Public Debt and Real Exchange Rate: the Case of the South American Countries within a framework of ”New Open Economy Macroeconomics”

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January 5, 2004

Abstract The aim of this article is to use the contributions of the New Open Macroeconomics in order to reconsider the relation between the external public debt and the real and nominal exchange rates of South American countries. By putting the current account in the center of the transmission system of the shocks, it makes it possible to bind the problems of the twin deficits and that of the transfer of the stock of foreign assets to the real exchange rate.

In the absence of ricardian equivalence, a fall of the taxes financed by debt involves the accumulation of a foreign debt and a depreciation of the nominal and real exchange rates in the long-term. The empirical study still remains to be made.

key-words Public debt, price rigidity, real exchange rate, overlapping generations.

Classification J.E.L. F41, H63, H3
1 Introduction

Between 1990 and 2002, the national debt of Argentina grew from 30% of the GDP to 150% of the GDP, which is equivalent to an annual growth rate of 14.3%. Beyond the particular case of Argentina, particularly touched by the economic crisis and whose GDP broke down, South America is, with Asia, the area where the national debt increased the most since the middle of the Nineties. Whatever their origins, these evolutions fall under the recurring problem of South American countries, that of the national debt.

Furthermore, the other problem which arises for these emergent countries is that of the evolution and the control of their exchange rate.

It is inflation related to the seigniorage which is in the center of the mechanisms binding public deficits and real exchange rate in the literature. The models of exchange rate crisis known as “first generation models” (following Krugman (1979), more particularly Sargent and Wallace (1982)) put, indeed, in light the role of the segniorage as a transmission channel of the debt to the exchange rate.

The capacity of this approach to explain this phenomenon proved reliable. Fischer, Sahay and Vegh (2002) show that there is a strong link between budget deficits and short-term as well as long-term inflation in the countries with strong inflation. It was the case of many South American countries. Nevertheless, the conditions of accumulation of the debt moved since the middle of the Nineties. After the exchange rate crises, those countries massively engaged into a fight against inflation. Such a goal increased as much the cost of the seigniorage. Indeed, the available data show a net deceleration of inflation and money supply during the nineties.¹

Within this framework, it is interesting to think about an another relation between the public debt and the exchange rate. Indeed, the preceding approach only retains the seigniorage as a transmission channel. It doesn’t take into account the reaction of private agents to the accumulation of a public debt.

The traditional approach, known as “ricardian equivalence” draws aside the possibility of an impact of the public debt on consumption and saving. The reason is that a fall of taxes financed by an increase of the public debt will be entirely saved by the households which anticipate an equivalent rise of future taxes. Nevertheless, authors like Khalid (1996), Corbo and Schmidt-Hebbel (1991) reject this assumption, particularly for the emergent countries, by postulating a constraint of liquidity.

It gives, at a theoretical level, but also at an empirical level, the possibility of exploring alternate transmission channels of the public debt to the exchange rate with the help of the households’ behavior (and not the government’s as it is the case in the exchange rate crisis). It is for instance what Normandin (1999) does when he binds the public and foreign deficits using a structure of overlapping generations in a model of inter temporal optimization. The New Macroeconomics in Open Economy (NMOE)² allows us to enrich the analysis

¹ cf. appendix for a survey of this trend
² The seminal article is Obstfeld et Rogoff’s "Exchange Rate Dynamic Redux"(1995). For
because prices viscosity will deeply modify and differentiate balances of short and long term.

The recent developments of the NMOE attempt to describe dynamics of the exchange rate with more realism. They integrate the contributions of the keynesian macroeconomics, mainly nominal rigidities and the markets’ imperfections within a framework of inter temporal optimization.

The contribution of this new literature is manifest for the analysis of the debt / real exchange rate relation for two reasons. First of all, its structure of inter temporal optimization allow us to found micro economically the macroeconomic relations between the various aggregates and thus to study in a much more precise way the impact of the tax policies than the traditional Mundell (1960) -Fleming (1962) model does.

In addition, the assumptions of the price viscosity (transitory rigidity, during one period, of the microeconomic prices) and of monopolistic competition, justify the fact that production is drawn by the short-term demand movements (Blanchard and Kiyotaki (1987))

A temporary shock has permanent effects on the real variables in the long run because of the accumulation of foreign credits it brings. This channel of transmission puts the current account in the center of the analysis (which in addition makes it possible to connect these problems to that of the “twin deficits”)

This theory was used a lot to analyze the consequences of monetary shocks, because it questions the neutrality of money in the long-term. But it was much less used to study the impact of tax policies and always within a framework where the government’s budget is balanced\(^3\).

Gannelli (2003) is the first that explicitly studies the impact of the public debt on consumption, production and the exchange rate within the framework of the NMOE. He uses a structure with overlapping generations in a model with two countries in order to make the debt have an impact on consumption by a wealth effect. The main transmission channel at work in this model is the interaction between consumption, demand of money and the price level.

In the absence of ricardian equivalence, a temporary decrease of taxes financed by a public debt involves the rise of the consumption and of the demand for the domestic currency in the short run, all things being equal. As the money supply is constant, the level of prices decreases and brings, according to PPP on the imported goods, an appreciation of the nominal exchange rate.

This effect, which is named ”Money Demand Effect” will dominate another effect, the ”Current Account Effect” which involves a depreciation of the nominal exchange rate by a deterioration of the current account. On the whole, one thus observes an appreciation of the real and nominal exchange rates in the short run.

The appreciation of the real exchange rate, as well as the increase in absorption, will create a deficit of the current account and then the accumulation of a foreign debt. This debt will weigh in the long run on the nominal exchange rate, by a wealth effect.

In this model, the fact that the PPP is valid in the long-term does not make it possible to bind this work to the conclusions of many empirical work which clarifies a positive correlation between stock of external credits and the real exchange rate (Faruqee (1995), Gagnon (1996), Lane and Milesi-Ferretti (1999, 2000)).

Among the most recent work, Lane and Milesi-Ferretti (2002, 2002) in particular, show that on average the countries which are net debtors have a more depreciated real exchange rate. The principal transmission channel is the relative price of the nonexchangeable goods for the emergent countries. The terms of trade, which seem to be a second transmission channel, have a much weaker explanatory capacity. This seems coherent insofar as what occurs in the small countries has a very weak impact on the prices of the exchangeable goods.

Our model studies the impact of a transitory fall of the taxes financed by public debt on the real and nominal exchange rate in the short run like in the long run.

It deals with a framework of a small open economy which produces nonexchangeable goods in imperfect competition and receives an equipment in exchangeable good. Its objective is to enrich the approach by Ganelli by integrating in the new open macroeconomics of the debt the theoretical contributions of the recent literature on the "transfer problem" of the stock of capital to the real exchange rate.

Indeed, the South American emergent countries, which are big net debtors, had and will have to undergo major negative wealth transfers.

For that, we integrate nonexchangeable goods in the analysis. In a small open economy, the domestic country does not have any impact on the price of the exchangeable goods. However, the structure of the NOEM rests on the existence of a monopolistic competition. It is thus necessary to introduce a nonexchangeable good in order to make it possible for the firms to have a strategic behavior. Moreover, like showed by Lane and Milesi-Ferretti "the transfert problem"seems to pass primarily through the prices of the nonexchangeable goods in the emergent countries.

It follows that the PPP is not respected in the short run as in the long run, which makes it possible to draw conclusions on the bond debt / real exchange rate.

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4 The short term differs from the long term by the rigidity of the microeconomic prices.

5 The permanent deviation from the PPP comes here from the existence of nonexchangeable goods. In the original model, the PPP is only valid in the long-run because of the short-run price rigidity.

Edwards and Savastano (1999), in a survey of the empirical literature on the PPP hypothesis in the emergent countries, conclude that the studies but also the data are still too rare to draw the conclusions on the validity of this assumption. Anyway, they quote the study of Goldfajn and Valdés (1996) which shows that the relation between the real exchange rate appreciation and the probability of its reversibility are strongly non-linear.
The results are the following. A temporary fall of taxes financed by external public debt involves, for plausible values of the parameters, the creation of a deficit of the current account and thus the accumulation of a foreign debt in the short run as well as an appreciation of the nominal and real exchange rates.

In the long run, this foreign debt involves the depreciation of nominal and real exchange rates because of a negative wealth effect. The channel of transmission will occur at the same time on the market of the nonexchangeable goods and on the market of the currency.

On the whole, we can find a positive relation between stock of net foreign credits and appreciation of the real exchange rate in the long run. This relation is all the stronger since the nonexchangeable goods represent a significant share of national consumption. The more the country is open to foreign trade, the smaller are the consequences of an acceleration of the public debt on the long term exchange rate. Indeed, when $\gamma$ tends towards 1, we find the effects enlightened by Ganelli.

2 Modèle

We integrate nonexchangeable goods in the structure of the article of Ganelli (2003) which is a model with overlapping generations and by using the assumptions of the small open economy.

The prices of the exchangeable goods ($P_{T,t}$) are related to their world price ($P^*_t$ is exogeneous and fixed) by "the Law of One’ Price” which postulates:

$$P_{T,t} = E_tP^*_t$$ (1)

the national interest rate is related to the world interest rate which is exogenic according to the following Non Covered Interest Rate Parity$^5$:

$$E_{t+1} = \frac{(1 + i_{t+1})}{(1 + i^*_t)}E_t$$ (2)

Because of those assumptions, there is no monopolistic competition and nominal rigidities left in the exchangeable goods sector. It is thus necessary to integrate a nonexchangeable goods sector. For it, we use the appendix of Obstfeld and Rogoff’s article (1995).

The number of people who were born at each period is standardized to 1. At each period, the agents have a constant and positive probability to die $(1 - q)$.

The consequence is that the size of the population is constant and equal to

$^5$The interest rate is a function of the price of the exchangeable goods because they are payed on foreign assets. So, the PPP on the exchangeable goods allows us to respect the Non Covered Interest Rate Parity
\[ \sum_{a=0}^{\infty} q^a = \frac{1}{1-q} \]

The consumption of the representative agent of age a is divided between the exchangeable good \( (C_{T,a,t}) \) and the nonexchangeable composite good \( C_{N,a,t} = \int_0^1 c_{N,a} (z)^{\theta} dz \) (where \( \theta > 1 \) is the elasticity of substitution between each couple of goods) consumption in the following way:

\[ C_{a,t} = (C_{T,a,t})^\gamma \cdot (C_{N,a,t})^{1-\gamma} \]  \hfill (3)

where \( \gamma < 1 \) represents the part of the exchangeable goods in the whole consumption.

We can deduce the general price level at the \( t \)-period:

\[ P_t = \frac{P_{T,t}^{\gamma} \cdot P_{N,t}^{1-\gamma}}{\gamma^\gamma \cdot (1-\gamma)^{1-\gamma}} \]  \hfill (4)

where \( P_{N,t} = \int_0^1 p_N (z)^{\theta-1} dz \) is the price of the nonexchangeable goods and where \( P_{T,t} = E_t P_T^* \) is the price of the exchangeable goods.

### 2.1 Production

Only the nonexchangeable goods are produced in the country. In addition, the agents receive an equipment in exchangeable good and can buy or sell an additional quantity of it abroad.

This assumption, which is the same as the one retained by Obstfeld and Rogoff (1995) is not illogical in so far as the local producers do not have any impact on the world price. Thus the quantity of exchangeable good produced in the country is fixed.\(^7\)

In the nonexchangeable goods sector, there is monopolistic competition - the firms produce a differentiated good and they have a monopoly on their market - and price rigidity in the short run (i.e. over one period).

The number of firms is standardized at one. One calls \( Z \in [0,1] \) both the firms and the good they produce.

The function of production is as follows:

\[ Y_{N,t} (z) = L_t (z) \]

\(^7\)c.f. the justification of those hypothesis for the case of the South American countries in the appendix.
where \( L_t(z) \) is the amount of labor employed by the firm \((z)\).

The demand addressed to the nonexchangeable goods sector firms is as follows:

\[
Y_{PC}^{N,t} = \left[ \frac{p_N(z)}{P_N} \right]^{-\theta} \left( C_{N,t}^{PC} + G_{N,t}^{PC} \right)
\]  

(5)

The demand for nonexchangeable goods by the private agents \((C_{PC}^{N,t})\) is the result of a choice which depends on their preferences and the relative prices.

The behavior of the government is the same one but its demand for nonexchangeable goods \((G_{PC}^{N,t})\) is exogenic (one does not know the government’s preferences).

The representative firm of the nonexchangeable goods sector maximizes its profit by taking the demand into account. It has a market power because of the imperfect competition and sells its production on the domestic market. Its program is as follows:

\[
\begin{align*}
\text{Max} & \quad \Pi_{N,t}(z) = p_{N,t}(z) Y_{N,t}(z) - W_t L_t(z) \\
\text{s.c} & \quad Y_{N,t}(z) = L_t(z) \\
Y_{N,t}(z) & = \left[ \frac{p_N(z)}{P_N} \right]^{-\theta} \left( C_{N,t}^{PC} + G_{N,t}^{PC} \right)
\end{align*}
\]

where \( \Pi_{N,t}(z) \) is the profit of the firm \((z)\).

With symmetrical balance, it proposes the following wage:

\[
W_t = \frac{\theta - 1}{\theta} P_{N,t}
\]

The wage depends on the price but also on the elasticity of substitution between nonexchangeable goods.

The profit of the whole sector is \( \int_0^1 \Pi_{N,t}(z) dz = \Pi_{N,t} \int_0^1 dz = \Pi_{N,t} \) and the whole production of the sector is \( Y_{N,t} = C_{N,t}^{PC} + G_{N,t}^{PC} \).

### 2.2 The households

#### 2.2.1 Insurance

\(^8\)It comes from a behavior of maximization of consumption under a budget constraint \( Z \) for the the private agent.

\(^9\)If one makes the assumption that the nonexchangeable goods sector firms are identical, i.e. that they have the same pricing policy and propose the same wages, they will propose the same price and thus \( P_{N,t} = p_{N,t}(z) \).

According to the price index formula, we have \( P_{N,t} = \left[ \int_0^1 p_{N,t}(z)^{\theta-1} dz \right]^\frac{1}{\theta-1} = P_{N,t} \left[ \int_0^1 (1)^{\theta-1} dz \right]^{\frac{1}{\theta-1}} = P_{N,t} \)
The agents have a positive probability to die. They do not wish to leave heritage and want to use all their wealth. Indeed, there is room for an insurance company which gives them a premium on their financial wealth at each period during their life and recovers the remainder of it on the day they die. As the aggregate financial wealth is the following

\[ v(t) = \sum_{a=1}^{\infty} q^a \frac{1}{q} \frac{M_{a-1,t-1}}{P_t} + \frac{P_{T,t}}{P_t} (1 + r_t) \sum_{a=1}^{\infty} q^a \frac{1}{q} F_{a-1,t} = \frac{1}{q} \left[ \frac{M_{t-1}}{P_t} + \frac{P_{T,t}}{P_t} (1 + r_t) F_t \right] \]

Under free entry and "zero-profit" assumptions, the firm equalizes the gains \((1 - q) \frac{1}{(1-q)} v(t)\) and the loss \(x q \frac{1}{(1-q)} v(t)\). So is the premium:

\[ x = \frac{1 - q}{q} \]

### 2.2.2 The consumer behavior

The representative agent of age \(a\) withdraws a utility from the consumption of exchangeable and nonexchangeable goods, of the detention of real money and of leisure. Preferences are homothetic and separable in each of their components. The equipment in time is standardized to one.

Its preferences are represented as follows:

\[ E(U_t) = \sum_{s=t}^{\infty} (\beta q)^{s-t} \left[ \log \left( \frac{C_{a+s-t,s}}{P_{a+s-t,s}} \right) + \chi \log \left( \frac{M_{a+s-t,s}}{P_{a+s-t,s}} \right) + \psi \log \left( 1 - \frac{L_{a+s-t,s}}{P_{a+s-t,s}} \right) \right] \]

where \(E(U_t)\) is the expected utility of the representative agent at the \(t\) period, \(\beta\) is the discount factor, \(C_{a+s-t,s}\) the whole consumption (exchangeable and nonexchangeable goods), \(M_{a+s-t,s}\) the amount of real money hold, \(L_{a+s-t,s}\) the amount of labor supplied and then \((1 - L_{a+s-t,s})\) the amount of leisure, everything for the agent of age \((a + s - t)\) at the \((s)\) period. \(\chi, \psi\) are positive parameters.

Its real budget constraint at the \(t\) period is as follows:

\[ \frac{P_{T,t}}{P_t} F_{a,t+1} + \frac{M_{a,t}}{P_t} + C_{a,t} = \frac{1}{q} \left[ \frac{M_{a-1,t-1}}{P_t} + \frac{P_{T,t}}{P_t} (1 + r_t) F_{a-1,t} \right] + \frac{W_{a,t} L_{a,t}}{P_t} + \frac{\pi_{N,t}}{P_t} + \frac{P_{T,t}}{P_t} y_T - \tau_{t,a} \]

The agent saves in bonds \((F_{a,t+1})\) and money \((M_{a,t})\) and consumes \((C_{a,t})\) the income which results from the "financial" wealth accumulated at the previous period \(\left( \frac{M_{a-1,t-1}}{P_t} + \frac{P_{T,t}}{P_t} (1 + r_t) F_{a-1,t} \right)\) and from the current income made of wages \(\left( \frac{W_{a,t}}{P_t} L_{a,t} \right)\), the per capita profit \(^{10}(\frac{\pi_{N,t}}{P_t})\) where \(\pi_{N,t} = (1 - q) \Pi_{N,t}\), the equipment in exchangeable goods \(\left( \frac{P_{T,t}}{P_t} y_T \right)\) where \(y_T\) is the (equal) endowment

\(^{10}\)The firm’s profits are also distributed between the agents independently from their age.
of exchangeable goods of each agent\(^{11}\), minus of a lump-sum tax \((\tau_{t,a})\). \(r_t\) is the real interest rate between \((t-1)\) and \(t\) periods \(^{12}\).

This constraint takes into account the insurance premium on the financial wealth \(\left(\frac{1-q}{q}\right)\).

The representative agent of age \(a\) maximizes its hoped utility under a budget constraint. By incorporating by age the individual first order conditions and by dividing the result by the size of the domestic population, we obtain the following per capita results with respectively the overall consumption \((8)\), the money demand \((9)\) and the labor supply \((10)\).

In addition, we reason in per capita values.

\[
C^{PC}_t = \left(\frac{1-q^2}{\chi + 1 + \psi \frac{P_{T,t}}{P_t}}\right) TW^{PC}_t
\]

\[
M^{PC}_t = \frac{1 + i_{t+1}}{i_{t+1}} C^{PC}_t
\]

\[
L^{PC}_t = 1 - \psi \left(\frac{P_t}{P_{N,t}}\right) \left(\frac{\theta}{\theta - 1}\right) C^{PC}_t
\]

where \(i_t\) is the nominal interest rate between \((t-1)\) and \(t\).

\(TW^{PC}_t\) is the the whole wealth given by :

\[
TW^{PC}_t = H^{PC}_t + (1 + r_t) \left(\frac{1}{(1 + i_t)} M^{PC}_{t-1} + F^{PC}_t\right)
\]

It is the sum of the "financial" wealth and of the "human" wealth \((H^{PC}_t)\) :

\[
H^{PC}_t = \sum_{a=0}^{\infty} (1-q) q^a \left[ \sum_{s=t}^{\infty} \alpha_{s,t} q^{s-t} \left(\frac{W_s}{P_{T,t}} + \frac{\pi_{N,s}}{P_{T,s}} + \frac{\tau_s}{P_{T,s}}\right)\right]
\]

\[
= \sum_{s=t}^{\infty} \alpha_{s,t} q^{s-t} \left(\frac{W_s}{P_{T,s}} + \frac{\pi_{N,s}}{P_{T,s}} + \frac{\tau_s}{P_{T,s}}\right)
\]

\(^{11}\)in such a way that the whole amount of exchangeable goods in the economy is : \(\nabla_T = \left(\frac{1-q}{q}\right)\).

\(^{12}\)the definition is the following because \(P^*_T\) is considered as constant :

\[
(1 + r_{t+1}) = \frac{P_{T,t}}{P_{T,t+1}} (1 + i_{t+1})
\]

\[
(1 + r_{t+1}) = \frac{E_t}{E_{t+1}} (1 + i_{t+1})
\]
which is the sum of the discounted future flows of income.

We can deduce the path of expansion of the overall consumption:

\[
C_{t+1}^{PC} = \frac{P_{T,t+1}}{P_{t+1}} \left( 1 - \frac{q \beta}{\chi + 1 + \psi} \right) (1 - q) H_{t+1}^{PC} + \left( \frac{P_{N,t}}{P_{N,t+1}} \right)^{1-\gamma} \left( \frac{E_{t+1}}{E_t} \right)^{1-\gamma} q \beta (1 + r_{t+1}) C_t^{PC}
\]

(13)

In addition, we know, from the procedure of optimization, the consumption of exchangeable goods and that of nonexchangeable goods at the t period according to the overall consumption:

\[
C_{T,t}^{PC} = \left( \frac{\gamma}{1 - \gamma} \frac{P_{N,t}}{P_{T,t}} \right)^{1-\gamma} C_t^{PC}
\]

(14)

\[
C_{N,t}^{PC} = \left( \frac{1 - \gamma}{\gamma} \frac{P_{N,t}}{P_{T,t}} \right)^\gamma C_t^{PC}
\]

(15)

So we can find the consumption of exchangeable goods and that of nonexchangeable goods:

\[
C_{T,t}^{PC} = \gamma \left( \frac{1 - q \beta}{\chi + 1 + \psi} \right) TW_t^{PC}
\]

(16)

\[
C_{N,t}^{PC} = (1 - \gamma) \left( \frac{P_{T,t}}{P_{N,t}} \right) \left( \frac{1 - q \beta}{\chi + 1 + \psi} \right) TW_t^{PC}
\]

(17)

It is noticed that inflation reduces the overall consumption (equations (8) and (10)). In the same way, the rise in the nonexchangeable goods relative price decreases the demand for these goods.

Inflation increases the labor supply too whereas the rise of the nonexchangeable good relative price reduces it, with the help of an income effect.

2.3 The government

The budgetary constraint in real terms is as follows:

\[
G_t + \frac{P_{T,t}}{P_t} (1 + r_t) D_t = \tau_t + \left( \frac{M_t - M_{t-1}}{P_t} \right) + \frac{P_{T,t}}{P_t} D_{t+1}
\]

(18)

Where \( G_t \) is the public expenditure at the t period, \( D_t \) the national debt accumulated between t and t-1. The debt is supposed to be held only by foreigners. There is no domestic financial market and bonds are exchanged only on the world financial market.

It can finance its expenditure and the national debt by a lump sum tax, monetary creation and the emission of a new debt.

13 By the same way, we can find the path of expansion of the exchangeable goods and nonexchangeable goods consumption.
2.4 Net Foreign Assets

The current account represents the constraint of the economy: It is the sum of the aggregated budgetary constraint of the households and of the government’s. So we find the net value of foreign assets accumulated over the period, following:

\[ V_{t+1}^{PC} - V_t^{PC} = \frac{\theta - 1}{\theta} \frac{\pi_{N,t}}{P_t} P_{t+1}^{PC} - C_t^{PC} - G_t^{PC} + \frac{\pi_{N,t}}{P_t} + \frac{P_{t+1}}{P_t} y_t + r_t V_t^{PC} \quad (19) \]

where \( V_{t+1}^{PC} = \frac{P_{t+1}}{P_t} (F_{t+1}^{PC} - D_{t+1}^{PC}) \) and \( V_t^{PC} = \frac{P_{t+1}}{P_t} (F_t^{PC} - D_t^{PC}) \) gives us the net amount of foreign assets.

It is noticed here that the national debt is held by foreigners. So it is the external national debt which is modelled here. In fact, the emission of this debt does not have any impact on the nominal and real interest rate.

2.5 The initial steady state

As the model does not produce a reduced-form solution, it is necessary to define a particular steady state around which we can log-linearize the model in order to study the macroeconomic impact of the debt. The log-linearization makes it possible to carry out an analysis in term of rate of variation \((\frac{\Delta \text{variable}}{\text{variable}_0})\).

Initial steady state is posed such as

\[ V_0 = D_0 = F_0 = G_0 = 0 \]

We obtain:

\[ C_0^{PC} = \frac{P_{N,0}}{P_0} Y_0^{PC} + \frac{P_{T,0}}{P_0} y_T \]

So the following per capita values are obtained for consumption, production and labor supply:\footnote{We can remark that they take a value at the initial steady state which is below the one they would take if we were in perfect competition, if \( \theta \to +\infty \). In that case they would take the following value:

\[ C_0^{PC} = \frac{P_{N,0}}{P_T} = c_{N,0}^{PC} = Y_0^{PC} = L_0^{PC} = \frac{(1-\gamma)}{(1-\gamma) + \psi} \]}

\[ C_0^{PC} = \frac{P_{N,0}}{P_T} = C_{N,0}^{PC} = Y_{N,0}^{PC} = L_0^{PC} = \frac{(1-\gamma)}{(1-\gamma) + \psi} \]

and the relative price of the nonexchangeable goods according to :

\[ \frac{P_{N,0}}{P_{T,0}} = \frac{1-\gamma}{\gamma} \]
2.6 Log-linearization

The notations are as follows:
\[ \dot{x} = \frac{dx}{\Delta t} \] represents the rate of variation of variable \( x \) in the short run.
\[ \ddot{x} = \frac{d^2x}{\Delta t^2} \] represents the rate of variation of variable \( x \) in the long run.

The log-linearized version of the model is summarized in the tables below.

According to the assumption of small open economy, the foreign variables are supposed to be fixed. Indeed, shocks on this economy have no impact on the macroeconomic variables of the rest of the world. So their rate of variation is set as null, \( \dot{x}^* = \ddot{x}^* = 0 \), and excluded from the analysis.

- In the short term

\[
\begin{align*}
\dot{p}_T &= \dot{e} \\
p &= (1 - \gamma)p_N + \gamma p_T \\
\bar{r} &= \bar{\tau} = 0 \\
y_N &= c_N \\
\bar{m} - \bar{p} &= \bar{e} + \frac{\bar{e} - \bar{e}}{\gamma (1 - \gamma)} - \frac{\bar{e}}{\gamma^2} \\
\psi &= (1 - \gamma)y_N - \bar{e} - g \\
c_T &= \bar{e} + (1 - \gamma)[p_N - p_T] \\
c_N &= \bar{e} + \gamma[p_T - p_N] \\
\lambda &= (1 - \gamma)\bar{e}
\end{align*}
\]

- In the long term

\[
\begin{align*}
\dot{p}_T &= \dot{e} \\
p &= (1 - \gamma)p_N + \gamma p_T \\
\bar{r} &= \bar{\tau} = 0 \\
l = gN = \frac{\bar{e} - \bar{p}_T}{\gamma(1 - \gamma)} \left( \bar{e} + \bar{p}_T - \bar{p}_N \right) \\
\bar{e} &= \frac{R_0}{\mu - \nu} \left( \psi + 1 \right) \left( \frac{1 - \nu}{\nu + 1 + \psi} \right) (1 - q) \left[ h + \bar{p}_T - \bar{p} \right] + q\beta R_0 \bar{e} \\
\bar{c}_T &= \bar{c} + (1 - \gamma)[p_N - p_T] \\
\bar{c}_N &= \bar{c} + \gamma[p_T - p_N] \\
h &= \frac{1}{(\psi + 1)} \left[ \bar{p}_N - \bar{p}_T \right] + \frac{(1 - \gamma)}{\theta(\psi + 1)} gN - \frac{q}{\mu - \nu} \bar{R} - \frac{1}{(\psi + 1)} \bar{\tau} \\
\bar{m} - \bar{p} &= \bar{e} - \frac{\bar{e}}{\gamma^2} \\
\bar{c} &= (1 - \gamma)\bar{y}_N - g + (R_0 - 1) \bar{v} \\
\lambda &= (1 - \gamma)\bar{c} - \bar{p}_N
\end{align*}
\]
The equations give us, respectively, the PPP on the exchangeable goods, the general price level index, the real interest rate, the labor supply and thus the production, the path of expansion of the overall consumption, the consumption of exchangeable goods and nonexchangeable goods according to the overall consumption, the expression of the human wealth, the money demand, the current account, the real exchange rate, everything in the long run.

3 Macroeconomic effects of the public debt

In this section, we consider a temporary reduction (i.e. over one period) of taxes financed by a new debt. We introduce nominal rigidities in the shape of viscous prices of nonexchangeable goods ($\tilde{p}_N = 0$). The consequence is that in the short-term the variations of the production in the monopolistic nonexchangeable goods sector cannot be analyzed as the result of the choices made on the supply side.

At the moment, it is necessary to note, in order to conclude the analysis, the log-linearized version of the budgetary constraint of the government.

$$\tilde{g} = \tilde{\tau} + \gamma \tilde{d}$$  \hspace{1cm} (20)

By assumption, the public expenditures are unchanged: $-\tilde{\tau} = \gamma \tilde{d}$

So, the budgetary equilibrium conditions in the long run give us:

$$\gamma (R_0 - 1) \tilde{d} = - (R_0 - 1) \tilde{\tau} = \tilde{\tau}$$  \hspace{1cm} (21)

The interests of the debt accumulated in the short run will come to enlarge the tax rate of long term.

In the short run, a temporary reduction (i.e. over one period) of the taxes financed by a public debt involves the increase in domestic consumption, the appreciation of the real exchange rate as well as the accumulation of a foreign debt. It is this foreign debt which will be the principal transmission channel between the public debt and the long term real exchange rate. In the long run, one finds the positive relation between stock of net foreign assets and the real exchange rate clarified by Lane and Milesi-Ferretti. The reason is that, parallel to the impact of a negative wealth effect on the money demand and thus on the general price level, the accumulation of a foreign debt will also lead to a modification of the demand for nonexchangeable goods and especially to a fall of the labor supply. Those tendencies will each involve a downward pressure on the price of nonexchangeable goods.

According to the log-linearized version of the general price level index,

$$\tilde{p} = (1 - \gamma) \tilde{p}_N + \gamma \tilde{c}$$

it is observed that the nominal exchange rate will have to depreciate to compensate for both the fall in the prices of nonexchangeable goods (as much
as their share in the economy is significant) and the rise of the general price level index

Thus, the real exchange rate will depreciate too.

\[ \hat{\lambda} = (1 - \gamma) [\bar{c} - \bar{p}_N] \]

### 3.1 Short term effects

The equations (8), (11) and (22) show that a temporary decrease in taxes, will increase the current consumption. Indeed, additional future flows of taxes related to the respect of the government budgetary constraint in the long-term appear as less important for people than the decrease of current taxes. The representative agent of age a is not certain to have to face them. So he will balance them by its probability of survival.

#### 3.1.1 Awaited results

The two effects enlightened by Ganelli (2003) are as follows:

- A “Money Demand” effect: The rise of consumption will result in an appreciation of the nominal exchange rate in the short run. Indeed, the rise of the demand for money for transaction which follows the rise of consumption, will involve, with constant money supply, a decrease of the general price level. Only an appreciation of the nominal exchange rate can reabsorb the difference between the general price level of short term and that which allows to balance the money market (prices are rigid in the short-term).

- A Current account effect: The rise of consumption will result in a depreciation of the nominal exchange rate in the short run. The mechanism is the following: the rise of the demand for foreign money for foreign trade which follows the rise of consumption will involve a direct downward pressure on the nominal exchange rate.

The consequences of the introduction of a nonexchangeable goods sector were indexed in particular by Hau (2000) and do not question the results of Ganelli. On the contrary, introducing a nonexchangeable goods sector will reinforce the “Money Demand” effect and will weaken the “Current Account” effect in such a way that it will always lead to an appreciation of the nominal exchange rate in the short run and the creation of a foreign debt.

The money demand effect is amplified. The fact that the domestic prices of the nonexchangeable goods are not related to their foreign prices by the PPP and are rigid in the short term will amplify the fluctuation required for the nominal exchange rate which follows the general price level variation (\[ \tilde{p} = (1 - \gamma) \bar{p}_N + \gamma \tilde{p}_T = \gamma \bar{c} \])

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In addition, a significant share of the nonexchangeable goods in consumption implies that a domestic growth will be mainly sent towards these nonexchangeable goods. We call this a "bias in favour of the domestic goods". In this case, the current account effect will be limited.

However, in Ganelli’s, the money demand effect already carried it on the current account effect. It seems that a shock of debt should thus always contribute to the appreciation of the nominal exchange rate in the short run.

3.1.2 Results

It is possible to reduce the system of log-linearized equations to two equations which respectively represent the current account effect and the money demand effect.

$$\bar{e} = \frac{\Gamma_2}{\Gamma_3} \left[ \frac{\Gamma_1 \gamma}{\Gamma_2 (\psi + 1)} \tilde{g} + \Gamma_5 \bar{c} + (R_0 - 1) \tilde{g} + \tilde{g} \right]$$ (22)

$$\bar{e} = -\frac{\Gamma_1 \gamma (R_0 - 1)}{(\psi + 1) \Gamma_4} \tilde{g} - \frac{\Gamma_6 \bar{c}}{\Gamma_4}$$ (23)

The composite parameters and their signs are summarized in the appendix.

It is interesting to note that these two equations establish a relation between consumption and the nominal exchange rate in the short term. In fact, they represent the two transmission channels described above. It will be noted that the equation (23) is the result of the Euler and money demand equations whereas the equation (22) results from the other equilibrium conditions of the model.

The current account effect (22) is the following. The rise of the exchangeable goods consumption automatically results in a rise of absorption and in particular in a rise of the demand for foreign currency. Nevertheless, a depreciation of the domestic currency can be likely to reorientate whole or part of this rise of consumption towards the nonexchangeable goods. Here, a rise of consumption always results in a depreciation of the nominal exchange rate ($\bar{e} > 0$) because the households want to consume a quantity of exchangeable good higher than their initial steady state equipment. The depreciation will nevertheless be all the weaker since the preference for the nonexchangeable goods is strong.

The money demand effect (23) is the following. The increase in the wealth and thus in the current consumption will encourage the households to modify the amount of money they wish to hold. But, money is not only held for transaction but also for its specific qualities, liquidity for example. The utility function (6) shows indeed that the detention of real money causes utility. Thus, a rise of consumption involves a rise of the demand for money for transaction reasons but it can be different for the money demand for monetary saving. If
the consequences of the shock are a strong improvement in the bonds income, it can result in a stronger decrease in the demand of money for monetary saving.

However, here the effects of a shock on the initial steady state depend on its characteristics and in particular on the real interest rate at the steady state \( (R_0) \). We can show that the real interest rate that allows this specific steady state is a function of the probability of survival \( (q) \) and of the discount factor \( (\beta) \). Nevertheless, for plausible values of \( q \) and \( \beta \) \( (\beta > 0.6 \text{ et } q > 0.8) \), it is possible to show that an increase in consumption involves an appreciation of the nominal exchange rate in the short run.

On the whole, by combining the equations, we obtain the following relation:

\[
\tilde{c} = \frac{1}{\Gamma_3} \left( \frac{1}{\Gamma_3 \Gamma_5} + 1 \right) \left[ -\Gamma_1 \gamma \left( \frac{1}{\theta - 1} + \frac{\Gamma_2 \Gamma_5 (R_0 - 1)}{\Gamma_6 (\psi + 1)} \right) \tilde{d} + \Gamma_2 (R_0 - 1) \tilde{g} + \Gamma_3 \tilde{g} \right]
\]

(24)

For possible values of \( q \) and \( \beta \), temporary fall of taxes in the domestic country, financed by a rise in public debt, involves an appreciation of the nominal and real exchange rates according to:

\[
\tilde{\lambda} = (1 - \gamma) \tilde{c}
\]

(25)

The equation (25) correctly translates the stylized fact according to which the variations of the nominal and real exchange rates are very close. It is the rigidity of the short-term prices which makes it possible to find this relation.

The consequence of the appreciation of the real exchange rate and the increase in absorption is of course the accumulation of a foreign debt.

The current account at the end of the first period

\[
\tilde{v} = (1 - \gamma) \tilde{y}_N - \tilde{c} - \tilde{g}
\]

where \( \tilde{g} = 0 \) by hypothesis, \( \tilde{y}_N = \tilde{c}_N \) and \( \tilde{c}_N = \tilde{c} + \gamma [\tilde{p}_T - \tilde{p}_N] = \tilde{c} + \gamma \tilde{c} \) becomes

\[
\tilde{v} = \gamma [(1 - \gamma) \tilde{c} - \tilde{c}]
\]

(26)

The appreciation of the real exchange rate \( (\tilde{\lambda} = (1 - \gamma) \tilde{c}) \) will deteriorate the current account by a fall of the domestic country competitiveness on the exchangeable goods. In addition, the rise of consumption will accentuate this deficit by an increase in absorption. As the stock of foreign assets is supposed to be null in the initial steady state, the domestic country accumulates a foreign debt until the end of the first period.
3.2 Long term effects

It is important to recall that the long term fluctuations of consumption, of production and thus, in fine, of the nominal exchange rate depend on the impact of the accumulation of a foreign debt on the constitution of a stock of net foreign assets by the domestic country. It is the wealth effect related to accumulation of a liability or a debt on the rest of the world which will link the shock on the public debt in the short term and the long term fluctuations.

However, according to the equation of the long term current account,

\[ \tilde{c} = (1 - \gamma) \tilde{y}_N + (R_0 - 1) \tilde{v} \] 

\[ \Leftrightarrow \tilde{v} = \frac{1}{(R_0 - 1)} \tilde{c} - \frac{(1 - \gamma)}{(R_0 - 1)} \tilde{y}_N \]

the counterpart of a modification of the external situation is a variation of consumption and labor supply.

These two phenomena, whose origin is a wealth effect, will result in a variation of the general price level (money demand effect) and prices of nonexchangeable goods (“transfert effect”). The transfert effect seems to be the transmission channel of the stock of net foreign assets to the real exchange rate evoked by Lane and Milesi-Ferreti (2000).

As the variation of the nominal exchange rate is the consequence of the variations of prices, if we look again at the equation of the long term general price level:

\[ \hat{p} = (1 - \gamma) \hat{y}_N + \gamma \hat{c} \]

The variation of the nominal exchange rate of long term is the consequence not only of the variation of the general level of the prices but also more particularly of the price level on the market of nonexchangeable goods.

\[ \hat{c} = \frac{1}{\gamma} \hat{p} - \frac{(1 - \gamma)}{\gamma} \hat{y}_N \] 

The variation of the long-term price of nonexchangeable goods is the consequence of the existence of a foreign liability or debt on supply and demand for nonexchangeable goods.

The variation of the nominal exchange rate results thus from two effects, the money demand effect which leads to a variation of the general price level and a transfert effect which leads to a variation of the prices of the nonexchangeable goods.
Thus according to the equation of the demand for money,

$$\hat{m} - \hat{p} = \hat{c} - \frac{\tilde{r}}{R_0}$$

(29)

where $\hat{m} = 0; \hat{r} = 0$

it comes that :

$$\hat{p} = -\hat{c}$$

(30)

If the domestic long term consumption decreases, that will involve a decrease in the money demand and thus, with a constant money supply, an increase in the general price level. It is the money demand effect.

Moreover, according to the equation of demand for nonexchangeable goods,

$$\tilde{c}_N = \hat{c} + \gamma [\hat{p}_T - \hat{p}_N]$$

it comes that variation of the price of nonexchangeable goods will depend on the impact of the wealth effect both on labor supply and on demand for nonexchangeable goods.

The variation of the price of nonexchangeable goods is thus :

$$\tilde{p}_N = \frac{1}{\gamma} (\hat{c}_N - \tilde{c}_N)$$

(31)

and then :

$$\tilde{p}_N = \frac{1}{1 + \frac{\psi \cdot \theta}{(1 - \gamma) \gamma}} \frac{(R_0 - 1)}{\gamma} \hat{c} + \hat{c}$$

(32)

On the one hand, the stress is put on the existence of a negative wealth effect which will involve a fall of consumption and then a decrease in the demand for money. The consecutive rise of prices, which balances the money market, requires a depreciation of the equilibrium long-run nominal exchange rate. This money demand effect is all the more significant since the part of the nonexchangeable goods in national consumption is large. It’s not surprising and joins Hau’s work (2000).

On the other hand, an income effect, combined with the fall of consumption, will modify balances on nonexchangeable good and labour markets. It results from this a fall of the price of nonexchangeable goods in the long run.

On the whole, the variation of the long-run nominal exchange rate will have to reabsorb the money demand effect, which pushes the general price level up and which is all the more strong since the country is slightly open ($\gamma$ close to zero), and the transfert effect which pushes the prices of nonexchangeable goods down.

$$\hat{e} = -\frac{(R_0 - 1)}{\gamma} \hat{p}$$

(33)
3.2.1 The real exchange rate

According to the formulation of the real exchange rate, the log-linearized version gives us the following relation:

$$\lambda = (1 - \gamma) [\bar{e} - \bar{p}_N]$$  \hspace{1cm} (34)

So we have:

$$\lambda = \frac{(R_0 - 1)}{[1 + \frac{\psi}{(\theta - 1)(1 - \gamma)}]} \frac{(1 - \gamma)}{\gamma} \hat{\gamma}$$  \hspace{1cm} (35)

Thus, we really have a positive relation between the stock of net foreign assets and the appreciation of the real exchange rate, such as given by Lane and Milesi-Ferretti (2000). The more the share of nonexchangeable goods is significant in the consumption of the domestic country, the more its real exchange rate depreciate. Indeed, there is no more deviation to the PPP of long term and the variation of the real exchange rate is null if the share of non exchangeable goods in domestic consumption tend to be null.

The deviation with the PPP makes it possible for depreciation of the real exchange rate and depreciation of the nominal exchange rate to coexist.

4 Conclusion

This model integrates the conclusions of the recent literature on the empirical relation between the stock of net foreign assets of an economy and the appreciation of its real exchange rate (Lane and Milesi-Ferretti in particular) within a framework of New Macroeconomics in Open Economy. This relation is indeed central to reconsider the relation between the national debt and the real exchange rate of a country.

A temporary fall of taxes, financed by public debt, will involve a depreciation of the long term real and nominal exchange rates under realistic assumptions concerning the parameters $q$ and $\beta$, by the creation of a deficit of the current account and the accumulation of a foreign debt.

The microeconomic rigidity of prices is fundamental for the analysis. Not only it simplifies some shocks (increase of the nonexchangeable goods sector supply in reaction to the increase in demand and thus increase of incomes and consumption) but it also prevents an adjustment by prices, and thus report all the adjustment on the nominal exchange rate. Lastly, the deviation with the PPP which it induces allows the movements of the real exchange rate. Those movements will be fundamental for the capacity of a transitory shock to involve a deficit or a surplus of the current account. The current account will be indeed the main driving belt between a transitory shock on the initial stationary state and the long term steady state.
Moreover, we show that the opening of the country to foreign trade limits the response of the economy to a domestic shock on its public debt. The fall in the prices of nonexchangeable goods in the long run, which is the consequence of the accumulation of a foreign debt, amplifies the variations of the nominal exchange rate which is necessary to balance the money market.

In so far as we observed a new rise of the public debt of the South American emergent countries\textsuperscript{15}, it can be interesting to see whether this approach is checked by the current evolution of the nominal and real exchange rates. This constitutes the continuation of our work.

5 Appendix

5.1 Justification of the model’s assumptions for the case of the South American emergent countries.

Although we did not lead empirical validation of this approach yet, the objective is to test the empirical range of it later\textsuperscript{16}. So we have used assumptions which are consistent with the commercial situation of most of these countries. We point out the main data which enable us to justify the structure of the productive sector.

The share of the agricultural produce and of minings, whose prices are given on international markets, accounts for more than 40\% of their exports in 2001. In addition, among the manufactured goods, which account for 60 \% of exports into 2001, domestic firms have no market power for more than half of these products.

- Iron and steel
- Chemicals
- Products of the car industry
- Means of transport.

On the whole, the South American countries have no impact on the price of 76\% of their exports. This validates our assumption to fix the price of the exchangeable good as exogenic and constant at its foreign level (perfect competition). The variations of the national demand do not have any impact on the world price according to the assumption of small country.
Figure 1: Rate of growth of the money supply in the South American countries -1991 -2002
5.2 Evolution of the money supply in the South American countries

5.3 Effects of the public debt on the nominal and real exchange rates in the short term

5.3.1 Value of the parameters

\[
\begin{align*}
\Gamma_1 &= \frac{R_0}{R_0 - q} (\psi + 1) \left( 1 - \frac{2\beta}{\psi + 1} \right) (1 - q) > 0 \\
\Gamma_2 &= \Gamma_1 (-1 + \frac{1}{\psi + 1} - \frac{\Gamma_1 (1 - \gamma) \psi (1 - \gamma) q \beta R_0}{\psi (1 - \gamma) (\psi + 1) (\theta - 1)}) + 1 + \frac{\Gamma_1 \psi \beta R_0}{(\psi + 1) (\theta - 1)} > 0 \\
\Gamma_3 &= (1 - \gamma) (q \beta R_0 + \Gamma_6 (R_0 - 1) \gamma) \\
\Gamma_4 &= (1 - \gamma) q \beta R_0 + \Gamma_2 [\gamma (R_0 - 1) - 1] \\
\Gamma_5 &= \frac{\psi R_0}{\psi + 1} + (R_0 - 1) \gamma > 0 \\
\Gamma_6 &= \gamma (R_0 - 1) \Gamma_3 - q \beta R_0
\end{align*}
\]

The signs of the coefficients \(\Gamma_4\) and \(\Gamma_6\) depend on two parameters, \(q\) and \(\beta\) which are the birth rate and the discount rate.

5.3.2 Effects of the public debt

The effects of the public debt on the nominal and real exchange rates are given by:

\[
\frac{\partial \bar{e}}{\partial d} = - \frac{\Gamma_1 \gamma}{(\Gamma_2 + \Gamma_5) \Gamma_3} \left[ \frac{1}{(\theta - 1)} + \frac{\Gamma_2 R_0 - 1}{\Gamma_6 (\psi + 1)} \right] \quad (36)
\]

where \(\frac{\Gamma_1 \gamma}{(\Gamma_2 + \Gamma_5) \Gamma_3} > 0\).

\[
\frac{\partial \lambda}{\partial d} = (1 - \gamma) \frac{\partial \bar{e}}{\partial d} < 0 \quad (37)
\]

But the sign of the coefficient is not completely defined. It depends on the values of \(q\) and \(\beta\).

Possible values of the parameters show us that a rise of the public debt in the view to finance a decrease of taxes will lead us to the accumulation of a foreign debt and a depreciation of the long run nominal and real exchange rates.

In the following example, the probability to survive \((q)\) is equal to 0.90, which is really below the real (child rate of death around 0.7%). For the discount rate, \(\beta\), we take

\[
\beta = \frac{1}{\Gamma + s}
\]

\[15\text{Cf. IMF’s World Economic Outlook 2003}

\[16\text{The empirical tests of this approach are still very few because of the particular characteristics of the initial steady state. Nevertheless see Bergin (2002) and Ghironi (1999b).}
Figure 2: Impact of a shock on the public debt on the real and nominal exchange rates for different rates of foreign trade

where $s$ is the saving rate (because the interest rate is endogeneous in the model). The saving rate is low in the south america countries, around 15% (13.4% according to Edwards (1995)), which gives us

$$\beta = \frac{1}{1 + 0.15} = 0.87$$

In this particular case, we obtain the following results which are a function of the opening of the country to foreign trade ($\gamma$):
Part of the foreign trade

Figure 3: Impact of a shock on the public debt on the current account for different rates of foreign trade

References


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