Optimal monetary policy in a New Keynesian model of the euro area: the role of the policy regime

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Abstract

We estimate a New Keynesian model of the euro area economy, also looking at US and German data for a comparison. Our main result is that, after the end of the dis-inflation process of the early 1980s, inflation appears to be a stationary process in the euro area, mainly driven by the output gap and with little or no intrinsic inertia. We argue that this inflation behaviour is also likely to prevail in the new environment of Stage Three of EMU, as it is the sign of a credible monetary policy regime. We also derive the monetary policy rule which is optimal under commitment given the estimated structure of the euro area economy. We find that, in an environment of credible monetary policy, the central bank should react relatively little to supply shocks, while we find a rather strong reaction to demand shocks which is related to the fact that the output gap is very persistent. Moreover, we find evidence that, broadly speaking, this is what the ECB has done since the start of Stage Three of EMU.

Keywords: Euro area, credibility, inflation inertia, optimal monetary policy rule, commitment.

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1 Introduction

Conducting monetary policy in the euro area is a unique challenge in the new environment of Stage Three of EMU. The uncertainty surrounding the structural parameters of the economy in the new monetary policy regime established after the introduction of the euro contributes to making monetary policy in the euro area arguably more challenging than elsewhere.

One major dimension in which the establishment of a new monetary policy regime might affect the structural parameters of the economy is by driving the degree of inflation inertia. If inflation is inertial, temporary innovations in the inflation process feed expectations about medium-term price developments and can become sustained if the central bank does not conduct a strong anti-inflationary monetary policy. This may happen, typically, when agents are uncertain about the central bank inflation target and ‘learn’ it in part by looking at actual inflation developments.\(^1\) By contrast, if inflation displays no inertia and inflation expectations are firmly bound to the central bank inflation target, temporary shocks to inflation are not a source for concern since they are not passed on to medium-term inflation developments. In this environment, the task of the central bank becomes ‘easier’ since, in the absence of cost push shocks, its monetary policy decisions only have to track developments in the equilibrium rate, without creating unnecessary fluctuations in the output gap (Goodfriend and King, 1997; Gali, 2001).

Against this background, the main objective of this study is to estimate a two-equation New Keynesian model of the euro area economy, similar to what was done for example by Peersman and Smets (1999), concentrating our attention on inflation inertia and the role of the monetary policy regime in determining it. Moreover, and in the same spirit as in Peersman and Smets (1999), we identify the optimal monetary policy rule for the euro area based on the estimated parameters for the euro area economy. In our case, however, given the forward looking nature of the model this is done under commitment. Our aim to derive a monetary policy rule for the euro area economy which should be optimal in the new environment created by the introduction of the euro in January 1999.

It should be emphasized that, although our objectives are similar to those of Peersman and Smets (1999), our analysis departs from that study in several dimensions. First, the model which we estimate is back-

\(^1\)For example, Erceg and Levin (2003) build a model in which rational private agents have limited information about the central bank inflation objective and have to use optimal filtering techniques to infer it from actual inflation developments.
ward and forward looking, while Peersman and Smets (1999) have a completely backward looking model. The focus on a forward looking model, in turn, allows us to consider optimal monetary policy rules under commitment, which makes sense only in a forward looking context. Furthermore, the data which we use cover the period up to the end of 2002, including the first four years of Stage Three of EMU, and cover the whole euro area, as opposed to five euro area countries over the 1975-1997 sample period as Peersman and Smets (1999). Finally, as noted above, our focus is on the role of the monetary policy regime in determining the degree of inflation inertia and therefore the optimal monetary policy rule under commitment which might be interpreted as an ‘optimal guideline’ for the monetary policy of the ECB during Stage Three of EMU.

The paper is structured as follows. In Section 2 we lay down and estimate a standard, small scale New Keynesian model of the euro area economy (also looking at the US and German data for a comparison). This is similar to what has been done in a large number of studies, mainly for the US but also for the euro area economy (see, e.g., Coenen and Wieland, 2000, and Benigno and Lopez Salido, 2002). By doing so, we identify a key structural break which occurred in the euro area after the mid 1980s, and is of the utmost importance as far as the degree of inflation inertia in the euro area economy is concerned. In Section 3, we derive the optimal monetary policy rule under commitment against the background of the parameters of the economy estimated in Section 2 and which we believe to be relatively unaffected by the change in monetary policy regime after the introduction of the euro. In particular, we look at both the optimal unconstrained rule as well as an optimal rule constrained to take the form of a Taylor rule with interest rate smoothing. Section 4 contains some sensitivity analysis of the results. Section 5 concludes.

2 A New Keynesian model for the euro area economy

2.1 Model specification

It has become common to evaluate monetary policy in a small scale model of the economy including a demand (IS) and a supply (Phillips) curve. The specification which we use in this paper is as follows:

\( x_t = \alpha_k x_{t-1} + \alpha_f E_t x_{t+1} + \beta \tilde{r}_{t-1} + \varepsilon_t^D, \)  
(1)

\( \pi_t = \gamma_k \pi_{t-1} + \gamma_f E_t \pi_{t+1} + k x_t + \varepsilon_t^S, \)  
(2)

where \( x \) is a measure of excess demand, \( \tilde{r} \) a deviation of the real interest rate from the steady state level, \( \pi \) is annualised and seasonally adjusted.
quarter-on-quarter consumer price inflation rate (the HICP in the euro area), and \( \varepsilon^D_t, \varepsilon^S_t \) are respectively a demand and a supply shock. The first equation is the IS curve, the second one the Phillips curve. Our aim is to estimate the model in (1)-(2) on euro area quarterly data from 1980:Q1 to 2002:Q4, also using US and German data for a comparison. A model similar to (1)-(2) has been already estimated on euro area data, in part or in full, by Coenen and Wieland (2000, 2002) and Gali, Gertler and Lopez Salido (2001) on quarterly data, and Smets (2000) on annual data. However, our estimate differs from previous attempts in some important respects, as will become clear shortly. The forward looking nature of the model represents a key point of departure from Peersman and Smets (1999).

The specification in (1)-(2) is the so-called ’hybrid’ model of inflation, as described for example by Gali and Gertler (1999). The importance of the backward looking element is captured by the coefficients \( \alpha_b \) and \( \gamma_b \); if \( \alpha_b = \gamma_b = 0 \), the model in (1)-(2) becomes purely forward looking.

A hybrid specification of the Phillips curve assumes that a fraction of price-setters is backward looking (see, e.g., Smets and Wouters, 2003), while a completely forward looking nature of price setting is typical of the New Keynesian approach. The fundamental reason behind the use of a hybrid model is empirical; most estimates have shown that a purely forward looking Phillips curve fits the data quite poorly. In particular, there is often a positive autocorrelation in the inflation data which is not accounted for by other determinants such as the output gap (Gali and Gertler, 1999).

In addition, we include both a backward and a forward looking component in the IS curve. The inclusion of a backward looking term is, again, motivated mainly by empirical reasons, and a theoretical New Keynesian IS curve includes only the forward looking term (Clarida, Gali and Gertler, 1999). The inclusion of a backward looking term in the IS curve can be rationalised, for instance, by the existence of habit formation in consumption, as emphasized by Amato and Laubach (2002). Moreover, and perhaps most relevant in our analysis, a backward looking term in the Phillips curve can reflect adaptive (or more in general not completely rational) expectations by economic agents (Roberts, 1997).

In this paper, we use the output gap (i.e., the deviation of log real GDP from a measure of trend) as our measure of excess demand \( x \). While the use of the output gap as a measure of the business cycle is standard in the literature, it has been criticised forcefully by Gali, Gertler and Lopez Salido (2001) for being an inappropriate measure of cyclical conditions.

\^{2}\text{In turn, if the output gap is a highly persistent series due to habit formation, also inflation may turn out to be a persistent process.}
in the euro area. This conclusion is based on the consideration that potential output is hit continuously by shocks and is therefore not likely to evolve in the smooth way assumed when building measures of the output gap based on statistical filters. Gali, Gertler and Lopez Salido (2001) propose a more direct measure of marginal costs, namely real unit labour costs. However, as will be shown later, we find real unit labour costs, either raw or detrended, to be a quite poor measure of cyclical conditions in the euro area, consistent with the recent findings of Jondeau and Le Bihan (2001) for the euro area data and Rudd and Whelan (2002) for the US data, while we obtain relatively good results for the output gap. Against this background, we decide to use the output gap as our measure of excess demand in the euro area.

When computing the output gap, the question arises of the statistical filter to remove the long run component from the raw log real GDP series. In the case of output, there is no consensus in the literature on the 'correct' filter to use, though a linear trend and the Hodrick-Prescott (henceforth HP) filter are commonly used. First and foremost, the selection of the statistical filter should reflect the behaviour of the raw series. In this respect, from a visual inspection the long-term behaviour of euro area (log) real GDP seems to be well approximated by a linear trend. Thus, also in the light of the well known end-period problem of the HP filter, we take out the long-term component of log output using a linear trend, and our main definition of the output gap, \( x \), is the deviation of log output from the linear filter.\(^3\) However, we also repeat the analysis using an output gap measure computed by de-trending output with the HP filter, which leads to similar (although not identical) empirical results, as we will show in the next section.

Coming to the interest rate gap term \( \bar{r} \), this is obtained as a four-quarter moving average (similar to Rudebush and Svensson, 1999) to take into account the transmission lags of monetary policy in a parsimonious way. In particular, the interest rate gap is defined as follows:

\[
\bar{r}_t = \frac{1}{4} \sum_{h=t}^{t-3} (i_h - E_h \pi_{h+1} - \pi_t),
\]

(3)

where \( i \) is the short-term (3-month) nominal interest rate (controlled by the central bank) and \( \pi_t \) is an estimate of the steady state level of the real rate, which can be time-varying. For simplicity, we compute the real rate using the ex post realized inflation rate one quarter ahead, i.e. assuming perfect foresight by economic agents.

\(^3\)This procedure is, of course, very common in the literature; see for example Fuhrer and Moore (1995).
To obtain an estimate of the equilibrium rate $\tau_t$, it has to be taken into account that the euro area experienced different policy regimes during our sample period. So, the equilibrium real rate may have changed in a non-smooth way due to policy regime switches, for example after the EMS collapse in 1992-1993. Indeed, the fiscal policy stance and exchange rate risk premia are factors which may have a significant bearing on real interest rates and may depend crucially on the policy regime (and were particularly affected by the collapse of the EMS in 1992-1993 and the subsequent convergence process towards the monetary union in the mid- and late 1990s).

A visual inspection of the real 3-month rate in the euro area, reported in Figure 1, suggests that this was indeed the case. The large fall in the real rate after 1992, in particular, was not followed by a boom in economic activity, which suggests a decrease in the equilibrium real rate rather than a fall in the interest rate gap. The visual analysis of the real 3-month rate also suggests that a linear trend would be inappropriate to remove the long-run component in this variable. Rather, in this case we apply an HP filter which seems to de-trend the series reasonably well. The fall in the real rate from the highs in the late 1980s (around 6%) to the lows in the early 2000s (around 2%) is worth emphasizing, and certainly cannot only reflect cyclical developments.

As regards the Phillips curve, our specification is relatively standard and does not deserve further comments. The only relatively controversial issue may be the timing of the effect of the output gap on prices, which is very uncertain (Batini and Nelson, 2001). However, the output gap $x$ is, irrespective of the estimation method, a strongly positively autocorrelated series, so the precise timing in equation (2) should not matter much for the estimate of the slope parameter $k$, and indeed this is what we find when we carry out a sensitivity analysis of the lag structure of the model.

We estimate the model in (1)-(2) using the generalised method of moments (GMM). We include two lags of all endogenous and exogenous variables in the list of the instruments. The results of the estimates are not overly dependent on the precise list of instruments, although we do not want to overemphasise this robustness as it is possible to find somewhat different results with a much shorter or longer instruments list. In addition, we estimate a backward looking version of the equations,

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4On the role of the exchange rate risk premia in nominal interest rates prevailing in euro area countries other than Germany, see also Coenen and Wieland (2000).

5Goodhart and Hofman (2001), for example, use the same methodology to de-trend the real short-term interest rate (as well as other asset prices) in their estimate of the IS curve in several industrialized countries.
2.2 Estimation results

Starting with the estimation of the IS curve, Table 1 reports the results of the estimates for equation (1) in the euro area and the US (the variables for the latter are defined in exactly the same way as the euro area, to ensure the full comparability of the results). The estimates of the parameters for the euro area equation are displayed in the first column on the left side of the table. These results are only partially satisfactory. Interestingly, there is evidence for both a backward looking and a forward looking component in the output gap in the euro area, with the backward looking component being somewhat more important. The sum of the coefficients associated to the backward and forward looking components ($\alpha_b + \alpha_f$) is not statistically different from one, which confirms previous findings that the output gap is a highly persistent series. The interest rate gap term has the expected negative sign, but is significant only at the 10% and not at the 5% significance level. The results for the US (second column on the left) are very similar to those for the euro area, with a slightly larger weight on the forward looking component ($\alpha_f$). The coefficient on the interest rate gap is slightly smaller and again only significant at the 10% significance level. Finally, both estimates display a significant amount of (negative) serial correlation. The J test and
other diagnostic tests (not reported for brevity) suggest that the model is well specified. Moreover, the J test suggests that the overidentifying restrictions are appropriate.

As the forward looking specification appears to be only in part satisfactory, we also estimate a more traditional backward looking specification of the IS curve with OLS (third and fourth columns from the left in Table 1). The results appear to be satisfactory especially for the euro area, where there is no sign of serial correlation. The coefficient of the interest rate gap term is now statistically significant even at the 1% confidence level, and is larger in absolute terms, at $-0.27$. There is again evidence that the output gap is a highly persistent series, with $\alpha_b = 0.95$. Interestingly, the results for the US are very similar to those for the euro area ($\alpha_f = 0.92$, $\beta = -0.23$), although there is evidence of some residual positive serial correlation in the US equation.

Table 1 – Estimates of the IS curve in the euro area and the US

<table>
<thead>
<tr>
<th></th>
<th>Euro area (GMM)</th>
<th>US (GMM)</th>
<th>Euro area (OLS)</th>
<th>US (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_b$</td>
<td>0.60* (0.10)</td>
<td>0.54* (0.05)</td>
<td>0.95* (0.03)</td>
<td>0.92* (0.04)</td>
</tr>
<tr>
<td>$\alpha_f$</td>
<td>0.39* (0.11)</td>
<td>0.47* (0.06)</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.12 (0.07)</td>
<td>-0.09 (0.05)</td>
<td>-0.27* (0.07)</td>
<td>-0.23* (0.08)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.97</td>
<td>0.97</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>J test</td>
<td>0.00</td>
<td>0.00</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>S. e. of regression</td>
<td>0.0032</td>
<td>0.0036</td>
<td>0.0068</td>
<td>0.0045</td>
</tr>
<tr>
<td>Durbin Watson</td>
<td>2.85</td>
<td>2.67</td>
<td>1.83</td>
<td>1.24</td>
</tr>
</tbody>
</table>

The results of the estimate of the euro area IS curve when the output gap is measured by using the HP filter, not reported here for brevity, are broadly similar to those displayed in Table 1 (for the OLS estimate, for example, we obtain $\alpha_b = 0.84$, $\beta = -0.24$).

Overall, we tend to prefer the backward looking specification of the IS curve, especially for the statistical significance of the interest rate gap term and for the absence of serial correlation in the euro area equation. The problems encountered in the forward looking specification of the IS curve confirm previous results in the literature (for example Fuhrer and Rudebusch, 2002, for the US data) that a backward looking specification may work better from an empirical perspective. Nonetheless, in

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6In the US, there is some positive serial correlation in the OLS estimate, which we impose in the model economy in the next section.
the next section we also check whether taking the ‘mixed’ backward and forward looking specification (as opposed to the only backward looking one) changes the features of the optimal monetary policy rule to a significant extent, and we find this not to be the case, which is a good sign for the robustness of the analysis.

We run a battery of diagnostic tests on the IS curves estimated with OLS, with good results. In particular, there is no indication of instability in the euro area equation, as can be seen for example in the recursive residuals series (Figure 2). We also estimate the equation up to 1998:Q4 and evaluate the out-of-sample forecasting performance since 1999:Q1 (the start of Stage Three of EMU), finding a satisfactory performance; in particular, the Chow out-of-sample forecast test is readily passed. This indicates that the establishment of the monetary union in 1999 appears to have implied no structural break in the transmission of monetary policy in the euro area, although it is clearly too early to come to a definitive conclusion on this matter.

Turning to the estimate of the Phillips curve in (2), previous attempts on euro area data (for example Coenen and Wieland, 2000) have used sample periods covering the early 1980s and, in some cases, also the 1970s. However, a simple visual inspection of the inflation series (see Figure 3) suggests the existence of a structural break in the mid 1980s, corresponding to the end of the disinflationary period in the early 1980s. Indeed, standard statistical tests promptly signal the existence of a break in the mid-1980s. It is also worth noting that in the full sample period,
from 1980:Q1 to 2002:Q4, it appears from standard statistical tests that inflation is a I(1) series. From 1986:Q1 to the end of the sample period, instead, inflation is a clearly stationary variable.

These observations seem to point to the idea of a policy regime change in the mid-1980s which had a very significant bearing on the behaviour of the inflation rate in the euro area. In particular, after the ‘restoration’ of monetary stability and the establishment of a broadly credible monetary regime (at least in most euro area countries) from the mid-1980s onwards, the inflation rate seems to be stationary and strongly correlated with the output gap, as Figure 4 suggests. Moreover, the inflation rate appears to be characterized by limited ‘intrinsic’ inertia (i.e. related to the coefficient $\gamma_b$); its persistence seems to be at least partly related to that in the output gap.\(^7\)

It is to be noted that the post-1986 inflation series displays a much less close correlation, if at all, with real unit labour costs (detrended by using the HP filter), the measure of excess demand favoured by Gali, Gertler and Lopez Salido (2001), as Figure 5 shows. Moreover, the time series behaviour of real unit labour costs do not seem to match what is the common perception of cyclical behaviour in the euro area (Jondeau and Le Bihan, 2001). For example, real unit labour costs rise strongly in

\(^7\)It should be clear that, of course, there were several regime changes in the sample period from the mid 1980s to date; for example the end of the ERM period in 1992-1993 certainly represented a policy regime change. What is claimed here is only that, according to our evidence, these regimes can be grouped as a single regime as far as the degree of intrinsic inflation inertia is concerned.

Figure 3: The quarterly inflation rate in the euro area
1992-1993, a period widely regarded as recessionary and of weak demand in the euro area. For all these reasons, we believe that the output gap is a better measure of excess demand conditions in the euro area to be used in the estimation of the Phillips curve in (2).

The results of the estimation of equation (2), reported in Table 2, tend to confirm these observations. In the first column from the left, we estimate equation (2) in the full sample period starting in 1980:Q1. This estimate delivers $\gamma_b = 0.45$ and $\gamma_f = 0.53$, which is broadly in line with the results in other studies (for example Gali, Gertler and Lopez Salido, 2001). However, the output gap is insignificant, which bodes ill for the economic interpretation of the equation. These results are confirmed when estimating a backward looking Phillips curve ($\gamma_f = 0$) over the same sample period by OLS (third column from the left). Inflation displays a strong autocorrelation, with a coefficient of 0.87, and the output gap remains insignificant.\footnote{This is also true when using the output gap based on the HP filter and real unit labour costs detrended with the HP filter as the measure of excess demand.} The results of the estimate change dramatically if one starts the sample period in 1986:Q1, i.e. taking out the first half of the 1980s (second column from the left). In this case, neither the backward looking nor the forward looking term is statistically significant; inflation appears to be stationary around a constant term and fluctuating around this mean in tandem with the output gap, which is strongly significant and with the right sign ($k = 0.42$). When estimating

Figure 4: Inflation and output gap in the euro area, 1986-2002
the equation in the backward looking version \((\gamma_f = 0)\) by OLS, we find the same result, with inflation displaying no statistically significant serial correlation and a strong dependency on the output gap (in this case, \(k = 0.36\)). Again, in the post-1986 regime, inflation appears to be a stationary variable around a constant mean and fluctuating with the output gap. In particular, inflation seems to have no intrinsic inertia in the post-1986 period; the persistence originally present in the inflation series, also in the post-1986 sample, appears to be due only to that in the output gap.\(^9\) The lack of inflation inertia is suggestive of a credible monetary policy regime in the post-1986 sample period (Goodfriend, 2002). The picture is completely different when one includes the early 1980s; over the full sample period, inflation is a highly inertial process with bears little relation to the output gap, indications which point to a monetary policy regime with poor credibility. This shows that the selection of the sample period is crucial in understanding the behaviour of the inflation process as it may be relevant for monetary policy during Stage Three of EMU.\(^10\)

\(^9\)It may be noted here that our results seem prima facie inconsistent with the evidence of 'type 1' euro area inflation persistence reported in Batini (2002). It should be noted, however, that the evidence in Batini (2002) – in particular that presented in Table 1 of that paper – is based on a purely time series model of inflation, i.e. excluding the output gap. Moreover, our partition of the sample period departs somewhat from that in Batini (2002), although not too much.

\(^{10}\)This conclusion is also consistent with the results of Erceg and Levin (2003) for
Table 2: Estimates of the Phillips curve in the euro area

Instruments (GMM): constant and two lags of all exogenous and endogenous variables, plus two lags of the interest rate gap. A constant term is included in all equations. Start of the sample period always indicated in parentheses in the heading; end of sample period is always 2002:Q4. Standard errors in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>GMM (80:Q1)</th>
<th>GMM (86:Q1)</th>
<th>OLS (80:Q1)</th>
<th>OLS (86:Q1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_b$</td>
<td>0.45* (0.08)</td>
<td>0.13 (0.14)</td>
<td>0.87* (0.05)</td>
<td>0.20 (0.12)</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>0.53* (0.09)</td>
<td>0.01 (0.19)</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>$k$</td>
<td>0.00 (0.03)</td>
<td>0.42* (0.12)</td>
<td>0.01 (0.02)</td>
<td>0.36* (0.08)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.82</td>
<td>0.44</td>
<td>0.81</td>
<td>0.45</td>
</tr>
<tr>
<td>J test</td>
<td>0.04</td>
<td>0.02</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>St. error</td>
<td>0.0093</td>
<td>0.0089</td>
<td>0.0027</td>
<td>0.0088</td>
</tr>
<tr>
<td>Durbin Watson</td>
<td>2.99</td>
<td>1.96</td>
<td>2.57</td>
<td>2.06</td>
</tr>
</tbody>
</table>

The estimates for the post-1986 period for the US (Table 3) deliver a broadly similar picture, although there seems to remain some inflation persistence, as shown by both the GMM and the OLS estimates (respectively first and second columns from the left in Table 2). Moreover, the coefficient on the output gap is smaller than in the euro area ($k = 0.18$ in the OLS estimate). A higher inflation inertia tends to suggest a monetary policy regime with less credibility than in the euro area, although cross country comparisons are to be interpreted with caution given that other factors may be at play. In addition, it should be emphasized that the degree of inflation inertia is relatively low also in the US case. In fact, an autocorrelation coefficient of 0.44 (OLS estimate) on quarter-on-quarter inflation data implies that a shock to inflation is almost completely reabsorbed in very few quarters.

Table 3: Estimates of the Phillips curve in the US and Germany (DE)

Instruments (GMM): constant and two lags of all exogenous and endogenous variables, plus two lags of the interest rate gap. A constant term is included in all equations. Start of the sample period always indicated in parentheses in the heading; end of sample period is always 2002:Q4. Standard errors in parentheses.

the US, which lend support to the idea that inflation inertia is a feature related to the (lack of) stability of the monetary policy regime, rather than an 'intrinsic' property of the US economy.

11 Adam and Padula (2003), for example, show that in the US lagged inflation appears to play a role which goes beyond explaining how actual inflation expectations are backward looking.
For a further comparison, in the two columns on the right side of Table 3 we report OLS estimates for a backward looking specification of the Phillips curve in Germany. This is an interesting test because the German inflation series does not show the discontinuity in behaviour of the euro area (and the US) series before and after the mid-1980s. Therefore, it seems reasonable to characterize the German experience in the full sample period 1980:Q1-2002:Q4 as a single, and credible, monetary policy regime. The estimates reported in Table 2 confirm this assessment. In fact, inflation displays little or no inertia in either the full or the post-1986 sample period, and is driven by a strongly significant output gap ($k = 0.28$ in the post-1986 estimate).\footnote{Also Coenen and Wieland (2000) and Benigno and Lopez Salido (2002) find no evidence of inflation inertia in Germany.} It is worth emphasizing that, apart from the constant term, the process driving inflation in the post-1986 period in the euro area is very close to the German one.

We carry out a number of diagnostic tests on the post-1986 OLS estimate of the euro area Phillips curve (third column from the left in Table 2), finding satisfactory results. The recursive residuals in Figure 6 suggest no instability during the post-1986 sample period. Moreover, also in this case we estimate the equation up to 1998:Q4 and then analyze the out-of-sample forecasting performance as from the start of Stage Three of EMU in 1999:Q1. Also in this case, we find evidence of a relatively good forecasting performance and no evidence of a structural break in the equation. It appears, overall, that the Phillips curve estimated with OLS in the post-1986 sample period provides a good basis for Stage Three of EMU.

We also carry out a robustness analysis of the results in Table 2 by using the output gap measure based on the HP filter in place of that based on the linear trend. The qualitative results of the estimate are the same, and in particular the change in monetary policy regime and inflation process before and after the mid-1980s continues to stand out in the euro area data. However, when estimating the Phillips curve

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\gamma_b$</td>
<td>0.38* (0.08)</td>
<td>0.44* (0.11)</td>
<td>0.21* (0.10)</td>
<td>0.10 (0.13)</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>0.20 (0.16)</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>$k$</td>
<td>0.13* (0.05)</td>
<td>0.19* (0.09)</td>
<td>0.18* (0.08)</td>
<td>0.28* (0.09)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.39</td>
<td>0.31</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>J test</td>
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<td>2.34</td>
<td>1.97</td>
<td>2.17</td>
<td>2.06</td>
</tr>
</tbody>
</table>
Recursive Residuals

Figure 6: Recursive residuals of the OLS estimate of the euro area Phillips curve

In the euro area in the sample period from 1986:Q1 with GMM, we find evidence of some, albeit limited, intrinsic inertia in inflation ($\gamma_0 = 0.33$). In addition, the coefficient for the output gap is lower ($k = 0.26$) and statistically significant only at the 10% level. Given the latter result, in the next section we take our estimates with the output gap derived from the linear trend as our 'baseline' model, and leave the estimate based on the output gap computed with the HP filter only for a sensitivity analysis (Section 4). Overall, the conclusion seems warranted that the inflation process shows little or no intrinsic inertia in the euro area if one excludes the first half of the 1980s from the sample period.13

It should be emphasized at this point that what we estimate in Table 2 is an *ex post* account of the inflation process in the euro area, which is not necessarily valid for the *ex ante* normative analysis which is the main objective of this paper. To understand this, assume for instance a standard New Keynesian Phillips curve as follows (omitting a serially

---

13 This result seems also consistent with the recent time series analysis in Levin and Piger (2003) for a large number of industrialized countries. Moreover, it should also be emphasized that, in the post 1986 sample period, inflation has no statistically significant inertia in Germany even using real GDP detrended with the HP filter as the measure of excess demand. To the extent that Germany might be considered as a relevant benchmark for the whole euro area in the environment of Stage Three of EMU (notably due to the very similar institutional arrangement), this finding reinforces the conclusion that inflation inertia is not a major issue for the euro area.
uncorrelated supply shock for simplicity):

\[ \pi_t = E_t \pi_{t+1} + k x_t \]  

(4)

Solving the model forward and applying the law of iterated expectations, we have:

\[ \pi_t = E_t \pi_{t+1} + k x_t = E_t E_{t+1} \pi_{t+2} + k (x_t + E_t x_{t+1}) = \ldots = E_t \pi_{t+j} + k E_t \sum_{s=t}^{j} x_s, \]

(5)

with \( j \) 'large'. Therefore, we may write a standard forward looking Phillips curve in (4) as the sum of long-term inflation expectations, or the central bank inflation target as perceived by price-setters, and the effect of the output gap term, current and expected in the future. If the price-setters' perception of the central bank inflation target is approximately constant during our sample period, and if the output gap is strongly serially correlated so that \( x_t \) is a good proxy for \( E_t \sum_{s=t}^{j} x_s \), we may observe ex post an inflation process behaving like:

\[ \pi_t = \pi + \hat{k} x_t, \]

(6)

where \( \pi \) is an approximately constant term. It should be recalled that Gerlach and Svensson (2003) and other authors have used an exponential trend to filter out a time-varying central bank inflation target (i.e., \( \pi = \pi_t \)) from the euro area inflation series. This was done, however, for sample periods including the early 1980s. We find that for the post-1986 sample period a constant inflation target turns out to be a valid approximation, although of course nothing more than an approximation.

From an ex ante, normative standpoint, however, the central bank cannot take inflation expectations as given, and its monetary policy rule has to enforce the inflation target. Therefore, for the central bank the relevant ex ante process is:

\[ \pi_t = E_t \pi_{t+1} + k x_t, \]

(7)

which the central bank has to take as its 'constraint' for optimal monetary policy under commitment.

At this point it is worth asking why, if the Phillips curve is a forward-looking one, we do not find inflation expectations to be significant in the estimated version in (6) (see the results in Table 2). The reason for that is simply that, being the output gap a very persistent variable, \( E_t \pi_{t+1} \) and \( x_t \) are almost collinear variables, and so the usual multicollinearity problem arises in the estimation. At the same time, the
empirical estimate $\hat{k} = 0.36$ (see Table 2) is obviously a biased estimate of $k$, as it also includes the effect of $x_t$ onto $E_t\pi_{t+1}$, rather than only on $\pi_t$.\footnote{This is, of course, a standard case of omitted variable bias.} A simple calculation shows that the 'true' value of $k$ for the euro area economy is about a half of the estimated $\hat{k}$, i.e. 0.18. Against this background, in the next section we take the inflation process in (7), with $k = 0.18$, to be the relevant process in order to evaluate the optimal monetary policy rule under commitment in the (credible) euro area monetary policy regime prevailing in Stage Three of EMU.\footnote{Note that with the version of the Phillips curve where the output gap is estimated based on the HP filter we have $k = 0.26$, which correcting for the omitted variable bias results in $k = 0.16$, very close to the baseline model.}

For the US, having found some evidence of inflation inertia with an autocorrelation coefficient equal to 0.44, we take the partially backward looking model as follows:

$$\pi_t = 0.44\pi_{t-1} + 0.66E_t\pi_{t+1} + kx_t$$

(8)

Again, it should be noted that inflation expectations $E_t\pi_{t+1}$ are not significant in the GMM empirical estimate in Table 2 because of its strong correlation with the current output gap $x_t$. Consequently, the estimated $\hat{k}$ is biased also in the US case, although the bias is smaller due to the higher degree of inflation inertia found for the US. The 'true' value of $k$ may be estimated to be around 0.11 in this case, which is a value in line with the existing empirical literature for the US (see for example Rudebusch and Svensson, 1999, who estimated $k = 0.14$ for the US economy, albeit in an entirely backward looking model).

A caveat should be added at this point: the model for the US serves only as a useful term of reference, and we do not claim it to be the 'best' representation of the US economy, nor do we want to draw any conclusions in terms of optimal monetary policy for the Fed. Our focus in fact is clearly on the euro area economy and the optimal monetary policy for the ECB.

3 Optimal monetary policy under commitment

In this section the two-equation model economy previously estimated is used to derive an optimal policy rule for the euro area. Moreover, we also show the optimal rule derived for the US as estimated in the previous section, which should provide an interesting reference. The policy-maker needs to take into account the constraints represented by the structure of the economy in order to perform its optimization exercise, which we assume to be carried out in a framework of credible commitment by the central bank. In other words, we assume that there is a commitment...
technology available and that the central bank makes optimal use of it. We limit our attention to the commitment case because it has been shown that the outcome under commitment is superior to that under discretion even when the average rate of inflation does not need to be reduced (Woodford, 1999a; Clarida, Gertler, Gali, 1999), due to the fact that the Phillips curve is forward looking.

The analysis of optimal monetary policy under commitment is carried out by considering two types of solutions, namely (1) optimal restricted simple rules (i.e., constrained to take a particular linear form), and (2) the optimal unconstrained rule. Generally speaking, we give emphasis to the results for the restricted simple rules because they are easier to interpret and realistic to internalize in agents’ expectations, while the optimal unconstrained rule is a very complex one and might be difficult to interpret, let alone to internalize in economic agents’ expectations.\footnote{The argument becomes forceful when agents do not have rational expectations, e.g. when they have to learn the monetary policy rule by inferring it from actual central bank behaviour.}

At the same time, we check whether the optimal restricted rule can be seen as a good approximation of the optimal unconstrained one.

3.1 The central bank problem

In this section we start the analysis by rewriting the short-run constraints facing the central bank in a form which is useful for solving the intertemporal problem. We then describe the loss function and show how to carry out the optimization exercise. The resulting optimal rules for the euro area and the US can finally be used for computing the optimal policy responses to demand and supply shocks and for comparing the path of the rule-driven interest rate with the actual development.

3.1.1 The relevant constraints for the central bank

The estimated parameters of the small New Keynesian model are used for constraining the optimization exercise of the policy maker. In the previous section we have seen that, for the baseline model for the euro area, $\alpha_b = 0.95$, $\alpha_f = 0$, and $\beta = -0.27$ (IS curve), $\gamma_b = 0$, $\gamma_f = 1$, $k = 0.18$ (Phillips curve), i.e. a backward looking IS curve and a forward looking Phillips curve. For the United States, we have $\alpha_b = 0.92$, $\alpha_f = 0$, and $\beta = -0.23$ (IS curve), $\gamma_b = .44$, $\gamma_f = .66$, $k = 0.11$ (Phillips curve), i.e. a backward looking IS curve and a Phillips curve with both backward looking and forward looking components. It is to be noted, in particular, that the coefficient which multiplies the output gap in the euro area is significantly larger than that of the US, in other words the inflation reaction to changes in output is higher in the euro area than in the US,
which implies a steeper short-run Phillips curve.

Against this background, we rewrite the previous two-equation model in a state-space representation form:17

\[
\begin{bmatrix}
\varepsilon_{t+1}^S \\
\hat{\varepsilon}_{t+1}^D \\
\hat{\bar{r}}_{t-2} \\
\hat{\bar{r}}_{t-1} \\
E_t x_{t+1} \\
E_t \pi_{t+1}
\end{bmatrix} = A \begin{bmatrix} z_{1t+1} \\ z_{2t+1} \end{bmatrix} + B \hat{\bar{r}}_t + \varepsilon_{t+1},
\]

where \( \bar{r} \) and \( \hat{i} \) represent the deviations from the steady state of, respectively, the real and the nominal interest rate (the latter being the control variable for the central bank). The steady state value for the nominal interest rate is \( \bar{r}_t = \bar{\pi} + \bar{\tau}_t \), where \( \tau_t \) is the slowly evolving time-varying natural rate of interest, and \( \bar{\pi} \) is the central bank inflation target.

Writing the model in a more compact format:

\[
E_t z_{t+1} = A z_t + B \hat{\bar{r}}_t + \varepsilon_{t+1},
\]

where \( z_t \) is the vector of predetermined variables (\( z_{10} \) given), \( z_{2t} \) the vector of forward looking variables, \( \hat{i}_t \) the policy instrument (control variable) and \( \varepsilon_{t+1} \) the vector of innovation to \( E_t z_{t+1} \) with covariance matrix \( \Sigma \).

For notational convenience let \( z_t = (z_{1t}, z_{2t}) \), so that:

\[
E_t z_{t+1} = A z_t + B \hat{\bar{r}}_t + \varepsilon_{t+1}
\]

3.1.2 The central bank loss function

We assume a rather conventional loss function, where the target for inflation is different from zero and where we also give a weight to fluctuations in the output gap and the nominal interest rate from the the equilibrium levels:18

\[
L_0 = E_0 \sum_{t=0}^{\infty} \delta^t \left[ \lambda_{\pi} (\pi_t - \bar{\pi})^2 + \lambda_{\tau} x_t^2 + \lambda_{\hat{i}} \hat{i}_t^2 \right],
\]

\[\text{17 Before writing the model in this form, we rewrite the IS curve at } t+1. \text{ Note also that we consider past real interest rates as state variables.}\]

\[\text{18 We follow the literature here, but it is clear that the loss function in (11) might be a relatively unprecise representation of the ECB’s preferences, given the overriding mandate to pursue price stability in the euro area and the subordinate nature of other objectives (see Smets, 2000).}\]
where \( 0 < \delta < 1 \) is a discount factor.\(^ {19} \)

In the state space form, the loss function can be rewritten as:

\[
L_0 = E_0 \sum_{t=0}^{\infty} \delta^t (z_t' Q z_t + 2 z_t' U \widehat{i}_t + \widehat{i}_t' R \widehat{i}_t),
\]

(12)

where \( Q, U \) and \( R \) are the matrices of the weights (see Appendix 2). The central bank minimizes (12) subject to (10).\(^ {20} \)

Two cases are considered. The first is the case in which the policy-maker commits to a simple decision rule of the form \( \widehat{i}_t = -F z_t \), where arbitrary restrictions may be placed on some elements of \( F \). The second is the unrestricted commitment case in which the central bank performs an initial optimization to determine the form of its optimal response and adheres to this derived rule in subsequent periods.\(^ {21} \)

We assume a 'conservative' central bank, both for the euro area and the US, i.e. a central bank for which inflation stabilization is relatively more important than output stabilization. Accordingly, and broadly in line with the existing literature, the weights in the loss function are set to be \( \lambda_\pi = 0.5; \lambda_y = 0.25 \). As noted earlier, we assume that the central bank can credibly commit to a policy rule. Finally, \( \pi \) is set to be equal to 1.8% both in the euro area and the US.

### 3.2 The optimal simple restricted rule

In this section we let monetary policy be specified by an interest rate feedback rule of the following form:

\[
\widehat{i}_t = \rho \widehat{i}_{t-1} + \phi_\pi (\pi_t - \overline{\pi}) + \phi_\pi x_t,
\]

(13)

i.e. a Taylor rule with interest rate smoothing. Before searching for the parameters which lead to the lowest loss, we need to limit our search

\(^{19}\text{Woodford (1999a) shows that the inclusion of the interest rate term as in the loss function in (11) is the correct quadratic approximation to the expected utility function when one takes account of the transaction costs that lead people to hold money balances.}\)

\(^{20}\text{Since the problem involves forward looking variables, except in rare circumstances we have to apply some numerical method to find the optimal unconstrained policy rule. To this aim, we use a routine written for Matlab described in Soderlind (1999).}\)

\(^{21}\text{In this problem the policy-maker in } t = 0 \text{ exploits the fact that the private sector formed expectations in } t < 0. \text{ There is always a temptation to exploit this, that is to set a policy in such a way that the Lagrange multipliers at } t = 0 \text{ for the forward looking variables is zero; but the commitment solution rules out this, except for the initial period. The 'timeless perspective' advocated by Woodford (1999a) is essentially to use the policy rule that comes out from solving the first order conditions only from some period } t > 0.}\)
within a range of values which ensure determinacy, so as to exclude coefficients which lead to more than one equilibrium price level (Woodford, 2003; see Appendix 1 for the derivation of the relevant conditions for determinacy).

The area on the right hand side of the frontier reported in Figure 7 shows the combinations of $\phi_n$ and $\rho$ (valid for values of $\phi_x$ going from 0.1 to 3) that guarantee the determinacy of the equilibrium. Conditioned on the model economy and on the parameters chosen for the loss function, the parameters $\phi_n$, $\rho$ and $\phi_x$ are obtained by performing a numerical analysis within a range of values consistent with a determinate solution.

For the euro area, the constrained interest rate rule which minimizes the loss function is found to be equal to:

$$\hat{i}_t = 0.7\hat{i}_{t-1} + 0.40(\pi_t - \bar{\pi}) + 0.40x_t$$  \hspace{1cm} (14)

Solving for the autoregressive part:

$$\hat{i} = 1.33(\pi - \bar{\pi}) + 1.33x$$  \hspace{1cm} (15)

For the US, the interest rate rule is equal to:

$$\hat{i}_t = 0.5\hat{i}_{t-1} + 1.00(\pi_t - \bar{\pi}) + 0.75x_t,$$  \hspace{1cm} (16)

or, solving for the autoregressive part:

$$\hat{i} = 2(\pi - \bar{\pi}) + 1.5x$$  \hspace{1cm} (17)

\footnote{We limit our search grid within values of $\rho$, $\phi_n$, and $\phi_x$ which have been shown by the literature to be empirically relevant.}
It should be emphasized that the main difference between the two feedback rules is represented by the coefficient for the inflation gap, which is somewhat higher in the US, while the interest rate smoothing coefficient is slightly smaller and the response to the output gap is very close to the euro area case. Compared with a standard Taylor rule, the response to the output gap is significantly stronger for both countries, which is a very common result in the literature (see, e.g., Rudebusch and Svensson, 1999; Ball, 2002). The response to the inflation gap is very close to the standard Taylor rule in the euro area economy, while it is somewhat stronger in the US due to the presence of some inflation inertia in the latter economy.

In order to better understand how this outcome depends on the specification of the model economy, we carry out several simulations by increasing the coefficient $\gamma_b$ in the Phillips curve. This is relevant because we want to evaluate the importance of the degree of inflation inertia – which, as noted, should depend crucially on the way expectations are formed and thus on the monetary policy regime in place – in determining the optimal monetary policy rule for the central bank in the euro area.

The results of the simulation are represented by the points indicated in Figure 7. More precisely, point 1 gives the combinations of $\phi_\pi$ and $\rho$ for the interest rate rule when the Phillips curve is completely forward looking (i.e. the baseline euro area case). Point 2 gives the combinations of $\phi_\pi$ and $\rho$ for the interest rate rule when there is a low degree of inflation inertia (i.e. the US case with inflation persistence at 0.44). Point 3 gives the combinations of $\phi_\pi$ and $\rho$ when the backward looking coefficient in the Phillips curve ($\gamma_b$) is set at 0.6 (and the forward looking coefficient $\gamma_f$ at 0.4). Finally, point 4 gives the combinations of $\phi_\pi$ and $\rho$ when $\gamma_b$ is equal to 0.8 ($\gamma_f = 0.2$).

These combinations show that the higher the degree of 'backward lookingness' in the Phillips curve, the higher the coefficient $\phi_\pi$ and the lower the interest rate smoothing term $\rho$ in the resulting optimal interest rate feedback rule. In case of a completely forward looking Phillips curve (euro area) the combination of $\phi_\pi$ and $\rho$ is such that the parameter $\phi_\pi$ is the lowest among those delivering a determinate equilibrium. By augmenting the degree of backward lookingness, Figure 7 shows that the optimal combination of parameters moves inside the frontier. Overall, the main lesson here is that central banks have to react more aggressively to inflation if the inflation process is intrinsically inertial. This is due to the fact that, under inflation inertia, supply shocks ($\varepsilon_\pi^S$) are at least in part passed on to medium-term price developments. Under these conditions, the optimal monetary policy rule is 'strictly inside' the area

22
of determinacy. By contrast, if the monetary policy regime is credible and there is no inflation inertia, the central bank should react to supply shocks as little as possible, i.e. only to the extent needed to ensure determinacy.

Figures 8 and 9 show the path of the actual and ‘rule driven’ short-run nominal interest rate for the euro area and the US over the sample period from 1986 to 2002. For both countries we obtain a relatively good track of the actual interest rate path, suggesting that the policies followed have indeed been close to the optimum. If we concentrate on the euro area, we can see that the behaviour of the actual interest rate has been almost coincident to the ‘rule driven’ optimal rate since the start of Stage Three of EMU (Figure 10), and the distance is slightly lower than that obtained by a standard Taylor rule. This implies a stronger reaction to the output gap and a slightly smaller weight to inflation with respect to a standard Taylor rule.

3.3 The optimal unconstrained rule

3.3.1 Derivation of the rule and the optimal interest rate path based on realized shocks

We now move to the optimal *unconstrained* minimization of the loss function (12), subject to the constraints (10) under commitment. The optimal monetary policy rule which can be derived under no restrictions will be complex and difficult to interpret. The optimal unconstrained
**Figure 9:** Actual and rule-driven nominal interest rate in the US, 1986-2002

**Figure 10:** Actual and rule-driven nominal interest rate, euro area, during Stage Three of EMU
rule, in fact, depends on the whole history of the output and inflation processes, including all demand and supply shocks. Therefore, we do not give it the strongest emphasis here. Nevertheless, it is important to evaluate its properties and to compare them with those of the optimal constrained rule, also in order to better assess whether the latter is a good approximation of the former.

As before, the central bank minimizes the following Lagrangian function (see Appendix 2):

$$E_0 \sum_{i=0}^{\infty} \delta^i \left[ z_i'Qz_i + 2z_i'U\hat{\nu}_i + \hat{\nu}_i'R\hat{\nu}_i + 2\mu_{i+1}'(Az_i + B\hat{\nu}_i + \varepsilon_{t+1} - z_{t+1}) \right]$$

(18)

where $\mu_t$ is the vector of Lagrange multipliers. After having solved for the first order conditions we use Soderlind (1999)’s Matlab program for obtaining the resulting expectations difference equations using a generalised Schur decomposition (Sims, 1995; Klein, 2000). The solution has two parts. First, the predetermined variables can be written as a function of their own lagged values and the shocks. Second, the forward looking variables are a linear function of the predetermined variables.

Generally speaking, the efficient feedback coefficients will be complex functions of the structural parameters of the model economy and the central bank preferences. The optimal interest rate in period $t$ is a function of the current value of the state vectors $(x_t, \pi_t, \hat{\gamma}_{t-1}, \hat{\gamma}_{t-2}, \hat{\gamma}_{t-3}, \varepsilon_t^D, \varepsilon_t^F)$ and a distributed lag of the state vector going back as far as the time when the initial optimisation took place, in our case 1986:Q1 (see Appendix 2 for the exact derivation of the formula).

For illustrative purposes, Figure 11 reports the path of the nominal interest rate $i_t$ computed on the basis of the optimal unconstrained rule from the early 1990s, based on the demand and supply shocks estimated in the baseline model of the economy (with the sample period starting in 1986:Q1). Overall, the optimal path is not too distant from the actual monetary policy decisions of euro area central banks over our sample period.

An interesting question is to what extent the path of the optimal rule depends on the parameters of the central bank loss function. Broadly speaking, we find that the optimal rule is very robust to plausible changes in the central bank loss function in (11). Figure 12 presents

\[23\] This is the case because, in our model, we have two sources of tension which make stabilization goals not to be achieved simultaneously. First, there are supply shocks in the Phillips curve and the central bank is assumed to care about output stabilization. Second, the central bank is assumed to have some concern about interest rate stabilization. On history dependence of the optimal policy rule in the presence of alternative stabilization goals, see Woodford (2003).
the optimal path of the nominal interest rate under four variants of the central bank loss function. In particular, variant (a) assumes a lower weight on the output gap stabilization ($\lambda_x = 0.2$, $\lambda_\pi = \lambda_i = 0.4$); variant (b) assumes an equal weight on inflation and output stabilization ($\lambda_x = \lambda_\pi = 0.4$, $\lambda_i = 0.2$); variant (c) assumes a very high weight on inflation stabilization ($\lambda_x = \lambda_\pi = 0.15$, $\lambda_i = 0.7$); finally, variant (d) assumes equal weights for output, inflation and interest rate stabilization ($\lambda_x = \lambda_\pi = \lambda_i = \frac{1}{3}$). Overall, the differences in the optimal path are minimal, suggesting that the specification of the loss function does not matter much as far as the optimal rule is concerned. This finding can be explained by the fact that the reaction to supply shocks is anyway very small under the optimal unconstrained rule (see the next subsection), and the optimal reaction to demand shocks does not hinge much on the parameters of the loss function, at least as far as the relative weights assigned to inflation and output stabilization are concerned.

As the rule is very complex and difficult to interpret when written down, we rather analyze the response of the nominal interest rate (in deviation from the steady state) to different shocks in order to provide an illustration of the properties of the rule. We first deal with cost-push shocks (i.e., $\varepsilon^u_t$), and then with aggregate demand shocks (i.e., $\varepsilon^d_t$).

### 3.3.2 Responses of inflation, output and the interest rate to shocks

**Supply shock**  In this section we study the reaction of the output gap, inflation and the interest rate to a 1% cost-push shock in the euro area.

![Figure 11: Optimal and actual nominal interest rate in the euro area](image_url)
and the US. In Figure 13, the numbers in the vertical axes represent the percentage deviation from the steady state, while in the horizontal axes the time-horizon is measured in quarters. The negative output reaction to a 1% cost-push shock is pretty similar in the euro area and the US. Thus, under an optimal monetary policy regime the output drop would be limited, with the highest effect felt one year after the shock. Figure 13 also shows the inflation reaction. In the euro area, after the initial 1% increase the inflation rate drops in the following quarter to a level lower than its equilibrium level and converges to the equilibrium afterwards. Due to the presence of higher inflation persistence in the US, the reaction of inflation to a US cost-push shock vanishes only after three quarters. In fact, inflation persistence in the US requires a stronger policy reaction to a cost-push shock, whereas in the case of a completely forward looking Phillips curve specification the interest rate needs to react only very little.

**Demand shock** As in the case of a cost-push shock, also a 1% aggregate demand (IS) shock leads to a very similar reaction of output gap in the euro area and the US (Figure 14). The inflation reaction is more volatile in the US, which is partly explained by the presence of autocorrelation in the US aggregate demand shock. The optimal policy reaction in both economies is much stronger than in the case of a cost-push shock and this depends on the backward looking specification of the IS curve.

Overall, interest rate reactions to cost-push and aggregate demand shocks suggest that the optimal policy conditional to the economy’s con-
Figure 13: Impulse responses to a 1% supply shock

Figure 14: Impulse responses to a 1% demand shock

The constraints estimated for the euro area and the US require the central bank to respond more aggressively to a shock coming from the aggregate demand rather than aggregate supply. It is worth emphasizing that this result has also been obtained in the case of restricted optimal rules where we find a higher coefficient on the output gap than on inflation compared with a standard Taylor rule, especially in the euro area. The key message here is that, if the monetary policy regime is credible and inflation has no inertia, it is, overall, optimal for the central bank to react more to demand shocks, and less to supply shocks. Intuitively, there is no reason to react strongly to a supply shock, if this has no bearing on inflation developments in the quarters ahead, while demand shocks affect inflation for some time, given the persistent nature of the output gap.
3.4 A comparison of policy reactions to supply and demand shocks in the restricted and unrestricted optimal rules

In order to assess the robustness of the results, we carry out a comparative analysis of the policy responses for the euro area in the two cases of the restricted and unrestricted optimal rules (Figure 15). Overall, we find that the response of the interest rate to both the demand and the supply shock is similar in the constrained and unconstrained rule – which implies that the former is a relatively good approximation of the latter – although there are some differences in the timing and to some extent also in the strength of the response. An aggregate demand shock leads to a much higher response on impact under the case of unrestricted optimal policy rule, but we also observe that the pattern becomes very similar after three quarters. A cost-push shock leads to a much similar response, although under the unconstrained case the policy response pattern is smoother. This reflects the fact that the optimal constrained rule has to ensure the determinacy of the equilibrium, and this requires a minimum reaction to an inflation shock as well. By constrast, the optimal unconstrained rule does not have to respect the same determinacy constraint, due to the fact that it depends on a much wider set of current and past state variables.

4 Some sensitivity analysis

In this section we carry out a sensitivity analysis of the results of the previous section, in particular as regards the implications of changes in
the model assumptions on the optimal restricted rule in the euro area economy.

Table 4 below reports the results for the optimal constrained rule in four cases. The first case (first column from the left) is the optimal rule derived in Section 3 under the baseline model of the euro area economy; the other three cases are plausible variants of the baseline one. The first variant (second column from the left) is the policy rule when $\lambda_s = 1$, $\lambda_x = \lambda_l = 0$, i.e. when the central bank only cares about inflation. This check seems important to understand whether and to what extent the policy rule depends on the specification of central bank preferences. It is found that the optimal policy reacts more strongly to inflation shocks, while the coefficient for the output gap is of the same magnitude as the baseline case. The second variant (third column from the left) is when the specification of the IS curve is both backward looking and forward looking, as in the GMM estimate for the euro area reported in Table 1 (Section 2). The interesting finding in this case is that the response to the contemporaneous output gap is significantly smaller, and the reaction to the inflation gap is also slightly smaller. Finally, we look at the model estimated using the output gap derived by applying a HP filter to the data (last column from the left). The notable feature of the optimal rule in this case is the significantly stronger reaction to the inflation gap compared with the baseline case, which is due to the fact that the Phillips curve has some, albeit still relatively small, intrinsic inertia in this specification (see Section 2).

**Table 4: Sensitivity analysis on the parameters of the optimal constrained monetary policy rule**

<table>
<thead>
<tr>
<th>Variant</th>
<th>Optimal</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.7</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>0.4 (1.4)</td>
<td>1.8</td>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>0.4 (1.5)</td>
<td>1.2</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Figure 16 below presents a graphical representation of the central bank optimal reaction to the inflation gap and the degree of interest rate smoothing under all considered variants. The determinacy contour was obtained under the assumption that $\phi_{\pi} = 0.6$.

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24 The determinacy contour was obtained under the assumption that $\phi_{\pi} = 0.6$. 

30
is estimated with the HP filter, which is very close to the solution for the US (point 2) when using a linear trend to compute the output gap. Point 4 is the solution when the IS curve is partially forward looking. Furthermore, point 5 is the optimal rule for a central bank which only cares about inflation, and this predictably leads to a stronger reaction to the inflation gap. In addition, we also include the two optimal rules for the high inertia cases (point 6 for $\gamma_h = 0.6$; point 7 for $\gamma_h = 0.8$) under the baseline loss function specification, which interestingly turns out to be quite close to the optimal rule for a central bank only caring about inflation stabilization.

Finally, perhaps the most important sensitivity test is the following: how should monetary policy have been conducted in the first four years of EMU were the variants to the model of the economy or the central bank loss function which we consider in this section to be taken seriously? In order to assess that, we consider the evidence already reported in Figure 8. In particular, we know already that actual monetary policy in Stage Three of EMU was very close to the theoretical (constrained) optimum derived in Section 3, under the baseline model of the euro area economy.

Figure 17 reports the level of the interest rate when the IS curve is also partly forward looking (variant (2)). As can be seen in the Figure, there is very little difference as regards interest rate developments. In practice, monetary policy would have been much the same, and the specification of the IS curve does not matter much in this regard.

Things are somewhat different, however, when considering other variants of our baseline case (Figure 18). When the output gap is estimated

Figure 16: Optimal restricted rule under the baseline case and some variants
with the HP filter, thereby allowing for some inflation inertia in the model and therefore for a stronger reaction to the inflation gap, the optimal rule would have implied a significantly higher interest rate in the 2000-2001 period. This is a period when, mainly due to a steep increase in energy and food prices, HICP inflation rose significantly above the 2% limit. A stronger reaction to inflation would have implied a higher interest rate, which did not materialize. The same conclusion may be reached if the central bank were to be characterized as only caring about inflation stabilization; also in this case the interest rate should have been higher in the 2000-2001 period. Similar considerations can be made for the 1998-1999 period, when inflation was unusually low in the euro area.

What can we conclude from this sensitivity analysis? First, the ECB does not seem to have followed a policy which having preferences only focused on inflation stabilization ($\lambda_{\pi} = 1$) would imply in this model set-up, which is hardly surprising. What is more open is how to interpret the evidence with the model where the output gap is estimated with the HP filter, where the reaction to the inflation gap is stronger than in the baseline case. There are, to our view, three reasons why the baseline case (with no inflation inertia) is more plausible and therefore more relevant as a normative tool for monetary policy decisions during Stage Three of EMU. First, as noted in Section 2, the estimation results where the output gap derived from a linear trend seem preferable from an empirical standpoint, also taking into account the known end-period problems of the HP filter. Second, there is other evidence (such as that derived from inflation expectations drawn from French bonds linked to HICP inflation) that the monetary policy regime in the euro area was credible.
Third, the fact that we find no evidence of inflation inertia in Germany, even using the output gap measured with the HP filter, is also another confirmation that the 'no inertia' baseline case is the correct one. In fact, one might argue that the experience in Germany is an even better guide for monetary policy during Stage Three of EMU than looking at the evidence in the euro area as a whole prior to the introduction of the euro, due to the very similar institutional arrangement.

5 Conclusions

This paper attempts to estimate a small scale, New Keynesian model of the euro area economy, and to derive a monetary policy rule for the ECB which is optimal against the background of the economy parameters. The main finding of the empirical analysis in the paper is that, taking out the first part of the 1980s and the 'restoration' of monetary stability in that period, it is possible to estimate a plausible model where inflation displays little intrinsic inertia. The model economy has thus all the features of the credible monetary policy regime which should prevail in the environment of Stage Three of EMU. Estimates do not suggest any instability in the model related to the introduction of the euro in the first quarter of 1999.

The optimal monetary policy rule under commitment derived taking into account of the parameters of the euro area economy is similar to a Taylor rule with interest rate smoothing and a significantly stronger coefficient on the output gap, as also suggested in the literature (Peersman and Smets, 1999). In particular, it is found that under the optimal
rule and the assumption of a credible monetary policy with little or no inflation inertia (which is supported by the euro area and even more the German data in our sample period), the reaction to supply shocks should be ‘small’ (i.e. no more than that required to ensure the determinacy of the equilibrium), while monetary policy should react strongly to demand shocks.\footnote{A caveat to this conclusion is the ‘robustness’ argument of Angeloni, Coenen and Smets (2003), who shows that – if the policy-maker wants to avoid really large losses – it might be better to err on the side of assuming more intrinsic inertia in inflation than is actually the case in reality.} This is, overall, the main message of this paper. Interestingly, we find evidence that actual monetary policy decisions have been close to the theoretical optimum identified by the normative analysis, which we take as a positive signal for both the validity of our model and the quality of the monetary policy decisions taken during the first four years of Stage Three of EMU.
Appendix 1: Determinacy of the constrained monetary policy rule

In this appendix we write down the conditions for the determinacy of the equilibrium. The general formula, which we rewrite below, has been derived by Woodford (2003).

In the case of interest rate rules that contain a smoothing parameter the solution to the system \(\text{det}(A - rI) = \det \begin{bmatrix} a_{11} - r & a_{12} & a_{13} \\ a_{21} & a_{22} - r & a_{23} \\ a_{31} & a_{32} & a_{33} - r \end{bmatrix} = \)

\(r^3 + a_2 r^2 + a_1 r + a_0\) is determinate if and only if the coefficients of the characteristic equation fulfills either:

- **Case I:** \(1 + a_2 + a_1 + a_0 < 0\) \hspace{1cm} (1)
  and \(-1 + a_2 - a_1 + a_0 > 0\) \hspace{1cm} (2)

or

- **Case II:** \(1 + a_2 + a_1 + a_0 > 0\) \hspace{1cm} (3)
  and \(-1 + a_2 - a_1 + a_0 < 0\) \hspace{1cm} (4)

\[a_0^2 - a_0 a_2 + a_1 + a_0 > 0\] \hspace{1cm} (5)

or

- **Case III:** \(a_0^2 - a_0 a_2 + a_1 + a_0 < 0\) \hspace{1cm} (6)
  and \(|a_2| > 3\) \hspace{1cm} (7)

where \(a_2 = -trA; a_1 = a_{11}a_{22} + a_{22}a_{33} + a_{11}a_{33} - a_{12}a_{21} - a_{23}a_{32} - a_{31}a_{13}; a_0 = -\text{det} A.\)

In our model, the matrix \(A\) is equal to:

\[
A = \begin{bmatrix}
1 & -k & 0 \\
\frac{\beta}{\pi}(\phi_x - 1) & a_{22} - r & a_{23} \\
a_{31} & a_{32} & a_{33} - r
\end{bmatrix}
\]

and the coefficients of the characteristic equation are:

\[
a_0 = -\text{det} A = -\alpha \rho < 0; \\
a_2 = -trA = -\left[1 + \alpha + \frac{\beta}{\pi}(\phi_x + k) + \rho\right] < 0; \\
a_1 = \alpha + \frac{\beta}{\pi}(\phi_x + \phi_x k) + \rho \left(\frac{\beta}{\pi}k + 1 + \alpha\right) > 0;
\]

We can immediately see that condition (2) is violated and condition (4) must instead hold, thus we can exclude Case I of the proposition. The remaining conditions are:
Figure 19: Area of determinacy according to different values for $\phi_x$.

\(\begin{align*}
(3) \quad 1 + a_2 + a_1 + a_0 > 0 & \implies \phi_{\bar{x}} > 1 - \rho. \\
(5) \quad a_0^2 - a_0 a_2 + a_1 + a_0 > 0 & \implies \phi_{\bar{x}} + \frac{\phi_{\bar{x}}}{\rho} (1 - \alpha \rho) - \rho (1 + \alpha) + \\
\quad & \frac{4}{\rho k} [\alpha^2 \rho (\rho - 1) - \alpha (\rho^2 - 1) + \rho + 1] > 0 \\
(7) \quad |a_2| > 3 & \implies [1 + \alpha + \frac{2}{4} (\phi_{\bar{x}} + k) + \rho] > 3
\end{align*}\)

the range of values of $\phi_{\bar{x}}$, $\phi_x$, and $\rho$ which lies in the region of determinacy is obtained numerically.

Figure 19 shows that the area of determinacy increases for higher values of $\phi_x$. 
Appendix 2: The solution of the optimal unconstrained problem

The policy-maker minimises the following loss function:

\[
L_0 = E_0 \sum_{t=0}^{\infty} \delta^t (z'_t Q z_t + 2z'_t u \widehat{\iota}_{t} + \widehat{\iota}'_t R \widehat{\iota}_t),
\]

under the constraints:

\[
E_t z_{t+1} = A z_t + B \widehat{\iota}_t + \varepsilon_{t+1}
\]

where \(z' = [\varepsilon^S, \varepsilon^D, r_{-3}, r_{-2}, r_{-1}, x, \pi]'\) is a 7x7 vector of all the backward and forward looking variables and \(\widehat{\iota}\) is the control variable. \(Q, U\) and \(R\) are the matrices of the weights: \[Q = \begin{bmatrix} 0 & \lambda_x & 0 \\ 0 & \lambda_x & 0 \\ \end{bmatrix}, \quad U = \begin{bmatrix} 0 \\ \end{bmatrix}, \quad R = [\lambda_i]\]

and \(A\) and \(B\) have been defined in the text. We can form the Lagrangean:

\[
L_0 = \min_{\{u_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \left[ z'_t Q z_t + 2z'_t u \widehat{\iota}_t + \widehat{\iota}'_t R \widehat{\iota}_t + 2 \mu'_{t+1} (A z_t + B \widehat{\iota}_t + \varepsilon_{t+1} - z_{t+1}) \right]
\]

where \(\mu_t\) is the vector of Lagrange multipliers. The first order condition for \(\widehat{\iota}_t\) is:

\[
-B' E_t \mu_{t+1} = U' z_t + R \widehat{\iota}_t
\]

The \(n = 7\) first order conditions for \(z_t\) are:

\[
\delta A' E_t \mu_{t+1} = \mu_t - \delta Q z_t - \delta U \widehat{\iota}_t
\]

We can write (2), (4) and (5) as:

\[
\begin{bmatrix} I_7 & 0_{7 \times 1} & 0_{7 \times 7} \\ 0_{7 \times 7} & 0_{7 \times 1} & \delta A' \\ 0_{1 \times 7} & 0_{1 \times 1} & -B' \\ \end{bmatrix} \begin{bmatrix} z_{t+1} \\ \widehat{\iota}_{t+1} \\ \mu_{t+1} \\ \end{bmatrix} = \begin{bmatrix} A & B & 0_{7 \times 7} \\ -\beta Q - \beta U & I_7 \\ U' & R & 0_{1 \times 7} \\ \end{bmatrix} \begin{bmatrix} z_t \\ \widehat{\iota}_t \\ \mu_t \\ \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \\ \end{bmatrix}
\]

and use the Generalised Schur decomposition to solve the system (Sims, 1995; Klein, 2000). The solution has two parts. First, the predetermined variables can be written as a function of their own lagged values and the shocks:

\[
\begin{bmatrix} z_{1t+1} \\ \mu_{2t+1} \\ \end{bmatrix} = M \begin{bmatrix} z_{1t} \\ \mu_{2t} \\ \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ \end{bmatrix}
\]
where $M$ is a $7 \times 7$ matrix, for the euro area is equal to:

$$
M = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0.39 & 2.09 & -0.54 & -0.54 & -0.14 & 2.24 & -0.26 \\
-0.03 & 0.86 & -0.03 & -0.03 & -0.06 & 0.85 & 0.02 \\
-1.27 & -1.17 & 0.06 & 0.06 & 0.08 & -1.63 & 0.85 \\
\end{bmatrix}
$$

Second, the forward looking variables are a linear function of the pre-determined variables:

$$
\begin{bmatrix}
\hat{z}_{2t} \\
\hat{i}_t \\
\mu_{1t} \\
\end{bmatrix}
= C \begin{bmatrix}
z_{1t} \\
\mu_{2t} \\
\end{bmatrix}
$$

(8)

$C$ is a $8 \times 7$ matrix, whose values for the euro area are:

$$
C = \begin{bmatrix}
0.81 & 0.37 & 0.04 & 0.04 & -0.03 & 0.66 & 0.12 \\
0.21 & 2.47 & -0.49 & -0.49 & -0.17 & 2.54 & -0.14 \\
1.22 & 0.55 & 0.07 & 0.07 & -0.04 & 0.99 & -0.81 \\
0.55 & 4.96 & -0.91 & -0.91 & -0.33 & 5.15 & -0.37 \\
0.07 & -0.91 & 10.7 & 10.7 & 0.06 & -0.89 & -0.04 \\
0.06 & -0.91 & 10.6 & 10.6 & 0.06 & -0.89 & -0.04 \\
-0.04 & -0.34 & 0.06 & 0.06 & 0.02 & -0.35 & 0.02 \\
0.99 & 5.15 & -0.89 & -0.89 & -0.34 & 5.75 & -0.66 \\
\end{bmatrix}
$$

For deriving the optimal interest rate in period $t$, consider the dynamic equation for the Lagrange multiplier, $\mu_2$, associated with the forward looking variables $z_2$, that is from (7):

$$
\mu_{2t+1} = m_{7,\{i=1,..,6\}} z_{1t} + m_{7,7} \mu_{2t}
$$

(9)

$$
= -1.27 \hat{z}_t^S - 1.17 \hat{z}_t^D + 0.06 \hat{r}_{t-3} + 0.06 \hat{r}_{t-2} + 0.08 \hat{r}_{t-1} - 1.63 x_t + 0.85 \mu_{2t}
$$

This difference equation can be solved backwards to yield:

$$
\mu_{2t+1} = m_{7,\{i=1,..,6\}} \sum_{j=0}^{t} m_{77}^j z_{1t-1-j}
$$

(10)

substituting (10) into the equation for $i_t$ in (8) we obtain the analytical expression for the optimal unconstrained rule:

$$
\hat{i}_t = c_{2,\{i=1,..,6\}} z_{1t} + c_{7} m_{7,\{i=1,..,6\}} \sum_{j=0}^{t} m_{77}^j z_{1t-1-j}
$$

(11)
From (11), we see that the optimal unconstrained rule is a linear function of all current state variables and of a distributed values of past state vectors going back as far as the time when the initial optimisation took place.
References


