Conditional Mean Jumps, Intraday Periodicity and Long Memory Volatility in High Frequency European Foreign Exchange Rates: Application of a FIGARCH model with a Bernoulli Jump Process

By

Young Wook Han+
Department of Economics and Finance
City University of Hong Kong, Hong Kong

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Abstract

This paper investigates the intriguing features of high frequency European exchange rates during 1996. In addition to the well know intraday periodicity and long memory property in the volatility process, the conditional mean jumps appear to be significant in the high frequency exchange rates so that the usual normality assumption is found not to be adequate. Hence, this paper relies on a normal mixture distribution that allows for Bernoulli jumps in the process of the high frequency exchange rates. This mixture distribution is found that the jumps are closely related to the intraday periodicity and that they appear to be important for the estimation of the long memory property in the volatility process of the high frequency exchange rates. In particular, the jump probability seems to induce higher long memory parameters.

JEL classifications: C22, F31, G15.

Keywords: High frequency European exchange rates, Normal mixture distribution, Bernoulli jumps, FIGARCH, Intraday periodicity, Long memory property.

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+ Correspondence: City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong. E-mail: efhanmsu@cityu.edu.hk; Phone: +852-2788-7312; Fax: +852-2788-8806.
1. Introduction

This paper is concerned with intriguing features of the 30-minute high frequency European spot returns series of the Swiss Franc (CHF), British Pound (BP), Deutsch Mark (DM), French Franc (FF) and Italian Lira (ITL) foreign exchange rates against the US Dollar ($) during 1996; Long memory property, Conditional mean jumps and Intraday periodicity. In particular, this paper mainly focuses on the conditional mean jumps in the high frequency exchange rates. In turn, the jumps may be caused by macroeconomic announcement, private information, central bank interventions and outliers like recording errors due to technical problems of computers, and thus they provide some strong implications for the modeling the high frequency returns.

The property of long memory and persistent volatility have become well documented features of high frequency foreign exchange rates such as the studies by Dacorgona et al. (1993), Andersen and Bollerslev (1997, 1998), Baillie et al. (2000, 2003), Baillie and Han (2002) and Han (2003b) among others. To estimate the long memory volatility parameter, this paper applies the Fractionally Integrated Generalized AutoRegressive Conditional Heteroskedasticity (FIGARCH) process of Baillie et al. (1996) in modeling the high frequency returns data. While numerous empirical studies for the analysis of high frequency returns dynamics have relied mostly on the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model provided by Bollerselv (1986) and Taylor (1986) independently, little of the literature has applied the FIGARCH model to represent high frequency exchange returns yet. Thus, in order to present the general appropriateness of the FIGARCH specification for high frequency returns, this paper presents the MA(1)-FIGARCH(1,δ,1) estimation results for the high frequency returns of the CHF-$, DM-$, $-BP, FF-$ and ITL-$ data.
The primary results from the MLE estimation of the basic FIGARCH model indicate that the FIGARCH model under the Gaussian normality assumption generally seems to match the dynamics of high frequency exchange rates and is a satisfying starting point to study the nature of the underlying features of the high frequency exchange rates. However, the use of the usual FIGARCH model with the normal distribution assumption leads to excess skewness and excess kurtosis which may be related to conditional mean jumps in the exchange rates. These jumps might lead to the “outliers” in the level and volatility process that can not be taken accounted for by the simple normal distribution model and (see, Hotta and Tsay, 1998). 3)

In fact, several previous studies have found that high frequency returns data can be characterized by several jumps or large shifts followed by random movements in the conditional mean process, which can be caused by public macroeconomic news, private information, central bank interventions and outliers like recording errors due to technical problems of computers. See for example, Goodhard and Figliuoli (1991), Goodhart, Hall, Henry and Pesaran (1993), Goodhart and Giugale (1993), Ghosh (1997), Andersen et al. (2002), Beine and Laurent (2003) and others. 4) In particular, the information arrivals may lead to the serial dependency in the conditional variance process since they are the mixing variables so that exchange returns can be generated by a mixture of distributions (Diebold, 1986; Taylor, 1986; Bollerslev and Domowitz, 1993).

Recently, Andersen et al. (2002) presented that US and Germany macroeconomic news affect foreign exchange rates including the CHF-$, DM-$ and $-BP exchange rates significantly and cause conditional mean jumps. They showed that the markets react to the jumps in asymmetric fashion. Andersen and Bollerslev (1998) and Chang and Taylor
(2003) also provided that the US macroeconomic announcements are one of the fundamental driving forces behind the volatility process of high frequency returns. And, Baillie et al. (2000) and Baillie and Han (2002) have showed that the high frequency DM-$ returns contains relatively high excess kurtosis which may be induced by the jumps.

Thus, this paper focuses on modelling the conditional mean jumps, the high excess kurtosis and the long memory property of the high frequency returns series of the European exchange rates because the jumps in the conditional mean process are of significant interest and the long memory property in the conditional variance process cannot be extracted without an appropriate specification of the conditional mean.

However, the basic FIGARCH model with the assumption of a symmetric normal distribution is unlikely to represent the asymmetric jump process and the high excess kurtosis of the high frequency returns and give appropriate estimation results. For this purpose, it appears to be quite useful to use the jump diffusion process proposed by Press (1967) in order to model the high frequency foreign exchange rates and properly account for the jumps. Since the statistical and economic motivations for the jumps and the long memory property are quite different, this paper invokes a combined MA(1)-FIGARCH(1,δ,1) model specification with Bernoulli jump process that accounts for the two features at the same time. Thus, this paper shows the combined model performs well and improves estimates of the long memory parameters because misspecification of the conditional mean without considering the jumps may provide distorted estimates of the long memory parameters.

This paper also demonstrates the dynamics of intraday periodicity in the volatility process of high frequency returns. The well known U-shaped intraday periodicity in the volatility process of the high frequency returns is generally attributed
to market activity across different financial centers around the world over 24 hours. As explained by Andersen and Bollerslev (1997, 1998), Baillie et al. (2000) and Baillie and Han (2002), the intraday pattern masks the long memory property in the volatility process of the high frequency returns. Thus, in order to eliminate the effects of intraday periodicity, this work applies the Flexible Fourier Form (FFF) method proposed by Gallant (1981, 1982). In particular, Baillie et al. (2000) have shown that the FFF method appears to be appropriate for representing intraday periodicity without inducing any non-linearity. Hence, after filtering the high frequency returns using the FFF method, this work employs the FIGARCH model combined with Bernoulli jump process to evaluate the potential impact of the strong intraday periodicity on the long memory property in the volatility process of the high frequency data.

The plan for the rest of this paper is as follows: section 2 presents a basic analysis of high frequency European returns series for the CHF-$, DM-$, $-BP, FF-$ and ITL-$ exchange rates. For the analysis of the high frequency returns, the application of the FIGARCH model of Baillie et al. (1996) is discussed. Section 3 of the paper then describes the normal mixture distribution model, the FIGARCH model combined with Bernoulli jump process in order to account for the conditional mean jumps, the high excess kurtosis and the long memory property in the high frequency returns. Section 4 briefly presents the effects of the intraday periodicity on the long memory property. This paper applies the Flexible Fourier Form (FFF) proposed by Gallant (1981, 1982) to filter the raw high frequency returns and eliminate the impacts of the intraday periodicity. Finally, section 5 provides a conclusion.

2. Basic Analysis of Long Memory Property
This section is concerned with a set of 30-minute CHF-$, DM-$, $-BP, FF-$ and ITL-$ high frequency spot exchange rates provided by Olsen & Associates of Zurich, consisting of Reuter FXFX quotes taken every 30 minutes for the complete calendar year of 1996. The sample period is 00:30 GMT, January 1, 1996 through 00:00 GMT, January 1, 1997. Each quotation consists of a bid and an ask price and is recorded to the nearest second.

Following the procedures of Müller et al. (1990), Dacorogna et al. (1993), Baillie et al. (2000) and Baillie and Han (2002), the spot exchange rate for each 30-minute interval is determined as the linearly interpolated average between the preceding and the following quotes. Hence the 30-minute exchange rates are defined as the difference between the midpoints of the logarithmic bid and ask rates. The sample used in subsequent analysis contains 262 trading days, each with 48 intervals of 30-minutes duration, which realizes a total of $T = 12,576$ observations for 262 days.

The five high frequency spot exchange rates series are presented in Figures 1(a) through 1(e). The time series are characterized by several large jumps or shifts followed by ostensibly random movement. The jumps in the high frequency exchange rates may have been caused by public macroeconomic news, private information and other outliers such as recording errors caused by computer problems. In particular, Andersen et al. (2002) have presented evidence that the US announcement surprises, the differences between expectations and realizations of macroeconomic fundamentals, produce conditional mean jumps so that the dynamics of exchange rates is linked to economic fundamentals and the markets react to the jumps in asymmetric fashion.

Figures 2 (a) through (e) plot the first 240 autocorrelations for the returns, squared returns and absolute returns of the raw 30-minute exchange returns of the CHF-
$, DM-$, $-BP, FF-$ and ITL-$ exchange rates for the year 1996. While higher order autocorrelations are not significant at conventional levels, the first order autocorrelation in the returns is small, negative but very significant for the high frequency returns. This may be due to the market microstructure effects suggested by Andersen and Bollerslev (1997, 1998) or the large jumps in the data as suggested by Goodhart and Figliuoli (1991), Goodhart and Giugale (1993) and Ghosh (1997).

The autocorrelation functions of the squared and absolute returns display the pronounced U-shape pattern associated with substantial intraday periodicity and decay very slowly at the hyperbolic rate, which is a typical feature of a long memory property. The two main features of the high frequency returns are more apparent in the autocorrelation functions of the absolute returns as presented by Granger and Ding (1996). These are in line with the findings of Wasserfallen (1989), Müller et al. (1990), Baillie and Bollerslev (1991), Dacorogna et al. (1993), Andersen and Bollerslev (1997, 1998), Baillie et al. (2000) and Baillie and Han (2002). The shapes and the amplitudes of the intraday periodicity in the autocorrelations of absolute returns appear to be quite similar across the currencies. This seasonal pattern is related to the similar trading activity and the opening and closing of European foreign exchange markets.

For the basic analysis of the high frequency returns, the FIGARCH model of Baillie et al. (1996) is applied to model the volatility process in the high frequency spot returns assuming that the innovations are normally distributed. In fact, this normal distribution assumption is justified by Weiss (1984, 1986) and Bollerslev and Wooldridge (1992) who showed that the QMLE (Quasi Maximum Likelihood) estimator under the normality assumption is consistent if the conditional mean and the
conditional variance are correctly specified even though the standard errors have to be adjusted.

The model postulated to describe the returns process is,

\[ y_{t,n} = \mu + \varepsilon_{t,n} + \theta \varepsilon_{t,n-1}, \quad (1) \]

\[ \varepsilon_{t,n} = z_{t,n} \sigma_{t,n}, \quad (2) \]

\[ \sigma^2_{t,n} = \omega + \beta \sigma^2_{t,n-1} + [1 - \beta L - (1 - \phi L)(1 - L)^\delta] \varepsilon_{t,n}^2, \quad (3) \]

where \( y_{t,n} = 1000^* \Delta \ln(S_{t,n}) \) and \( S_{t,n} \) is the 30-minute spot foreign exchange rates, \( z_{t,n} \) is i.i.d.(0,1) and the returns are specified as following an MA(1) process to account for the strong negative first autocorrelation. The conditional variance process \( \sigma^2_{t,n} \) in equation (3), is represented by a FIGARCH \((1,\delta,1)\) process to represent the long memory property, following the work of Baillie et al. (1996). The above FIGARCH \((1,\delta,1)\) process is sufficiently general that it can model very slow hyperbolic rate of decay in the autocorrelations of squared returns.

The FIGARCH process has impulse response weights, \( \sigma^2_{t,n} = \omega/(1 - \beta) + \lambda(L)\varepsilon^2_{t,n} \) which for large lags, \( \lambda_k \approx k^{\delta - 1} \), is essentially the long memory property or “Hurst effect” of hyperbolic decay.\(^6\) The attraction of the FIGARCH process is that for \( 0 < \delta < 1 \), it is sufficiently flexible to allow for intermediate ranges of persistence. The simpler FIGARCH \((1,\delta,0)\) process is of the form,

\[ \sigma^2_{t,n} = \omega + \beta \sigma^2_{t,n-1} + [1 - \beta L - (1 - L)^\delta] \varepsilon^2_{t,n}, \quad (4) \]
Equations (1) through (3) are estimated by using non-linear optimization procedures to maximize the Gaussian log likelihood function,

$$\log(\xi) = -(T/2)\ln(2\pi) - (1/2)\sum_{t=1, n=1}^{261, 48}[\ln(\sigma^2_{t,n} + \epsilon^2_{t,n}\sigma^2_{1,n})], \quad (5)$$

Since most return series are not well described by the conditional normal density in equation (5), subsequent inference is consequently based on the Quasi Maximum Likelihood Estimation (QMLE) technique of Bollerslev and Wooldridge (1992),

$$T^{1/2}(\hat{\theta}_T - \theta_0) \rightarrow N\{0, A(\theta_0)^{-1}B(\theta_0)A(\theta_0)^{-1}\}, \quad (6)$$

where \(A(.)\) and \(B(.)\) represent the Hessian and outer product gradient respectively; and \(\theta_0\) denotes the true parameter values.

The estimated parameters of the above model for the five high frequency raw returns series are presented in Table 1. The estimated long memory volatility parameters, \(\delta\), are very significant and quite similar across the high frequency returns series ranging from 0.27 through 0.29, implying that the long memory property in the volatility process appears to be consistent across the European currencies. For the all currencies, the robust Wald test statistics \(W_{\delta=0}\) of a stationary GARCH model under the null hypothesis versus a FIGARCH model under the alternative hypothesis have clearly rejected the null for the all currencies when compared with the critical values of a chi squared distribution with one degree of freedom for the five high frequency returns
data. Thus, the FIGARCH model appears to be more appropriate to represent the high frequency returns data than the regular GARCH model.

However, the estimated kurtosis statistics for the high frequency returns are found to be relatively large compared with those of the daily data, which implies the fat-tailed distributions of the high frequency returns even after the long memory property is removed effectively. The excess kurtosis property of the high frequency exchange rate returns may be related to the conditional mean jumps caused by the monetary and financial information or some outliers. These jumps might lead to the level and volatility “outliers” which can not be taken into account for by the simple normal distribution as Hotta and Tsay (1998) presented.

Furthermore, the Box-pierce test statistics represented by Q2(50) and Q3(50) for the squared and the absolute residuals from the basic MA(1)-FIGARCH (1,δ,1) model for the all high frequency returns reject the null hypothesis of no serial correlation significantly suggesting that the conditional variances of the high frequency returns series are significantly correlated. This may be due to the strong intraday periodicity in the squared and absolute returns. Thus, the basic MA(1)-FIGARCH (1,δ,1) models under the assumption of Gaussian normal distribution seem to be inappropriate to represent the high frequency returns series properly.

3. Bernoulli Jump Process and Long Memory Property

This section is concerned with modeling jumps and high excess kurtosis of the high frequency European spot returns series. And several empirical studies such as those of Goodhart et al. (1993), Ghosh (1997), Almeida et al. (1998), Melvin and Yin (2000), Andersen et al. (2002) and Beine and Laurent (2003) have suggested that the
jumps in the conditional mean process of high frequency exchange rate series appear to be linked with public macroeconomic news, private information, central bank interventions and some outliers caused by technical errors. These jumps or the large changes in the movements of the high frequency exchange rate series may cause asymmetry and high excess kurtosis in the process.\textsuperscript{9} In particular, Andersen et al. (2002) have presented that high frequency foreign exchange rates response to news asymmetrically and the asymmetry may be driven by the uncertainty in the underlying state of the economy. Thus, the basic MA-FIGARCH model with the assumption of the normal distribution seems to be inappropriate to represent the asymmetric jump process of the high frequency returns.

This study employs the jump diffusion process proposed by Press (1967) in order to account for the conditional jumps in the 30-minute high frequency spot returns. Initially, Press (1967) proposed a jump diffusion model for stock prices under the assumption that the logarithm of the stock price follows a Brownian motion process on which i.i.d. normal distributed jumps are included. Jorion (1988) used a Press-type model to find some statistical evidence of jumps in the US$-DM exchange rate for the post 1971 freely floating period.\textsuperscript{10}

This jump diffusion model has subsequently been widely employed to model features of Exchange Rate Mechanism (ERM) of European Monetary System (EMS) such as the jumps resulting from realignments of the ERM bands and high excess kurtosis. See Vlaar and Palm (1993), Nieuwland et al. (1994), Neely (1999), Baillie and Han (2001) and others. Baillie and Han (2001) have particularly considered a model in discrete time, providing a relatively simple formulation to investigate a target zone model, while Chung and Tauchen (2001), Ball and Roma (1993) and others have favored continuous
time models and have used jump diffusion processes for the analysis of foreign exchange rates in the EMS under a target zone model.

This study uses a Bernoulli distribution to model the stochastic jumps in the conditional mean process of the 30-minute high frequency spot rates series. The main characteristic of the Bernoulli process is that over a fixed time period \((t)\), either no information impacts on the price or one relevant information arrival occurs with probability \(\lambda\). While stochastic jumps are generally modelled by a Poisson distribution (see e.g., Akgiray and Booth, 1988; Ball and Torous, 1985; Hsieh, 1989; Jorion, 1988; Nieuwland et al., 1991), the Bernoulli jump process appears to be more an appropriate model for the jumps caused by new information arrivals. This is due to the following aspects highlighted by Ball and Torous (1983) and Vlaar and Palm (1993): i) for practical reasons, no more than one “abnormal” information can be expected to arrive during a very short period as in high frequency data, ii) if the returns are computed for finer time intervals, the Bernoulli model would converge to a Poisson model, iii) satisfactory empirical analyses are available since maximum likelihood estimation of the relevant parameters is practically and economically implementable, and iv) the Bernoulli process is simple in calculation without requiring the infinite sum and the truncation process required by the Poisson process.

Also, Vlarr and Palm (1993) and Baillie and Han (2001) have shown that the results from the Poisson distribution and the Bernoulli distribution are very similar in the case of the analysis of a target zone model. Thus, this study uses the Bernoulli jump process to account for the conditional mean jumps and combines it with FIGARCH model to analyze the impact of jumps in the conditional mean process on the long memory property in the conditional variance process of the high frequency returns series.
For the Bernoulli distribution, the jump intensity ($\lambda$) is forced in the (0,1) interval by estimating a parameter $j$ in the expression, $\lambda = [1 + \exp(j)]^{-1}$. The jump size is given by the random variable $v$, which is assumed to be NID($v$, $\delta^2$). The same FIGARCH model as in section 2 is used for the long memory volatility process. Since the statistical and economic motivations for the jumps and the long memory property are quite different, this work chooses a model specification that accounts for the two features at the same time.

Hence, this work investigates the high frequency returns of CHF-$, DM-$, FF-$, $-BP and ITL-$ exchange rates by employing the Bernoulli jump diffusion process to consider the conditional mean jumps and the excess kurtosis, combined with the FIGARCH model to capture the long memory property in the conditional variances. The inclusion of the jump process may reduce the influence of the conditional mean jumps on the MA(1)-FIGARCH(1,$\delta$,1) specification.

The combined MA(1)-FIGARCH (1,$\delta$,1) model with Bernoulli jump process is,

$$y_{t,n} = \mu + \lambda v + \varepsilon_{t,n} + \theta \varepsilon_{t,n-1}, \quad (7)$$

$$\varepsilon_{t,n} = z_{t,n} \sigma_{t,n}, \quad (8)$$

$$\sigma^2_{t,n} = \omega + \beta \sigma^2_{t,n-1} + [1 - \beta L - (1 - \phi L)(1 - L)^\delta] \varepsilon^2_{t,n}, \quad (9)$$

The 30-minute high frequency returns are still specified as following an MA(1) process, with a jump intensity of $\lambda$ and a jump size of $v$. The volatility process is a FIGARCH(1,$d$,1) conditional variance model as developed in section 2.
The log likelihood function for the combined model has the following form,

\[
(\xi) = -(T/2) \ln(2\pi) + \sum_{t=1,262, n=1,48} \ln \left\{ \frac{1}{h_{t,n}} \right\} \exp \left\{ -\frac{(\epsilon_{t,n} + \lambda v)^2}{2h_{t,n}^2} \right\} + \lambda \left( \frac{\delta^2}{h_{t,n}^2 + \delta^2} \right)^{1/2} \exp \left\{ -\frac{(\epsilon_{t,n} - (1 - \lambda)v)^2}{2(h_{t,n}^2 + \delta^2)} \right\} \quad (10)
\]

The form of the likelihood function for the Normal-Bernoulli jump processes is basically the same as that developed by Vlaar and Palm (1993) and Baillie and Han (2001). Asymptotic standard errors are calculated from the QMLE of Bollerslev and Wooldridge (1992) as in section 2.

The estimated model parameters for the high frequency series are reported in Table 2. The estimated parameters \( (\lambda) \) for the jump intensity \( (\lambda) \) in the high frequency returns are all significant at the conventional level of significance, implying that the Bernoulli jump process appears to be quite appropriate to account the conditional mean process. The jump intensities \( (\lambda) \) calculated from the estimated \( (j) \) are 0.18, 0.12, 0.17, 0.16, and 0.25 for the CHF-$, DM-$, FF-$, $-BP and ITL-$ high frequency series respectively. Within the one year series of 12,576 30-minute observations, the corresponding implied numbers of the jumps are 2264, 1509, 2138, 2138 and 3144 of the CHF-$, DM-$, FF-$, $-BP and ITL-$ high frequency returns series respectively.

In particular, given the fact that there have been approximately total 720 public macroeconomic news released from US and German during 1996\(^{11}\), part of the jumps in the DM-$ exchange rates may have taken place by the other reasons like private information, noisy trading of dealers and outliers. One interesting issue concerns the interpretation of the jumps and whether or not they can be explained by private information of traders, noisy trading and some outliers as well as by the public

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macroeconomic news. The interpretation of the parameters ($\lambda$) indicate the probability of a jump occurring during 1996 in high frequency foreign exchange rates, suggesting that the DM-$ foreign exchange market seems to be more stable during 1996 while the ITL-$ market appears to be unstable.

The estimated parameters ($\nu$) which represent the impacts of the jumps on the mean process are found to be insignificant for the all high frequency returns. This is in consistent with the general pattern of the very quick and effective exchange rate conditional mean adjustment after the jumps as reported by Andersen et al. (2002). But the effects of the jumps ($\delta^2$) on the volatility process are estimated to be very significant and much greater than those on the mean process, suggesting that the effects on the volatility process are more important and more significant.

In particular, the results in Table 2 show that the estimated long memory parameters ($\delta$) of the high frequency returns are 0.03, 0.07, 0.06, 0.05 and 0.05 for the CHF-$, \text{DM-}$, FF-$, \text{S-BP}$ and ITL-$ high frequency returns respectively and they are all very significant.\(^{12}\) Interestingly the values of the estimated long memory parameters are found to be quite smaller than those from the basic MA(1)-FIGARCH (1,$\delta$,1) models without considering the jumps. This suggests that the high long memory property of the high frequency returns may be significantly affected by the jumps in the conditional mean process and the higher values of the long memory parameters can be because the jumps are not accounted for in the conditional mean process. These estimation results are quite in consistent with the findings of Beine and Laurent (2003) who used the same Bernoulli-normal FIGARCH model for the investigation of the daily exchange rates of DEM-$, \text{FRF-}$, GBP-$ and JPN-$.\(^{12}\)
As Beine and Laurent (2003) presented, these results seem to be quite understandable given that the conditional mean jumps are fully accounted for in the mixture distribution since the jumps may be spuriously associated with the additional volatility. Thus, the higher long memory property seems to be related to the asymmetric conditional mean adjustments to the jumps, which is much more gradual and persistent than the conditional variance adjustments and the jumps seem to be the possible driving forces behind the long memory property in the volatility process of the high frequency returns as presented by Andersen and Bollerslev (1998) and Andersen et al. (2002).\textsuperscript{13}

And similarly to the findings of Nieuwland et al. (1994), Vlarr and Palm (1993) and Beine and Laurent (2003), the estimation results show that the skewness and kurtosis statistics are also reduced significantly for the high frequency returns after the jumps are accounted for. Thus, it confirms that this mixture distribution generally outperforms the simple normal distribution. And, it seems relevant to introduce the possibility of jump in the conditional mean process of the high frequency returns. In particular, the values of \(v\) are found to be negative for DEM, FF, BP and ITL and corresponding to the negative skewness, implying that on average, the jumps shift the density to the left. And, the reduced excess kurtosis confirms that accounting for a non-uniform flows of information can reduces the excess kurtosis.

While the \(Q_2(50)\), the Box-Pierce test statistics for the squared residuals are significantly reduced and they accept the null hypothesis except the returns of the $-BP and ITL-$ exchange rates, the Box-Pierce test statistics, \(Q_3(50)\) from the absolute residuals for all currencies are similarly decreased but they still reject the null hypothesis of no serial correlation in the conditional variance process. This seems to be due to the intraday periodicity in the conditional variance process as presented by
Figures 3(a) through (e) which represent the correlograms of the standardized residuals of the high frequency returns from the combined MA(1)-FIGARCH(1,δ,1) model with Bernoulli jump process. Interestingly, the correlograms show that the effects and amplitudes of the intraday periodicity are generally reduced for all five currencies. This indicates that the intraday periodicity may be linked to the conditional mean jumps which are caused in foreign exchange markets over the world.

4. Conditional Mean Jumps, Long Memory Property and Intraday Periodicity

In order to model the high frequency returns series more properly, this study applies the approach of Andersen and Bollerslev (1998), Baillie et al. (2000, 2003) and Baillie and Han (2002) and uses a two-step estimation method. The intraday periodicity is first removed by applying the Flexible Fourier Form method of Gallant (1981, 1982). Then the combined MA(1)-FIGARCH (1,δ,1) model with Bernoulli process presented in Section 3 is applied to model the high frequency filtered returns.

The intraday periodicity is first removed by applying the Flexible Fourier Form (FFF) approach of Gallant (1981, 1982) (see Baillie et al. (2000) and the Appendix in Baillie and Han (2002) for the details of the FFF approach). The 30 minute returns are filtered by the estimated intraday seasonality, $P_{t,n}$ from the FFF method, which are defined as

$$y_{t,n} = \frac{R_{t,n}}{P_{t,n}},$$

(11)

where $R_{t,n} = 1000*\Delta[\ln(S_{t,n})]$ and $S_{t,n}$ is the 30-minute spot foreign exchange rates.
The high frequency filtered returns are then modeled using the same combined Bernoulli jump process-FIGARCH model represented by equation (7) through equation (9) in section 3. The parameters of the estimated models for the high frequency filtered returns are reported in Table 3. In particular, the Box-Pierce test statistics, Q2(50) and Q3(50) of the squared and the absolute standardized residuals from the combined model can not reject the null hypothesis of no serial correlation in the variance process for the all high frequency returns except the case of the ITL-$ exchange rates. These results can be confirmed by the correlograms of the standardized residuals from the combined model presented in Figures 4 (a) through (e).

Many of the intrinsic features of the 30-minute high frequency returns are captured quite well by the model. The strong intraday periodicity in the standardized residual of the absolute returns is reduced very significantly for all the high frequency returns, so the FFF method appears to be appropriate for removing the intraday periodicity as in Andersen and Bollerslev (1997,1998), Baillie et al. (2000) and Baillie and Han (2002).

Similarly to those in section 3, the effects of the jumps ($\delta^2$) on the volatility process are estimated to be very significant and much greater than those on the mean process while the impacts of the jumps on the mean process represented by the parameters ($\nu$) are found to be insignificant for the all high frequency returns due to the general pattern of the exchange rate conditional mean adjustment after the jumps. Interestingly, the estimated parameters ($\nu$) of ITL-$ is changed to positive after the intraday periodicity is accounted for and is corresponding to the positive skewness. This implies that the periodicity may shift the density to the right. And, the estimated ($j$) is significant at the conventional level of significance for the five high frequency returns.
series, suggesting that the jumps are still very significant and the Bernoulli process appears to be quite successful in capturing the conditional mean jumps properly.

The jump intensities ($\lambda$) calculated from the estimated ($j$) are 0.12, 0.09, 0.09, 0.10 and 0.12 and the corresponding implied numbers of the jumps are 1509, 1131, 1131, 1257 and 1509 for the CHF-$, DM-$, FF-$, $-BP and ITL-$ high frequency returns series respectively. These values of the jump intensities for the five high frequency returns are reduced slightly compared with those from the combined model for the raw returns data containing the intraday periodicity in section 3 implying that the filtered returns after the intraday periodicity is removed may be more stable than the raw returns. This may be because the log-transformation of the FFF filtering eliminates the extreme outliers in the high frequency returns as mentioned in Andersen and Bollerslev (1998).

The estimated long memory parameters of the high frequency returns are 0.07, 0.11, 0.07, 0.08 and 0.07 for the CHF-$, DM-$, FF-$, $-BP and ITL-$ high frequency returns respectively and they are all very significant. The values of the estimated long memory parameters for the five high frequency returns are generally similar to those from the original raw returns without filtering the intraday periodicity in section 3. The results suggest that the intraday periodicity may not affect the long memory property in the conditional variance process significantly.

Thus, the Flexible Fourier Form method generally appears to quite appropriate in removing the intraday periodicity in the high frequency returns of European exchange rates without inducing any distortions as presented in Baillie et al. (2000). And, the combined MA(1)-FIGARCH (1,$\delta$,1) model with the Bernoulli process seems to be very successful in accounting for both the conditional mean jumps in the conditional mean
process and the long memory property in the conditional variance process of the filtered high frequency European returns.

5. Conclusions

This paper has considered one year of high frequency European exchange rate returns data for the CHF-$, DM-$, FF-$, $-BP and ITL-$ and has investigated the conditional mean jumps, the high excess kurtosis, the intraday periodicity and the long memory volatility process of the high frequency returns. Special attention has been devoted to the conditional mean jumps and normal mixture distribution in the MLE estimation procedure.

The general results indicate that the usual normality assumption appears not to be appropriate for the high frequency exchange returns data mainly due to the important number of jumps in the conditional mean process, that the major parts of the jumps seem to be related to public macroeconomic announcements, private news and central bank interventions in the foreign exchange markets and that the normal mixture distribution model, MA(1)-FIGARCH(1,d,1) model with Bernoulli jump process is found to be quite successful in representing the conditional mean jumps, excess kurtosis and the long memory volatility property of the high frequency returns.

In particular, the results present that the intraday periodicity is significantly weaker when the jump process is included so that it may be related to the conditional mean jumps and that the estimated long memory parameters are found to be much smaller than those from the basic MA(1)-FIGARCH (1,d,1) model without accounting for the jumps providing statistical evidence that the specification of the conditional mean process without considering the jumps seem to increase the exchange rate
volatility and may distort the estimates of the long memory parameters, much in line with the finding in the empirical literature..

The Flexible Fourier Form (FFF) method is also found to be successful in removing the strong intraday periodicity without inducing any significant distortions as in Baillie et al. (2000). Thus, the filtered high frequency currency market data can be represented by the combined MA(1)-FIGARCH(1,d,1) model with Bernoulli process surprisingly well after the intraday periodicity is removed by using FFF method.

This study seems to be helpful in deepening our understanding of the exchange rates and their long memory volatility property. In particular, the results suggest that such a representation model is a possible alternative modeling strategy to long memory model with a normal distribution or to account for the jumps in the exchange rate dynamics.
ACKNOWLEDGEMENT

The author gratefully acknowledges support from City University of Hong Kong, Strategic Research Grant (SRG) # 7001493 and #7001573. The author is also grateful to Olsen and Associates for making available their real time exchange rates and to Money Market Services (MMS) International for their news announcement data.
ENDNOTES

1. Recently, Dacorona et al. (2001) provided a summary of high frequency financial time series data with details of the analysis, modelling and inference in foreign exchange markets.

2. The studies of Baillie et al. (2000, 2003), Baillie and Han (2002) and Beltratti and Morana (1999) have presented that the FIGARCH model seems to provide better descriptions of high frequency return volatility than the GARCH(1,1) model. Similarly, some papers such as Baillie Osterberg (2000), Beine et al. (2002 a,b) and Han (2003a) have used the FIGARCH model for the analysis of daily exchange returns.

3. Frances and Ghysels (1999) showed that the excess kurtosis in the stock market volatility can be due to neglected additive outliers (AOs).


5. Since it has become very common to remove atypical data associated with slower trading patterns during weekends, returns from Friday 21:00 GMT through Sunday 20:30 GMT are excluded. However, returns for holidays occurring during the sample are retained in order to preserve the number of returns associated with one week.

6. Beine, Laurent and Lecourt (2002) and Han (2003a) have presented the cumulative impulse functions of FIGARCH models and compared them with those of GARCH and IGARCH models. The FIGARCH model seems to be better represent the hyperbolic decay of the impulse functions than GARCH model.

7. From the analysis of the daily exchange rates of the same five currencies which sampled from March 14, 1979 to December 31, 1998, the estimated kurtosis from the MA(1)-FIGARCH model are 4.81, 4.50, 6.24, 4.85 and 4.76 for the CHF-$, FF-$, DM-$, $-BP and ITL-$ respectively.

8. The autocorrelation functions of the squared and absolute standardized residuals from the basic MA(1)-FIGARCH models represent that the U-shaped intraday periodicity are apparent in the volatility process masking the long memory property. The correlograms are not reported to reserve the space, but they are available on the request to the author.

9. DeGeoeij and Marquering (2002) presented that the macroeconomic news announcement shocks affect the conditional volatility process of stock and bond returns and that the asymmetric volatility process in the Treasure bond market can be explained by the macroeconomic news shocks.
10. Akgiray and Booth (1987) and Nimalendran (1994) have examined empirically the jump diffusion process for the analysis of stock prices and portfolios and have found that the jump diffusion model was useful for multiple announcements.

11. The data for the public macroeconomic news of US and German is provided by MMS (Money Market Service) International. During 1996, 40 different kinds of US macroeconomic indicators have been released with approximately 420 observations, and 27 different types of Germany public news have been announced with approximately 300 observations.

12. The analysis of the daily data which sampled from March 14, 1979 to December 31, 1998 for the FF-$, DM-$ and ITL-$ returns series generally showed the similar results that the estimated long memory parameters are reduced after the Bernoulli jump process is included in the model to account for the jumps. The detailed results for the analysis of the daily data are available by the request on the author.

13. Several papers also found that the decrease in long memory when accounting for structural changes in exchange rate dynamics (see, Diebold and Inoue 1999; Granger and Hyung 1999; Beine and Laurent 2001; Bouble and Laurent 2001).

14. There are some other methods for modeling the intraday periodicity. The first one is to use seasonal dummy variables as in Baillie and Bollerslev (1991). The second one is to use an intraday time scale proposed by Dacorogna et al (1993). The third one is to use the multiplicative factors approach to model the intraday seasonality as in Taylor and Xu (1997) and Chang and Taylor (1998, 2003).

15. The correlograms of the filtered high frequency returns of the CHF-$, DM-$, FF-$, $-BP and ITL-$ exchange rates present that the strong intraday periodicity is reduced very significantly but the long memory property is still remained for the three currencies. The detailed results are available from the author on the request.
REFERENCES


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geographical model for daily and weekly seasonal volatility in the foreign
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DeGoeij, P., Marquering, W., 2002. Do Macroeconomic Announcements Cause

Econometric Reviews 5, 51-56.

Department of Finance, Stern School, NYU.


Han, Y.W., 2003a. Central bank interventions and long memory property in the daily


Table 1: Estimated MA(1)-FIGARCH (1, δ, 0) Model for Raw 30-minute Returns

\[ y_t = 1000* \Delta \ln(S_{t,n}) = \mu + \varepsilon_{t,n} + \theta \varepsilon_{t,n-1} \]

\[ \varepsilon_{t,n} = z_{t,n} \sigma_{t,n}, \text{ where } z_{t,n} \text{ is i.i.d.(0,1) process} \]

\[ \sigma_{t,n}^2 = \omega + \beta \sigma_{t,n-1}^2 + [1 - \beta L - (1 - \varphi L)(1 - L)^\delta] \varepsilon_{t,n}^2 \]

where \( t = 1, \ldots, 262 \) and \( n = 1, \ldots, 48 \).

<table>
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<tr>
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Keys: \( S_{t,n} \) is the 30-minute spot exchange rates from 00:30 GMT, January 1, 1996 through 00:00 GMT, January 1, 1997 for a total of 262 days. QMLE asymptotic standard errors are in parentheses below corresponding parameter estimates. The quantity \( \ln(L) \) is the value of the maximized log likelihood. The sample skewness and kurtosis refer to the standardized residuals. The Q(50), Q2(50) and Q3(50) statistics are the Ljung-Box test statistics for 50 degrees of freedom to test for serial correlation in the standardized residuals, squared standardized residuals and absolute standardized residuals.
Table 2: Estimated MA(1)-Bernoulli jump process- FIGARCH \((1,\delta,1)\) Model for Raw 30-minute Returns

\[ y_{t,n} = 1000^* \Delta[\ln(S_{t,n})] = \mu + \lambda v + \varepsilon_{t,n} + \theta \varepsilon_{t,n-1} \] where \( \lambda = [1 + \exp(j)]^{-1} \)

\[ \varepsilon_{t,n} = z_{t,n} \sigma_{t,n} \] where \( z_{t,n} \) is i.i.d.(0,1) process

\[ \sigma_{t,n}^2 = \omega + \beta \sigma_{t,n-1}^2 + [1 - \beta L - (1 - \phi L)(1 - L^\delta)] \varepsilon_{t,n}^2 \] where \( t = 1,...,262 \) and \( n = 1,...,48 \).

<table>
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| \( \ln(L) \) | -13755.535 | -10377.550 | -100595.581 | -10010.661 | -11971.152 |
| Skewness   | 0.106 | -0.039 | -0.043 | -0.021 | -0.015 |
| Kurtosis   | 4.361 | 5.148 | 5.167 | 4.794 | 4.368 |
| Q(50)      | 88.240 | 70.753 | 63.673 | 57.865 | 68.647 |
| Q2(50)     | 93.871 | 82.589 | 89.169 | 172.843 | 166.357 |
| Q3(50)     | 274.231 | 294.747 | 213.406 | 288.781 | 240.155 |

Keys: the same as Table 1 except that a jump intensity of \( \lambda \), where \( 0 < \lambda < 1 \) and \( \lambda = [1 + \exp(j)]^{-1} \), and is specified by the Bernoulli process. The jump size is given by the random variable \( v_t \), which is assumed to be NID\((v, \delta^2)\).
Table 3: Estimated MA(1)-Bernoulli jump process- FIGARCH (p,δ,q) Model for Filtered 30-minute Returns

\[ y_{t,n} = \frac{R_{t,n}}{P_{t,n}} = \mu + \lambda v + \varepsilon_{t,n} + \theta \varepsilon_{t,n-1} \text{ where } \lambda = [1 + \exp(j)]^{\delta/\lambda} \]

\[ \varepsilon_{t,n} = z_{t,n} \sigma_{t,n} \text{ where } z_{t,n} \text{ is i.i.d.}(0,1) \text{ process} \]

\[ \sigma_{t,n}^2 = \omega + \beta \sigma_{t,n-1}^2 + [1 - \beta L - (1 - \varphi L)(1 - L)^{\gamma}] \varepsilon_{t,n}^2 \text{ where } t = 1,\ldots,262 \text{ and } n = 1,\ldots,48. \]

<table>
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\[ \ln(L) \] | -14067.588 | -10527.383 | -11074.756 | -10140.791 | -12510.049 |
| Skewness | 0.008 | -0.095 | -0.108 | -0.032 | 0.001 |
| Kurtosis | 4.874 | 5.246 | 7.843 | 5.700 | 6.304 |
| Q(50)   | 78.638 | 71.528 | 63.836 | 59.318 | 92.699 |
| Q2(50)  | 58.911 | 26.311 | 61.603 | 60.842 | 137.284 |
| Q3(50)  | 64.707 | 40.530 | 89.194 | 75.931 | 100.902 |

Keys: the same as Table 1 except that \( R_{t,n} = 1000^\delta \Delta[\ln(S_{t,n})] \) and \( P_{t,n} \) is the estimated intraday periodicity from the Flexible Fourier Form method and that a jump intensity of \( \lambda \), where \( 0 < \lambda < 1 \), \( \lambda = [1 + \exp(j)]^{\delta/\lambda} \) and is specified by the Bernoulli process. The jump size is given by the random variable \( v_t \) which is assumed to be NID(v, δ²).
Figure 1: 30-minute Spot Exchange Rates during 1996

(a) DM-$

(b) CHF-$

(c) FF-$
Figure 2: Correlograms of 30 minute Raw Returns

(a) CHF-$

(b) DM-$

(c) FF-$
Figure 3: Correlograms of Standardised Residuals of 30-minute Raw Returns from Bernoulli Jump Process-MA(1)-FIGARCH Model

(a) CFH-$

(b) DM-$

(c) FF-$
Figure 4: Correlograms of Standardised Residuals of 30-minute Filtered Returns from Bernoulli Jump Process-MA(1)-FIGARCH Model

(a) CHF-$

(b) DM-$

(c) FF-$
(d) $-BP$

(e) ITL-$\$