis-à-vis direction of changes in habits stock, consumption and leisure. However, unlike in part 2.3.1 above, households respond to a rise in consumption tax by accumulating financial wealth. This implies that households will tend to exhibit shallow adjustments in leisure relative to the decrease in consumption in the case of consumption tax along the long run adjustment path, than in the case of labour income tax. Most of the changes in leisure occur at the impact as the households’ efforts to maintain a given standard of living prevent them from changing consumption too aggressively. Subsequently, greater consumption changes occur following the impact, as
households delay changing consumption in response to a relatively strong effect of habits stock change on the marginal utility of consumption.

The assumption of separability of habits and leisure in the utility function does not change these results. As mentioned above, if we relax this assumption, leisure changes will be closer in nature to consumption changes. However, the impact effect of the lower importance of consumption in determination of habits, $\eta$, will remain. This implies that once again consumption adjustments will be delayed relative to leisure adjustments.

2.4.2. Case B: $-\phi > \lambda$ and $0 < \frac{\eta \phi U_{ch}}{w^2 (1-\eta)^2 U_{ch} - \eta U_{ll}} < \lambda$.

Case B corresponds to the case of moderately fast moving habits. As in case A above, at impact, household habits adjust incompletely toward a new higher steady state. However in contrast with the case A, consumption rises. The impact adjustment in consumption is incomplete, since a moderately high speed of adjustment in habits stock implies a strong smoothing incentive for household optimisation. Case B is associated with the change in the main variable of smoothing from leisure to consumption. As in response to an increase in the labour income tax, leisure falls at the impact part of the way toward the lower steady state.

Over time, consumption and labour supply rise to the new steady state values. However, the benefit of increased labour supply can be taken either as consumption today or future consumption. In this case in order to maintain habitual standard of living households postpone consumption over time. The resulting savings are transferred into higher asset holdings by the household. Figure 7 below provides the details.
As can be seen from Figure 7, consumption tax and labour income tax differ, in cases A and B, in their effects on consumption and asset position of the households. Strong preferences for consumption smoothing induced by fast moving habits and a relatively strong importance of habits in consumption vis-à-vis leisure implies that agents smooth consumption more in response to consumption tax change. This effect is amplified by the assumption on the separability of leisure and comprehensive habits stock in the instantaneous utility function. The latter reason implies that in case B leisure can act as a stronger shock absorber than consumption. Hence, the households increase their labour supply in response to a change in the relative price of consumption, while they will increase their leisure in response to the change in the real after-tax wage.

2.4.3. Case C: \(-\phi > \lambda\) and \(0 < \lambda < \frac{\eta \phi U_{ll} (1 - \eta) U_{ch} - \eta U_{ll}}{w^2 (1 - \eta)^2 U_{ll}}\).

Case C corresponds to the environment of slow moving habits. This implies that habits stock increases over time with incomplete adjustment to the new steady state at the impact. Slow moving habits imply in case C that consumption becomes the predominant channel for intertemporal smoothing allowing leisure to strongly respond to the shocks. As a result of this, consumption jumps up towards the new steady state at the impact, while leisure overshoots (downward) the new lower steady state. The initial jump in consumption is relatively weak vis-à-vis the complete adjustment to the new steady state, since habits tend to adjust slowly. This implies that habits continue to rise over time along a relatively flat slope, while consumption increases at a faster rate. At the same time leisure falls below the steady state which implies that over time households will adjust their labour supply downward. The
dual effect of rising consumption and leisure requires drawing down the financial wealth of the households.

**Figure 8.** ABOUT HERE

Figure 8 above illustrates the dynamics of these adjustments. In contrast to the response to a rise in income tax, consumption tax yields the opposite direction effects for all variables involved. The reason for this is once again the reversal of the smoothing instrument from consumption (in part 2.3.3 above) in favour of leisure.

**Part 3.** Comparative Analysis of Models 1 and 2 Under the Specific Functional Form Assumption.

In the following we assume the specific utility function that is logarithmic in both consumption and leisure net of a proportional share of the habits stock. The details of assumption and solutions are supplied in the Appendix 3 below.

As shown in equations (A.3.7), (A.3.9) and (A.3.11) under the log-log utility function specification, the only case arising in Model 1 is that of the case B. As discussed in part 1 above, case B corresponds to the situation where the speed of adjustment in habits stock is relatively high. In this case, relative reduction of the impact of future habits stock movements on the marginal utility of consumption implies that the households are less willing to adjust their consumption at the impact and are more willing to adjust their leisure than in the case of slow moving habits. However, due to absence of habits effect in leisure, the households use their labour supply to smooth consumption over time. The result is that the amelioration of
conclusion that consumption adjustment in response to higher taxation occurs in both cases of labour income and consumption taxation.

Solving for the deviations in consumption, leisure and foreign asset position from their steady state levels in response to the tax policy changes we obtain the relative results shown in the Table 1 below. The results in Table 1 confirm our intuitive discussion provided in Part 1 above. With relatively fast adjustment in the habits stock corresponding to the case B, leisure acts as the main shock absorption variable of choice. However, in the case of consumption tax, habits effects of consumption are reinforced by the direct price effect of taxation. Since the price of leisure is not affected directly by the consumption tax, the leisure adjustments are more dramatic in the case of consumption tax increase. In the case of labour income tax increase, the negative income effect of lower after tax wage is ameliorated by the substitution effect in favour of leisure, so that the depth of leisure adjustments in case of labour income tax is lower than in case of consumption tax. For the same reasons, consumption adjusts more in case of consumption tax than in case of labour income tax changes. The result is that consumers are forced to finance higher adjustment in leisure, through a greater downward adjustment in their asset holdings in the case of changes in consumption tax than in the case of labour income tax. This is what we mean by saying that consumption tax in presence of

Table 1. Comparison of Model 1 responses to changes in the tax policies (Case B alone)

<table>
<thead>
<tr>
<th></th>
<th>(\Delta C_{1,ss}^{rc} &lt; \Delta C_{1,ss}^{el} &lt; 0)</th>
<th>(0 &lt; \Delta I_{1,ss}^{el} &lt; \Delta I_{1,ss}^{rc})</th>
<th>(\Delta B_{1,ss}^{rc} &lt; \Delta B_{1,ss}^{el} &lt; 0)</th>
</tr>
</thead>
</table>

40
habitual consumption alone magnifies the negative effects of habits on the variables of choice.

As shown in the Appendix 3 below, model 1 predictions are consistent in general case with the results obtained in case of specific utility function assumption. However, the explicit specification of preferences shown in the Appendix 3 allows us to solve for the effects of habits parameters, \( \gamma \) and \( \lambda \), on the responsiveness of choice variables, \( C_{ss}, B_{ss} \) and \( l_{ss} \) to changes in the tax policies. Table 2 below summarises the results derived in the Appendix 3.

To the best of our knowledge, no study of habits formation mechanism in consumption have so far been able to determine the above effects of the habits parameters values on the overall model responses in consumption, leisure and asset holdings. The effects of both the speed of adjustment in habits, \( \lambda \), and the strength of habits in the utility function, \( \gamma \), on consumption are generally consistent in our model with the traditional models of habits formation. Stronger habits effects in the model arise from either slower adjustment speed of the habits stock (lower \( \lambda \)) or higher disutility effect of habits in consumption (higher \( \gamma \)). In either case, the stronger are the habits effects, the lower is the household’s willingness to adjust consumption.

However, the effects of these two habits effects parameters on leisure and asset holdings depend on whether leisure is a strong shock-absorbing variable (as in the case of changes in the labour income tax) or a weak one (as in the case of consumption tax change). In the

<table>
<thead>
<tr>
<th></th>
<th>Consumption Tax, $\tau_c$</th>
<th>Labour Income Tax, $\tau_l$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Consumption Response</td>
<td>Leisure Response</td>
</tr>
<tr>
<td></td>
<td>$dC_{10}\over d\tau_c$</td>
<td>$dL_{10}\over d\tau_c$</td>
</tr>
<tr>
<td>Effect of $d\gamma &gt; 0$</td>
<td>(–)</td>
<td>(–)</td>
</tr>
<tr>
<td>Effect of $d\lambda &gt; 0$</td>
<td>(+)</td>
<td>(+)</td>
</tr>
</tbody>
</table>

former case, stronger habits in consumption will increase the leisure responsiveness to changes in the labour income tax rate, while the opposite happens in the later case. From the intuitive discussion in Part 1 and the results of Table 2 above, the effects of habits parameters on leisure adjustment in response to changes in the tax rates are matched by the effects of the parameters changes on the asset holdings of the consumers.

In Appendix 3 we provide the details of the analysis of relative effects of consumption and labour income tax on the choice variables of the model in presence of comprehensive habits in both the consumption and leisure expenditures. For simplicity of comparison, we focus on case 2A alone that corresponds closely to that of case B in Model 1. Once again, confirming the intuitive analysis presented in Part 2, we can establish that changes in the consumption
tax have stronger effect on the choice variables adjustments than the changes in labour income tax. Table 3 below summarises the results of our model.

Table 3. Comparison of Model 2 responses to changes in the tax policies (Case 2A only).

\[
\begin{align*}
\Delta C_{2,ss}^{tc} &< \Delta C_{2,ss}^{zl} < 0 \\
0 &< \Delta I_{2,ss}^{zl} < \Delta I_{2,ss}^{tc} \\
\Delta B_{2,ss}^{tc} &> 0 > \Delta B_{2,ss}^{zl} \\
\text{mod } \Delta B_{2,ss}^{zl} &< \Delta B_{2,ss}^{tc}
\end{align*}
\]

As argued in Part 2 above, changes in consumption tax generate negative income effect and a substitution effect that acts in the direction of ameliorating income effect for the dominant smoothing variable of choice. In the case A, such variable is leisure. However, unlike in Model 1, Model 2 is characterised by habitual dependence for both consumption and leisure choices so that the main shock-absorbing variable in the model is the asset holdings of the household. Hence, the labour income tax increase results in a weaker adjustment in foreign assets position than a rise in the consumption tax. As the relative price of consumption rises due to an increase in consumption tax, households prefer stronger adjustments in consumption to stronger variation in leisure. In other words, distortion of the relative price of consumption means that higher consumption tax reinforces household’s motivation to smooth leisure caused by habitual dependence in leisure. Hence, in relative terms, case A is the case of stronger habits in leisure than in consumption.

Unfortunately, as argued in the Appendix 3 below, relatively little can be said under the assumed specification about the effects of habits parameters on the responses in leisure, consumption and asset holdings to tax changes. Short of conducting numerical exercises, the
model 2, in contrast with model 1, does not yield any conclusive results. The latter exercise is limited by the lack of empirically justifiable choices of parameter values for $\gamma$, $\eta$ and $\lambda$ that characterise comprehensive habits in consumption and leisure expenditure.

**Conclusions.**

As mentioned in the introduction, the central feature of the traditional habits formation models is the exogeneity of labour supply decisions by the households. Following Boyer (1978), the vast majority of habit formation studies assume inelastic supply of labour. As a result of this, traditional literature completely ignores both the direct effects of macroeconomic shocks on labour-leisure trade-off and the link between the shocks’ effects on consumption and the presence of leisure in the optimisation set of the households.

Several recent papers touch upon the issue of endogenous labour supply in the presence of habits in consumption. Faria (2001) derives steady state solutions for leisure and consumption in a standard exogenous habits model. However, his work does not provide an in depth discussion of the effects of leisure-consumption trade-off on consumption adjustments. Furthermore, his work does not discuss the model’s dynamics. Graham (2003) similarly extends the traditional habit formation model to include labour supply decisions. Using dynamic simulations he shows that under certain specific form assumptions, habit formation models perform less convincingly in the presence of endogenous labour supply than do standard habits models. Once again, the study lacks an analytical discussion of the model’s dynamics.
To the best of our knowledge at present, no literature addresses the fundamental features of labour-consumption trade-off in household optimisation in the presence of a habit formation mechanism either in consumption alone, or in other components of the utility function. Our paper attempts to close this gap.

Following the rationale presented above, we first develop a model of habitual consumption decisions in the presence of endogenous labour supply. As expected, we analytically obtain the results similar to those in Graham (2003) numerical approximations. Furthermore, with respect to the effects of taxation changes, our benchmark model captures the results that mirror those established in habit formation literature (e.g. Mansoorian, 1993). In the presence of habits in consumption alone, leisure acts as the main shock-absorbing variable, with discontinuous adjustments to the new steady state at the impact. Furthermore, in this setting, consumption tax is qualitatively equivalent to labour income tax. An increase in either one of the tax rates will result in case of slow moving habits, in downward overshooting in consumption, discrete increase in leisure and strengthening of the asset position by the households. In the case of fast moving habits, consumption incompletely adjusts downward at the impact, and continues to decline over time toward the new steady state, while leisure discretely rises. This implies that over time households reduce their financial wealth.

Following the results from the benchmark model, we proceed to develop a model in which habits are determined jointly by consumption and leisure. The main motivation for this exercise lies in the failure of the consumption-habit model to generate any persistence in labour supply. In addition, a simple extension of the traditional model to include endogenous labour supply, provided in Part 1, fails to generate any changes in the dynamics of the household consumption decisions relative to the benchmark models. For example, independent of the speed of habits stock adjustment to the new steady state, the model shows
co-movement between consumption and labour supply. The reason for this is that habits in consumption imply that leisure and financial assets act as the main buffers in the households’ response to exogenous shocks. Thus Model 1 fails to account for the possibility that consumption and leisure may co-move along the business cycle or in response to tax policy changes.

Model 2 introduces a new idea of comprehensive habits. Instead of basing a habitual standard of living on consumption history alone, we propose to model habits as evolving according to a law of motion that accounts for both real consumption and real leisure expenditure over time. This is precisely what we term comprehensive habits.

Intuitively, when consumers smooth both consumption and leisure, several forces determine their response to the exogenous shocks. First, they will substitute between consumption and labour supply as the main instruments for smoothing their intertemporal utility. Second, changes in real wage will have different effects in such environments from changes in the real price of consumption. The latter has stronger habit smoothing effect on consumption. In contrast, the former involves both substitution effects in leisure and the effect in terms of habit smoothing through the leisure component of habits. As agents substitute away from leisure, they also reduce the pressure of the negative effects of habits by lowering the impact of leisure in the habits stock. At the same time, since the speed of habits adjustment to the steady state is linked to the effects of habits on the marginal utility of consumption, marginal utility of consumption will be lower when the leisure component of comprehensive habits falls. This implies that agents may find it less costly to vary either leisure or consumption depending on the speed of habits adjustment relative to the responsiveness of the marginal utility of consumption to changes in habits stock.
As shown in Model 2, based on the speed of adjustment in habits relative to the importance of habits in the marginal utility of consumption, we can distinguish three environments.

In case A, households operate in the environment of fast moving habits. The habits stock adjusts to the new steady state at a rate that is above the rate of change in the marginal utility of income. Additionally, high speed of adjustment in habits stock implies that the marginal utility of consumption is strongly influenced by changes in the habits stock. As labour income tax increases, households, interested in maintaining habitual standard of living, are more willing to use leisure as the main smoothing instrument over consumption. The result is to increase leisure at the impact. However, with the smoothing motive being strong, households will not adjust consumption to the sufficiently low levels required to offset a rise in leisure. Thus households will draw down their financial wealth along the adjustment path of rising leisure and falling consumption.

A similar situation arises in the case of an increase in the rate of consumption tax. Both consumption and leisure exhibit qualitatively similar dynamics. However in this case while smoothing motivation remains strong, consumers respond to a rise of the relative price of consumption induced by tax increase by significantly lowering consumption in excess of the adjustment required to compensate households for a rise in leisure. This arises due to leisure becoming a stronger smoothing variable relative to consumption in response to consumption tax changes. As the result, households accumulate assets along the transition path.

In the case of slower adjusting habits, marginal utility of income grows faster than habits stock, while the overall effect of habits on the marginal utility of consumption remains relatively high. Then, in response to an increase in the labour income tax, consumption becomes the main instrument of smoothing. Leisure exhibits incomplete adjustment to the
new lower steady state, while consumption overshoots the new steady in a downward direction. Over time, consumption rises towards the steady state, while labour supply continues to rise. The dual fall in consumption and leisure, however, are too shallow to fully compensate for the loss of after tax income. This requires a reduction in the households’ financial wealth in the long run.

In response to a change in consumption tax, the effects of tax policy shock differ in consumption and asset position of the households. Strong preferences for consumption smoothing induced by the fast moving habits and relatively strong importance of habits in consumption vis-à-vis leisure imply that agents smooth consumption more in response to consumption tax change. Thus leisure now acts as a stronger shock absorber than consumption when the income effects of labour tax are replaced by the substitution effects of consumption tax. Hence, households increase their labour supply (as in the case of the response to the income tax increase discussed above), while increasing their consumption at the same time. The increase in labour supply is sufficiently strong to compensate for the higher consumption expenditure, so that along the adjustment path, savings rise and thus households accumulate financial assets.

Finally we consider the case of slowly adjusting habits combined with a relatively low effect of habits on marginal utility of consumption. By the assumption of separability, habits have no direct effect on the marginal utility of leisure. As a result, in response to changes in labour income tax, households continue to view leisure as the main smoothing component in the utility function, so that consumption overshoots its higher steady state level at impact, while leisure falls down part of the way toward the new steady state level. Overtime, as consumption and leisure both fall, assets are accumulated.
In the case of consumption tax changes, as before, households view consumption as the main smoothing component, allowing for only an incomplete upward adjustment in consumption. Leisure absorbs the shock at impact and falls below the steady state level of demand. Overtime, an upward adjustment of leisure and consumption requires reduction in the households’ wealth.

Hence, overall, in the case of both consumption and leisure contributing to the habit stock, consumption and labour income taxes are no longer qualitatively equivalent in their effects on the household decisions. As shown in this paper, differences in households responses to different tax policies depend on the environment described by the speed of habits adjustment to the steady state relative to the effect of habits on both the marginal utility of income and the marginal utility of consumption. In some cases, labour supply changes procyclically with changes in consumption, while in other cases, leisure changes reinforce adjustments in consumption.

Similarly, asset positions of the households can either be procyclical with consumption changes or counter-cyclical in the presence of comprehensive habits. This result does not hold in the traditional model of habit formation in consumption.

Furthermore, in all cases in the presence of comprehensive habits, leisure follows a gradual adjustment path. The speed of adjustment, as well as the depth of changes in each variable of interest, vary across various environments and tax policies. The singular prediction of the traditional model of habits in consumption in presence of endogenous labour supply is that consumption will exhibit excess smoothening, thus failing to capture an empirically plausible degree of variation along the adjustment path. This prediction breaks down in the case of comprehensive habits. As shown in our model, comprehensive habits introduce time
dependency into both consumption and leisure decisions of the households. This implies that depending on the model parameters both consumption and leisure may overshoot their targets. The implied volatility of consumption around the steady state in our model is thus subject to more variation than in the traditional model of habit formation.

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