Currency Crises and the Informational Role of Interest Rates

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Abstract

This paper modifies the currency crisis framework of Morris and Shin (1998) by letting market prices adjust freely to economic conditions. An endogenous interest rate is shown to play a dual role: it sends a public signal about the state and also affects the strategic interaction of informationally heterogeneous agents. Due to the latter role of the interest rate, virtually perfect common knowledge of the fundamentals can coexist with non-trivial uncertainty about agents’ aggregate behaviour. As a result, equilibrium uniqueness is attained even when a necessary condition for it within the Morris-Shin framework, regarding the relative precision of public and private information, does not hold. The dual role of market prices reveals a new perspective on policy alternatives aimed at curbing sudden and violent speculative attacks. The paper also suggests a rectification of the existing approach to structural empirical analysis of currency crises.

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1 Introduction

In order to explain drastic shifts in financial market behaviour that seem to be unrelated to the underlying fundamentals, economic theory makes often use of a self-fulfilling feature of rational beliefs. That feature is the result of strategic complementarity: when economic agents anticipate a financial crisis, they attempt to shield themselves from its consequences and in the process precipitate it; in contrast, if the crisis is not seen as imminent, agents' behaviour preserves the status quo. In a representative agent model, self-fulfilling beliefs generate multiple equilibria and can thus rationalise puzzling phenomena. There is, however, a downside: the particular prevailing outcome is to be attributed to economically meaningless variables which act as coordination devices and are frequently referred to as sunspots. Since sunspots are inevitably associated with forces outside the theoretical model, multiple equilibria accounts are of limited use for policy analysis.

In a series of articles, Stephen Morris and Hyun Song Shin abandon the representative-agent paradigm and demonstrate that potentially self-fulfilling beliefs might actually have a unique equilibrium realisation. In addition to publicly available information about the economy, agents in the Morris-Shin (henceforth, MS) framework hold diverse perceptions of the world due to their access to private signals. This leads to uncertainty of market participants about their peers' beliefs and actions and does not allow for any role of sunspots. Furthermore, the ensuing unique equilibrium accounts for seemingly unwarranted financial crises and, by establishing a deterministic link between economic fundamentals and market behaviour, places policy analysis on a firm footing.

Its insights notwithstanding, the contribution of the MS framework has been considered to be of limited applicability. The framework’s main results hinge on a premise that the two types of information (private and public) have independent sources. The premise is hard to accept if one believes that publicly observed prices are the outcome of heterogeneous agents’ collective actions and, as such, aggregate at least some of these agents’ private knowledge.

I address this criticism of the MS framework by letting a market price generate an endogenous link between financial market participants’ private and public information. I then prove that equilibrium uniqueness in the new framework requires weaker restrictions on the quality of public knowledge and is consistent with dramatic revisions of agents’ behaviour triggered by small changes in the fundamentals. Policy analysis can thus be conducted in the spirit of previous work by Morris and Shin, ie via straightforward comparative statics, while cast in a more realistic context. A central authority that actively participates in financial markets is seen to be potentially in a position to
manage a crisis by influencing the informational content of prices.

The paper’s analysis is cast in a context of speculative currency attacks. The model introduces an endogenous interest rate in the global games setup of Morris and Shin (1998), which generalizes the informational structure of the second-generation approach to currency crises.

There is evidence that the European exchange rate mechanism (ERM) could have been successfully attacked for some time before 1992-3 when speculative pressure led to changes in a number of the constituent regimes (Eichengreen and Wyplosz (1993)). There is also evidence that, without such pressure, the strength of the European economies and the long-run political stake in a European monetary union would have preserved the ERM regimes from the late 1980s throughout the 1990s (Eichengreen (2001), Obstfeld (1996)). The second-generation approach to currency crises reconciles such accounts by focusing on the interdependence of the optimisation problems of two distinct foreign exchange market players: the private sector and the central authority managing the exchange rate regime. Strategic complementarity in these players’ actions is shown to lead to multiple equilibria.

Suppose for concreteness that the authority’s options are to either maintain an exchange rate peg or devalue. If devaluation expectations are high, private speculators attack the currency and cause, for example, an increase of the domestic interest rate and/or an outflow of the authority’s foreign reserves. Under such pressure, the authority’s incentives to devalue intensify and the private sector’s expectations might be warranted. Unchanged economic fundamentals might, however, also justify low devaluation expectations which, by producing a low domestic interest rate and a small reserve outflow, would preserve the authority’s incentives to maintain the peg. By identifying the possibility for contrasting market behaviour in the same exogenous environment, the second generation approach delivers a stylised account of events like the ERM crisis. Nothing in the model indicates, however, which of the multiple equilibria should materialise at a given point in time: the prevailing one is chosen by a sunspot.

Using analytical tools of the global games literature, Morris and Shin argue that multiple equilibria within the second-generation approach are a consequence of the overly simplistic assumption that private agents hold all their knowledge of the underlying fundamentals in common. The assumption is not innocuous because the strategic complementarity in foreign exchange markets is based on the dependence of the authority’s

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1 The term “global games” stands for a strategic incomplete-information environment and was coined by the seminal paper Carlsson and van Damme (1993). An overview of the evolution of the global games literature is provided in Morris and Shin (2000).

2 Obstfeld (1994) and Bensaid and Jeanne (1997), for example, adopt that approach.
incentives on speculators’ aggregate behaviour. Thus, an attack would materialize only if a sufficiently high fraction of the speculators believe with a sufficiently high probability that a sufficiently high proportion of their peers believe with a sufficiently high probability, and so on, that the fundamentals and the magnitude of the attack would lead to a devaluation. Symmetrically for the absence of an attack. In other words, a particular aggregate action is an equilibrium one only if there is a sufficient degree of common knowledge that the action is justified by the fundamentals. If private agents are realistically allowed to have their own “window on to the world”, i.e. to receive private signals about the fundamentals, there would be a limit to the attainable degree of common knowledge. When that degree is insufficient to rationally justify more than one actions, there is failure of common knowledge and equilibrium uniqueness ensues.3

Failure of common knowledge is a necessary condition for equilibrium uniqueness in the MS setup: a condition that holds only if private information is sufficiently more precise than public information.4 The higher is the relative precision of private information, the bigger is its importance in agents’ beliefs. When actions are rationalized by beliefs with a big idiosyncratic component, there is strategic uncertainty (ie uncertainty on the part of each individual concerning the action of others) which impairs agents’ coordination capacity and eliminates the role of sunspots.

The ultimate objective of the MS setup is to account for violent speculative attacks within equilibrium uniqueness. This type of attacks occurs when a small change in the state of the economy induces an overwhelming majority of private traders to bet against a currency. Rationality in turn implies that such a phenomenon can be observed only when agents hold accurate, and thus similar, information about economic fundamentals. In the case of well informed agents, however, the requirement on the relative precision of private and public information, which is to deliver failure of common knowledge, is rather strong. For sufficiently accurate beliefs, equilibrium uniqueness prevails in the MS setup only if the superiority of private signals induces agents to virtually ignore their public signals. The condition is satisfied due to an assumption that market prices are constant and that the two types of information have independent sources.

The equilibrium uniqueness result of the MS setup hinges on the just-mentioned assumption whose realism is, however, questionable. Foreign exchange transactions

3 “Failure of common knowledge” has been used in the global games literature with a slightly different connotation. In this paper, the phrase describes a situation in which uncertainty prevents self-fulfilling beliefs from generating multiple equilibria. Refer to Morris and Shin (1999, 2000) for a precise definition and rigorous analysis of (degrees of) common knowledge.

4 An outburst of currency crisis research built on and expanded the original MS framework but preserved its key features: Morris and Shin (1999), Corsetti et al (2000) and Goldstein (2000) are representative examples.
produce *publicly observable* prices that respond to and, thus, at least partially, aggregate market participants’ private beliefs. Rose and Svensson (1994), Eichengreen et al (1996), for example, view the evolution of interest differentials during the ERM crisis as expressing foreign exchange traders’ perception of the sustainability of the European currency zone. An increase in private signals’ precision would thus translate into an improved precision of public signals in the form of prices. It should then be expected that, if the former signals enrich non-trivially individual information sets, so would the latter ones.5

Motivated by the above observations, I develop an Endogenous Public Signal (EPS) model as a modification of the framework of Morris and Shin (1998): a modification in which market prices play an informational role.6 In taking opposite sides of foreign exchange transactions, agents settle at an interest rate at which there is no incentive to recontract even when new information, revealed in equilibrium, is taken into account. Agents in the EPS model communicate their beliefs by transacting in the market and simultaneously learn about others’ beliefs. As the private signals’ precision improves, so does the precision of the signal sent by the publicly observable interest rate: virtually perfect common knowledge of the fundamentals is established when the informational noise is infinitesimal. In the MS setup, this would violate a necessary condition for unique equilibrium behaviour of well informed agents.

In the EPS model, however, the endogenous interest rate governs the strategic interaction among agents and makes it possible to attain a unique equilibrium under a weaker restriction on the level of common knowledge of the fundamentals. At any level of the fundamentals, the central authority’s incentive to abandon the exchange rate regime increases with the domestic interest rate and the outflow of official reserves. For a given flow of reserves, an upward revision of private devaluation expectations would translate into a higher domestic interest rate and might consequently be self-fulfilling. A higher domestic interest rate might, however, coexist with an unchanged aggregate level of devaluation expectations: in such a case, the domestic-currency assets are more attractive and private agents’ optimal investment positions generate a larger inflow of official reserves. Strategic uncertainty is thus established because different aggregate behaviour of the private sector, with different implications for the exchange

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5Grossman (1989) is the classic reference on the role of prices in a heterogeneous-agent rational-expectations environment. Rey (2000) and Atkeson (2000) point out the issue when commenting on a paper within the MS approach.

6Dasgupta (2002) endogenises the source of common information in a conceptually different fashion. He does not consider the capacity of prices to instantaneously aggregate agents’ idiosyncratic information. Instead, he adopts a dynamic setup of sequential learning in which past aggregate actions of the private sector provide a public signal about the current state of the economy.
rate regime, can be expressed by the same interest rate. The uncertainty is non-trivial in
the EPS model as long as there is some, even infinitesimal, noise in the private signals
that makes each agent unsure about others’ devaluation expectations and about the
flow of reserves. Consequently, even under virtually perfect common knowledge of the
fundamentals, there might be failure of common knowledge of the impact of private
sector’s overall behaviour on the central authority’s incentives. Ultimately, this would
lead to only one rationally justifiable private-sector action in each state of the world.
Produced by the endogeneity of the interest rate, the just-described strategic uncertainty
is absent in the MS setup.

The conditions for equilibrium uniqueness in the EPS model are in terms of parame-
ters that capture characteristics of the economy’s institutional infrastructure and finan-
cial markets. A non-trivial region of the parameter space delivers a unique equilibrium,
in which the domestic interest rate is a strongly non-linear function of the economic fun-
damentals. It is thus possible to determine unambiguously how the likelihood of sudden
and violent currency attacks depends, for example, on changes in transaction costs or
on the authority’s commitment to the exchange rate regime. It is also seen how the
incidence of speculative attacks can be influenced by a large market participant who is
successful in manipulating the informational content of prices. Finally, by demonstrat-
ing that seemingly unwarranted interest rate spikes are not necessarily a symptom of
multiple equilibria, the EPS model provides a critique of structural empirical analyses
of currency crises. Such analyses embrace the second-generation theoretical approach
and attribute to sunspots any drastic shift in interest rates coexisting with unchanged
behaviour of the fundamentals.\textsuperscript{7}

The contribution of the EPS model is best understood if compared directly to the
MS framework. Hence, in Section 2, I develop a setup which incorporates both models
as special cases. The following two sections consider the models sequentially while
assuming that agents’ private signals are extremely precise. Section 3 states and briefly
discusses the main results of the MS setup. Section 4 derives the EPS model equilibrium,
provides the intuition behind the conditions for equilibrium uniqueness and conducts
comparative statics. In Section 5, I conclude by summarising the paper’s message and
then pointing to a direction for future research suggested by the analysis.

\textsuperscript{7}Such analysis is conducted in Jeanne (1997) and Jeanne and Masson (2000).
2 The Model

The model relates to a small open economy which maintains an exchange rate peg and is referred to as the domestic economy. There are only two currencies: domestic and foreign. The foreign price level and interest rate are fixed and purchasing power parity holds.

The economy evolves over two periods. In the first period, the exchange rate, the domestic currency’s value of the foreign currency, is fixed and normalised to unity. If the peg is abandoned, the currency is devalued in the second period to a new parity $E > 1$. $E$ is a constant that is publicly known at the beginning of the first period.\(^8\)

I now describe the two types of players: the private agents and the domestic central authority.

2.1 Private Agents

The private agents form a continuum and are uniformly distributed on the real unit interval. They act on the foreign exchange market where two assets are traded in the first period: one-period default-free discount bonds denominated in the foreign and domestic currency. The associated interest rates are denoted respectively by $i^*$ and $i$ and the relative price of the two assets is normalised to unity. In order to facilitate the flow-of-funds analysis, it is assumed that the assets are traded against their currency of denomination and in the region of the world in which that currency is issued.

Agents enter the first period with no liquid wealth and have to borrow if they want to invest. The value of the funds an agent can borrow cannot exceed unity (in either currency).\(^9\) The value of the aggregate investment in the domestic (respectively, foreign) asset can thus range from $-1$ to $1$: at the former extreme, all agents borrow domestically (respectively, abroad). Since borrowing is conducted at nominally riskless interest rates, I assume that each agent possesses wealth that is liquid only in the second period and is sufficient to cover any repayment obligation.\(^10\)

Agents trade bonds in the first period in order to maximise the expected real value

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\(^8\)The assumptions that (i) a revaluation is not possible and (ii) the (shadow) devaluation rate is a fixed commonly known number are made for expositional simplicity. Relaxing these assumptions would not affect the principal conclusions of the paper.

\(^9\)The analysis is not affected by the exact value of the upper limit on borrowing as long as this value is finite.

\(^10\)Denote this wealth’s second-period real value by $W$. Straightforward algebra shows that, if $W \geq (E - 1)(1 + i^*)$, each agent is guaranteed to have enough funds to honour his/her period-two obligation. Note that, for a fixed $W$, an agent’s repayment capability changes with the currency in which his/her bond is denominated. Modifying agents’ borrowing constraint in order to accommodate this consideration would unnecessarily burden the exposition without affecting the model’s main thrust.
of their net payoff in period two. Agents’ decisions are based on their rational beliefs about the state of the economy. The state is realised in the first period and is fully described by an exogenous “fundamental” random variable, \( \Theta \), and a stochastic shock to the supply of the domestic currency bond, \( U \). For simplicity, \( \Theta \) and \( U \) are assumed to be mutually independent. In the currency crisis literature, \( \Theta \) typically represents the domestic economy’s relative productivity or employment rate. In turn, \( U \) is to be thought of as driven by (not modelled) domestic residents who invest or save at home.

Part of agents’ beliefs are formed by interpreting exogenous signals about the state and possess a publicly observed (ie common) and a private (ie idiosyncratic) component. \( \Theta \) is assumed to be a priori uniformly distributed on the real line. Before agents trade, their perceptions of \( \Theta \) are sharpened by a public signal, denoted by \( y \), which is the realisation of:

\[
Y = \Theta + \sqrt{t} Z^y
\]

where \( Z^y \) is a standard normal variable that is independent of \( \Theta \) and \( U \) and \( t > 0 \). The exogenous public component of beliefs is completed by the prior:

\[
U \sim N (\mu_u, \sigma^2_u)
\]

Unless explicitly stated otherwise, \( \mu_u = 0 \) for the rest of the analysis. The assumption is inconsequential in the MS setup. Allowing for \( \mu_u \neq 0 \) in the paper’s EPS model leads to purely algebraic complications without altering agents’ strategic interaction and, thus, the main message of the paper.

The idiosyncratic component of agents’ beliefs leads to heterogeneous perceptions of the likelihood of a devaluation. That component incorporates each agent \( j \)’s private signal about \( \theta \), which is denoted by \( x_j \) and is a realisation of:

\[
X_j = \Theta + \sqrt{\varepsilon} Z_j
\]

where \( \varepsilon > 0 \) and \( Z_j \) is a standard normal variable that is independent of \( \Theta \), \( U \), \( Z^y \) and \( Z_k \) for \( k \neq j \). An agent \( j \) is identified by his/her private signal and is henceforth referred to as an \( x_j \)-agent.

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11 Unless explicitly stated otherwise, random variables are denoted by upper case letters; the corresponding lower case letters are used to denote the variables’ specific realisations.

12 The assumption is adopted for algebraic tractability and is inconsequential when: (i) the signals about \( \theta \) are sufficiently more precise than \( \Theta \)’s prior and (ii) the latter prior has a continuous density.

13 A working version of the paper, available from the author upon request, analyses the EPS model’s equilibrium for a general \( \mu_u \). A conclusion of that analysis is used in Figure 1 below in order to make a quantitative illustration.
Given \( i, i^*, \) and \( E, \) there exists a level of devaluation probability, \( \Pr^d, \) at which the return on the foreign currency asset is equal to the expected real return on the domestic asset:\(^4\)

\[
\Pr^d = \frac{1 - \frac{1+i^*}{1+E}}{1 - \frac{1+i}{1+E}} \equiv \iota \in [0,1]
\]

Agents who perceive a devaluation probability equal to \( \Pr^d \) play a key role in equilibrium and I denote their private signal by \( x^p. \) Given that \( \iota \) depends only on the relative values of \( i \) and \( i^*, \) I henceforth refer to it as the interest differential.\(^15\) \( i \) and \( i^* \) affect the analysis only through \( \iota. \)

### 2.2 The Central Authority

In the first period, the central authority observes noiselessly the state of the economy,\(^16\) holds foreign reserves and acts as a counterparty to all foreign exchange transactions. By intervening (or not) on the foreign exchange market, the authority can influence the cost of borrowing in the domestic currency. The domestic interest rate policy in period one is what distinguishes the MS and EPS frameworks and is described at the beginning of Sections 3 and 4.

In the second period, the authority decides on the exchange rate, at which it again stands ready to trade one currency for the other. The decision is assumed to depend on developments in the first period. The authority’s temptation to abandon the peg increases with the domestic interest rate, a rise in which would typically put pressure on domestic banks, and with the outflow of reserves, which might influence the authority’s capacity to service debt obligations. Finally, the authority’s tolerance to pressure on the peg increases with \( \theta. \)

Denoting the outflow of reserves by \( R, \) the period two decision of the central authority is formalised by assuming that it devalues if and only if:

\[
\tau \iota + R > a(\theta)
\]

\(^{14}\)The equation in (4) is obtained after rearranging the uncovered interest parity condition: \( 1+i^* = (1+i) \left( (1-\Pr^d) \ast (1+\Pr^d) \right) \)

\(^{15}\)\( \iota = 0 \) when the domestic interest rate is at its lowest no-arbitrage value: \( i = i^*. \) In turn, \( \iota = 1 \) when \( i \) is at its highest no-arbitrage value: \( i = (1+i^*) E - 1. \)

\(^{16}\)The authority’s beliefs constitute all the information that is relevant for private agents’ actions. The assumption that the authority has perfect knowledge about the state is thus made without loss of generality.

\(^{17}\)The mechanism via which these three factors influence the authority is not explicitly modelled. The second-generation literature discusses at length the relevance of fundamentals, interest rates and reserve flows for currency crises: see for example Obstfeld (1994), Obstfeld and Rogoff (1995) and Bensaid and Jeanne (1997).
\( \tau > 0 \) captures the relative importance of the interest differential and reserve outflows; \( a(\cdot) \) is a continuous function with the following properties:

\[
0 < b_1 \leq \frac{da(\theta)}{d\theta} \leq b_2 < \infty
\]  

(6)

where \( b_1 \) are \( b_2 \) are constants.\(^{18}\) The left-hand side (LHS) of inequality (5) can be interpreted as an indicator of foreign exchange market conditions. The right-hand side (RHS) sets a threshold value of that indicator, beyond which the currency is devalued.

2.3 Sequence of Events, a Recapitulation

In the first period, the exchange rate is fixed. The state of the economy is realised and observed only by the central authority. The atomistic agents hold common prior beliefs about the state random variable and receive private signals about its particular realisation. Then agents borrow and invest and, in the process, settle at an interest differential, whose value depends on the authority’s intervention in the foreign exchange market.

In the second period, after observing the aggregate investment position of the private sector, the central authority decides whether to preserve or abandon the peg. Finally, private agents make and/or receive payments according to their period one contracts.

3 The MS Setup

Suppose that the interest rate policy rule of the authority requires that it intervene on the foreign exchange market in order to keep the interest differential fixed. Then, the model outlined above delivers the thrust of the MS setup. In this section, I assume that \( \iota \) is a constant on the unit interval, state the assumption’s equilibrium implications and focus on the intuition behind them. The interested reader is referred to Morris and Shin (1998) or (1999) for all the underlying proofs.

Recall equation (4) for the definition of an \( x^p \)-agent: in the only strategy that is sustainable in equilibrium, this agent is pivotal. All agents with private signals smaller than \( x^p \) bet against the domestic currency: they issue the maximum possible amount of domestic debt, convert the obtained currency at the domestic authority’s window and

\(^{18}\) \( b_1 \) assures that changes in the fundamentals affect non-trivially the central authority’s incentives. In turn, \( b_2 \) rules out a drastic revision of the authority’s incentives that is due solely to the fundamentals: if there is a sudden and violent currency crisis, it is due to the way market participants interact.
invest the proceeds in the foreign asset. According to equation (3), this generates an outflow of reserves equal to $\Phi \left( \frac{x^p - \theta}{\sqrt{\varepsilon}} \right)$. Symmetrically, agents with signals bigger than $x^p$ generate an inflow of reserves equal to $1 - \Phi \left( \frac{x^p - \theta}{\sqrt{\varepsilon}} \right)$. The net outflow of reserves is then:

$$ R = 2 \Phi \left( \frac{x^p - \theta}{\sqrt{\varepsilon}} \right) - 1 \in [-1, 1] $$

(7)

Given a particular $\iota$ and a value of the pivotal agent’s private signal, $x^p$, inequality (5) and equation (7) identify a unique critical value of the fundamentals, $\theta^{cr}$: the peg is abandoned if and only if $\theta < \theta^{cr}$, where:

$$ a (\theta^{cr}) = \tau \iota + 2 \Phi \left( \frac{x^p - \theta^{cr}}{\sqrt{\varepsilon}} \right) - 1 $$

(8)

Using equations (1) and (3), equation (4) implies that:

$$ \Phi \left( \frac{\theta^{cr} - \left( \frac{\iota}{1+\varepsilon} x^p + \frac{\varepsilon}{1+\varepsilon} y \right)}{\sqrt{\frac{\varepsilon}{1+\varepsilon}}} \right) = \iota $$

(9)

where the LHS is the devaluation probability, as perceived by an $x^p$-agent. From this agent’s point of view, $\left( \frac{\iota}{1+\varepsilon} x^p + \frac{\varepsilon}{1+\varepsilon} y \right)$ and $\frac{\varepsilon}{1+\varepsilon}$ are respectively the posterior mean and variance of $\Theta$.

Recall that $R \in [-1, 1]$ and define

$$ \underline{\theta} \equiv a^{-1} (-1 + \tau \iota) $$

$$ \bar{\theta} \equiv a^{-1} (1 + \tau \iota) $$

(10)

Inequality (5) implies that, if $\theta \in (-\infty, \underline{\theta})$ or $\theta \in (\bar{\theta}, \infty)$, the peg is respectively abandoned or preserved for any aggregate investment position of the private agents. In the global games literature, the latter two regions are referred to as the dominance regions on the support of the fundamentals.

For a given public signal, $y$, and a given interest differential, $\iota$, an equilibrium is defined by a pair $\{x^p, \theta^{cr}\}$ that solves equations (8) and (9) simultaneously. By the definition of the dominance regions, an equilibrium $\theta^{cr}$ belongs to $[\underline{\theta}, \bar{\theta}]$. Observe that $U$, the stochastic shock to the supply of the domestic asset, does not enter the determination of equilibrium: in order to keep $\iota$ fixed, the central authority’s intervention insulates the private sector’s demand for the domestic asset, and thus the outflow of reserves, from the supply shock.

$^{19}$Throughout the paper, $\Phi (\phi)$ stands for the standard normal CDF (PDF).
As discussed in the introduction, the model developed in Section 2 could account for sudden and violent speculative attacks only if private agents’ information about the fundamentals is sufficiently precise. For this reason, papers adopting the MS setup often focus on the limit in which the noise in private signals is extremely small: $\varepsilon \to 0$.\(^{20}\) In that limit, three relative configurations of $\varepsilon$ and the precision of public information, $t$, capture the setup’s main message.

**Case 1. No uncertainty:** $\varepsilon = 0$ and/or $t = 0$. The resulting equilibrium is standard for the second-generation approach to currency crises.

For $\theta < \bar{\theta}$ (recall (10)), all agents borrow domestically and invest abroad in order to shield themselves from the peg’s inevitable collapse. For $\theta > \bar{\theta}$, agents’ only rational decision is to invest domestically. Strategic complementarity sets in if $\theta \in (\bar{\theta}, \bar{\theta})$. If all agents decide to invest abroad, the resulting attack on the peg generates a reserve outflow that is too high for the authority to bear and the peg collapses. Conversely, if agents decide to invest domestically, the peg survives and justifies their decision. For intermediate values of $\theta$, agents’ actions are thus independent of its particular realisation and are best thought of as coordinated by a sunspot.

**Case 2. Both signals are of an extremely high and comparable precision:** consider the limit $\varepsilon \to 0$ and assume that $\lim_{\varepsilon \to 0} \frac{t}{\varepsilon} = \kappa > 0$, where $\kappa$ is an arbitrary constant. In this case, both signals influence non-trivially the formation of agents’ beliefs and the set of equilibria converges to the one in Case 1.

At any level of its precision, the public signal plays a dual role: on the one hand, it leads to better knowledge of the fundamentals; on the other hand, it anchors the heterogeneous beliefs and establishes public awareness that they are similar. In the limit $\varepsilon \to 0$, there is common knowledge of the fundamentals: each agent knows that all agents know, and so on, that $\theta$ belongs to any given interval containing the public signal.\(^{21}\) Thus, when $y \in (\bar{\theta}, \bar{\theta})$, there is common knowledge that the two equilibria, emerging in Case 1 when $\theta \in (\bar{\theta}, \bar{\theta})$, are rationally justifiable: a sunspot can lead to either one. For $y > \bar{\theta}$ or $y < \bar{\theta}$, the corresponding unique equilibrium is derived with an analogous argument.

**Case 3. The high precision of private signals renders public signals redundant:** consider the limit $\varepsilon \to 0$ and assume that $\lim_{\varepsilon \to 0} \frac{\sqrt{\varepsilon}}{t} = 0$. In this case, beliefs are determined predominantly by their idiosyncratic component even if the public signal is extremely precise. Equilibrium uniqueness prevails in all states of the world: a

\(^{20}\)Since $\varepsilon$ cannot be negative, “$\varepsilon \to 0^+$” is replaced by “$\varepsilon \to 0$” in order to reduce the notational clutter. Since the model’s equilibrium might be discontinuous at $\varepsilon = 0$ (compare Case 1 to Case 3 in the present section), “in the limit $\varepsilon \to 0$” is most usefully thought of as “for a sufficiently small but strictly positive $\varepsilon$”.

\(^{21}\)Strictly speaking, the public signal is to belong to the interior of that interval.
single equilibrium pair \( \{x^p, \theta^c\} \), where \( \theta^c \in (\underline{\theta}, \bar{\theta}) \), arises for any \( y \). An infinitesimal worsening of the state, which pushes \( \theta \) below \( \theta^c \), induces a massive attack: an outflow of reserves that topples the peg.

Let us suppose that \( \theta \in (\underline{\theta}, \bar{\theta}) \). For \( \varepsilon \to 0 \), equation (3) implies that virtually all agents believe that \( \theta < \bar{\theta} \). Furthermore, if all agents holding such beliefs were to attack, the attack would be justified ex post by a collapse of the peg. However, each agent needs to form a belief not only about \( \theta \) but also about other agents’ beliefs about \( \theta \). For the assumed relative precision of public and private information, there are not enough agents who believe that there are enough agents who believe (and so on) that \( \theta < \bar{\theta} \). Thus, there is failure of common knowledge of the fundamentals, which translates into failure of common knowledge that an attack would succeed. As a result, an attack cannot materialise: only investment in the domestic asset is rationally justifiable. Symmetrically, for \( \theta \in (\underline{\theta}, \theta^c) \), only an investment in the foreign asset is justifiable.

Contrasting Case 3 against the other two cases underscores the gist of Morris and Shin’s contribution to the currency crisis literature. A massive currency attack that topples the peg but is decoupled from the macroeconomic environment could emerge under equilibrium uniqueness if idiosyncratic signals dominate private agents’ beliefs. The limiting condition in Case 3 formalises the meaning of dominance.

### 3.1 Aggregate Uncertainty

In the MS setup, failure of common knowledge of the fundamentals would be driven trivially by sufficient noise in the private signals that is the same across agents.\(^{22}\) Such noise creates aggregate uncertainty (uncertainty that remains even if all agents share their information sets) and obscures the ultimate impact of any private sector strategy. This could prevent agents from rationally justifying different actions for the same perception of the fundamentals and could thus lead to equilibrium uniqueness. An analogous result is obtained in the homogeneous-agent second-generation environment.

In contrast to strategic uncertainty, which arises under informational heterogeneity and is the driving force of equilibrium uniqueness in Case 3 of Section 3, aggregate uncertainty simply reduces the quality of private agents’ information and cannot, by itself, decouple a speculative attack’s intensity from the state of the economy.\(^{23}\) As argued in the introduction, such a decoupling requires that the rational agents are well

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\(^{22}\)Such noise can be modelled by assuming that \( Z_j \) from equation (3) is distributed \( N(\zeta, 1) \), where \( \zeta \) is a random variable that is the same across agents.

\(^{23}\)See Jeanne (1997) for an illustrative example.
informed. Consequently, if the objective is to capture seemingly unwarranted revisions in
the outlook of a currency regime and if the analysis is of a qualitative nature, aggregate
uncertainty can be assumed away without loss of generality. This was done in the
current Section 3 and in all the currency crisis analyses within the MS setup that I am
aware of. Aggregate uncertainty is also absent in the EPS model to which I now turn.

4 The EPS Model

In this section, I study an Endogenous Public Signal (EPS) model that represents the
contribution of the paper. The framework is the same as the one described in Section
2 with the qualification that, in the first period, the central authority’s interest rate
policy rule stipulates no intervention on the foreign exchange market. Thus, market
clearing implies that the interest differential would have to adjust in order to induce an
aggregate investment position of the private sector that matches the stochastic shock on
the domestic asset market, $U$. The following equation specifies how that shock translates
into the exogenous component of the domestic asset supply, $S$:

$$S = 2\Phi(U) - 1$$

(11)

The support of $S$ is thus the interval $[-1, 1]$, which, as noted in Section 2.1, also defines
the range of the private sector demand for the domestic asset. This leads to market
clearing in any equilibrium of the EPS model. Further, as illustrated in Section 4.1,
equation (11) simplifies the exposition tremendously and allows for algebraic tractability.

In the EPS model, the domestic asset supply is vertical and its position is fixed by
the realisation of $U$; in the MS setup, the supply is horizontal and its position is fixed
by the target value of the interest differential. A setup between these two extremes
would not have a closed form solution but would produce an interest differential that,
by reacting to supply and demand on the domestic asset market, emits an endogenous
public signal similar to the one identified in Section 4.1. The logic of Section 4.3 would
then apply and would deliver the main message of the EPS model.24

In order to allow for a direct parallel between the MS and EPS models, I assume
that there is no exogenous public signal in the latter one: ie it is analysed after setting
$t = \infty$ in equation (1). The assumption allows for underscoring the implications of
an environment in which the relative precision of private and public information is an

---

24 Even though the paper’s main message does not hinge on the extreme assumption underlying
equation (11), useful insights would be provided by an analysis of how the implications of the model
from Section 2 evolve with the sensitivity of the domestic asset supply to the interest differential. Such
an analysis is beyond the scope of this paper.
equilibrium outcome.

4.1 Equilibrium in the EPS Model

Agents exhaust their borrowing constraints and, if they issue a bond in one currency, they do so in order to invest the proceeds in a bond denominated in the other currency. An investment strategy can be viewed as setting an agent’s position in the foreign asset as a function of that agent’s private signal. As will become clear below, the endogenous public signal is a parameter in that function.

I assume that the space of equilibrium-implementable strategies contains only monotone functions of the private signal. The adopted restriction of the strategy space is intuitively reasonable and is consistent with the MS setup. Further, the resulting setting is general enough for the paper’s main objective: to provide insights as to why the strategic interaction of informationally heterogeneous agents could lead to equilibrium uniqueness when publicly observable prices aggregate extremely accurate private information about the fundamentals. I prove in Appendix 1 that, within the assumed strategy space and under an additional minor technical assumption, an equilibrium-implementable strategy in the EPS framework is necessarily of the type that led to equation (7) in Section 3: a decreasing step function of the private signal. A comparison between the EPS and the MS setups is thus straightforward.

For a given interest differential, the threshold value of the private signal is equal to \( x^p \), the private signal of an agent who perceives the devaluation probability given by equation (4). Agents with public signals higher (lower) than \( x^p \) invest in the domestic (foreign) asset. Thus, domestic-asset market clearing in the EPS setup implies:

\[
2\Phi\left( \frac{\theta - x^p}{\sqrt{\varepsilon}} \right) - 1 = s(u)
\]  

(12)

where, on the basis of equation (7), the LHS represents the net demand for the domestic asset. The RHS stands for the supply of that asset, expressed as a function of the shock \( u \). Equations (11) and (12) then imply that the value of the pivotal agent’s private signal is also equal to the realisation of the random variable \( X^p \):

\[
X^p \equiv \Theta - \sqrt{\varepsilon} U
\]  

(13)

I derive below that the equilibrium interest differential is associated with a single value of \( x^p \). In conjunction with equation (13), this implies that the market price sends an endogenous public signal about the fundamentals.
In equilibrium, an $x_j$-agent’s information set is fully defined by equations (2), (3), (13) and knowledge of $x_j$ and $x^p$. With such an information set and using the properties of the binormal distribution, an $x_j$-agent deduces the following posteriors of the two state variables:

\[
\Theta \sim N \left( x_j \frac{\sigma_u^2}{1 + \sigma_u^2} + x^p \frac{1}{1 + \sigma_u^2}, \epsilon \frac{\sigma_u^2}{1 + \sigma_u^2} \right)
\]

\[
U \sim N \left( \frac{x_j - x^p}{\sqrt{\epsilon}} \frac{\sigma_u^2}{1 + \sigma_u^2}, \frac{\sigma_u^2}{1 + \sigma_u^2} \right)
\]  

(14)

For any $\{\theta,u\}$, an equilibrium in the EPS setup is defined by the vector \(\{x^p, \iota, \theta^{cr}\}\) that solves simultaneously equation (12) and the following two equations:

\[
a(\theta^{cr}) = \tau \iota + 2\Phi \left( \frac{x^p - \theta^{cr}}{\sqrt{\epsilon}} \right) - 1
\]  

(15)

\[
\Phi \left( \frac{\theta^{cr} - x^p}{\sqrt{\epsilon}} \frac{\sigma_u^2}{1 + \sigma_u^2} \right) = \iota
\]  

(16)

Equation (15) is exactly the same as equation (8); equation (16) is the EPS analog of equation (9) and incorporates the first line in (14) after setting $x_j = x^p$.

An equilibrium is a solution of a fixed-point problem, solved by each atomistic agent. Tautologically, the interest differential, at which agents trade, is publicly known to be supported in equilibrium. For such an $\iota$, an agent conjectures an equilibrium investment strategy, ie a value for the pivotal agent’s private signal: say, $x^p_1$. Given $\iota$ and $x^p_1$, equation (15) determines the critical value of the fundamentals, $\theta^{cr}(x^p_1, \iota)$, and thus splits the state space into two regions: the peg survives if and only if $\theta > \theta^{cr}(x^p_1, \iota)$. Then, using his/her private signal and the value of $x^p_1$, the agent deduces a devaluation probability. If the latter probability is greater than $\iota$, the agent issues a domestic currency bond and invests in the foreign currency. The opposite investment position is optimal if the perceived devaluation probability is lower than $\iota$. Following the same logic, the agent deduces the investment decision of any other agent, receiving any possible private signal. Since the perceived devaluation probability decreases in the private signal, the private sector investment position is summarised by the value of a pivotal agent’s private signal, $x^p_2$, which solves equation (16) for $\theta^{cr}(x^p_1, \iota)$. For an equilibrium, $x^p_1 = x^p_2 = x^p$. Finally, clearing on the foreign exchange market requires that the prevailing interest differential is such that the resulting $x^p$ satisfies equation (12) for the given $\theta$ and $s$ (equivalently, $u$).
Note that $x^p$ is determined uniquely by the state, via equation (12) alone. Further, equation (15) defines $\theta^c$ as an increasing function of $\iota$. Given a pair $\{\theta, u\}$ and the implied $x^p$, equilibrium multiplicity prevails if at least two different pairs $\{\theta^c, \iota\}$ solve equations (15) and (16). When the latter is true, the prevailing equilibrium value of $\iota$ is assumed to be picked by a sunspot variable that is independent of all the random variables in the system. It is straightforward to demonstrate that each $\iota$ is associated with a single $x^p$ even under multiplicity of equilibria.\footnote{This is done by following Morris and Shin (1999) in its proof of equilibrium uniqueness under a step-function investment strategy.}

The assumption of no intervention by the central authority on the foreign exchange market implies that there are two state variables in the EPS setup: $\Theta$ and $U$. In contrast, there is effectively only one in the MS setup: $\Theta$. The different dimension of their respective state spaces does not obscure, however, the comparison of the two models. First, in both models, the optimal behaviour of private agents is driven exclusively by their heterogeneous beliefs about the fundamentals, $\theta$, and about the aggregate outcome of others’ investment decisions: recall equations (8)-(9) and (15)-(16). Furthermore, the latter outcome is fully defined by the observable $x^p$ and the value of $\theta$. The role of $U$ in the EPS setup is thus to prevent the equilibrium from establishing informational homogeneity by fully revealing $\theta$: recall equation (13). Second, just as in the MS setup, there is no aggregate uncertainty in the EPS setup: knowledge of all the private signals reveals $\theta$ and, since $x^p$ is publicly known, knowledge of $\theta$ reveals $u$ via equation (12). Third, a second state variable in the authority’s reaction function would simply produce aggregate uncertainty in the MS setup.\footnote{The statement assumes that there would be no private signals about a second state variable in the MS setup. Otherwise, the setup would be isomorphic to the one discussed in Section 3.} As discussed in Section 3.1, such uncertainty cannot influence the qualitative message of the setup without hindering its capacity to account for drastic speculative attacks under equilibrium uniqueness.

Further similarities between the MS and EPS models can be found if one focuses on the state variable they share: the fundamentals, $\Theta$. Noise in the private signals about the fundamentals generates strategic uncertainty and, abstracting from aggregate uncertainty, is a necessary condition for equilibrium uniqueness in both the MS and EPS models. In the context of the former model, this was demonstrated by Case 1 of Section 3. To demonstrate the point in the context of the EPS model, I consider a homogeneous-agent version of it in Appendix 2. The key assumption in that version is that the beliefs of an arbitrary $x_j$-agent from the original model, as expressed in (14), are held in common by all, now identical, private agents. In order to parallel Case 1 of Section 3, I also impose exact common knowledge of the fundamentals by setting $\varepsilon = 0$. (Since
agents are identical, this is done without loss of generality when the limit of interest is $\varepsilon \to 0$.) The resulting setup lacks strategic uncertainty because all investment decisions are the same, and thus publicly known. The perfect synchronization of agents’ actions restores sunspots’ role as coordination devices and equilibrium multiplicity ensues.  

**4.2 Equilibrium Uniqueness in the EPS Model**

Paralleling Section 2, I analyse the EPS model under the assumption that private signals are extremely precise, ie that $\varepsilon$ is close to but always bigger than zero. Note that, according to equation (13), the variance of the noise in the public signal about $\theta$ is equal to $\varepsilon \sigma_u^2$. Thus, recalling equation (3), the relative precision of private and public information is derived to be equal to $\sigma_u^2$, the volatility of the domestic asset’s supply shock.

Since $\sigma_u^2$ does not vary with $\varepsilon$, the EPS equilibrium analysis is conducted in an informational setting that is identical to the one of Case 2 in Section 3. In the latter case, an increase in the exogenously set precision of private and public signals, attained via a decrease in $\varepsilon$, is the sole factor leading to equilibrium multiplicity. The number of possible equilibria is independent, for example, of the value of $\tau$: the parameter influencing the maximum level of official reserve losses that the authority can incur without devaluing. As noted in Section 3, the multiplicity result is due to the fact that in Case 2, and in contrast to Case 3 in the same section, there is common knowledge of the fundamentals.

When the noise in private signals is sufficiently small, virtually perfect common knowledge of the fundamentals is established within the EPS model as well. This fact notwithstanding, the following proposition states that parameters in the latter model, which are unrelated to the exogenous signals’ properties but capture characteristics of the economy’s institutional infrastructure and financial markets, are crucial for the existence of equilibrium multiplicity.

**Proposition 1** There exists $\varepsilon_0 > 0$ such that, for any $\varepsilon \in (0, \varepsilon_0)$ and any $x^p$, there is exactly one solution to equations (15) and (16) in terms of $\theta^p$ and $\nu$ if and only if $\{\tau, \sigma_u^2\} \in (0, 2) \times \left[\frac{\tau^2}{4-\tau^2}, \infty\right]$.

Thus, equilibrium uniqueness emerges if and only if (i) the authority’s exchange rate decision is not too dependent on the interest differential and (ii) the supply of the

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27 The result is derived in Appendix 2. Numerical simulations of the homogeneous-agent version of the EPS model, which illustrate equilibrium multiplicity, are available from the author upon request.
domestic asset is sufficiently volatile. The intuition behind the proposition is provided in Section 4.3, whereas the proof is relegated to Appendix 2.

Figure 1: Equilibrium interest differential as a function of the public signal

In the EPS context, an abrupt speculative attack is characterised by a drastic increase in the interest differential that is triggered by a small change in the economic environment. The model’s capacity to account for such an attack within equilibrium uniqueness is demonstrated in Figure 1, which plots the equilibrium interest differential as a function of the public signal.\textsuperscript{28} The result is formalised in Section 4.4, which provides further discussion of the EPS equilibrium implications.

At this stage, it is important to recall equation (13), which implies that:

\[ \text{plim}_{\varepsilon \to 0} (X^p - \Theta) = 0 \]  

(17)

In other words, when private signals are extremely precise, the graph in Figure 1 may be thought of as depicting the equilibrium interest differential as a function of the fundamentals.

4.3 Equilibrium Uniqueness in the EPS Model, an Interpretation

Proposition 1 states that the EPS model, in contrast to Case 2 of Section 3, does not produce equilibrium multiplicity solely on the basis of an increase in private signals’

\textsuperscript{28} In order to emphasise the interest differential’s abrupt rise from low levels, I have used $\mu_u < 0$ in Figure 1. More precisely, the model is parameterised as follows: $a(\theta) \equiv \theta$, $\tau = 1$, $\sigma_u^2 = 0.51$, $\mu_u = -0.68$, $\varepsilon = 10^{-10}$. 
precision (ie a decrease in $\varepsilon$) that leads to virtually perfect common knowledge of the fundamentals $\theta$. The reason behind this result is seen if one first considers a modification of the EPS model which helps dissect the role of strategic uncertainty. The modification is obtained by assuming that the outflow of reserves, $R$, does not enter the authority’s reaction function (5). The resulting equilibrium condition can then be written as:

$$a(\theta^{cr}) = \tau\Phi\left(\frac{\theta^{cr} - x^p}{\sqrt{\varepsilon x u^2}}\right) \equiv \tau\iota(\theta^{cr}; x^p, \sigma^2_u, \varepsilon)$$  \hspace{1cm} (18)$$

which is the result of setting $R = 0$ in (5), treating the latter inequality as an equation and then combining it with (16) by substituting out $\iota$. For any value of the public signal, $x^p$, an equilibrium is fully characterised by a critical value of the fundamentals, $\theta^{cr}$, solving equation (18).

Observe that the pivotal agent in the modified EPS framework needs not take into account other agents’ actions: the private sector participates in the determination of the pressure on the peg only by setting the interest differential, about which there is no uncertainty. Thus, the $x^p$-agent’s problem is isomorphic to the one faced by the representative agent in the second-generation approach to currency crises. The self-fulfilling feature of beliefs is seen as follows. If agents’ pessimism intensifies, ie if they conjecture a higher $\theta^{cr}$ for a given $x^p$, they would put pressure on the peg by settling on a higher interest differential. The pessimism would be rationally justified if the action it generates increases sufficiently the authority’s incentive to devalue: this is more likely when the weight of $\iota$ in the authority’s reaction function, $\tau$, is higher. Following the just-outlined logic, a high $\tau$ would allow agents to also rationally justify a high degree of optimism (ie a low $\theta^{cr}$) for the same perception of the fundamentals (ie for the same $x^p$): multiple equilibria would emerge. Likewise, the smaller are the values of $\varepsilon$ and/or $\sigma^2_u$, the more precise is private agents’ information regarding $\theta$, the more certain they are about the ultimate implications of their actions, and the more likely they are to rationally justify different actions for the same $x^p$. To recapitulate: equilibrium multiplicity emerges in the modified EPS framework (ie there exist more than one values of $\theta^{cr}$ solving (18) for the same $x^p$) when $\varepsilon$ and/or $\sigma^2_u$ are sufficiently close to zero and/or $\tau$ is sufficiently high.
Consider now the counterpart of equation (18) within the original EPS model:\footnote{The first line of equation (19) is the result of substituting \( \lambda \) out of equations (15) and (16).}

\[
\begin{align*}
a(\theta^{cr}) &= \tau \Phi \left( \frac{\theta^{cr} - x^p}{\sqrt{\varepsilon \frac{\sigma_u^2}{1 + \sigma_u^2}}} \right) + \left[ 2 \Phi \left( \frac{x^p - \theta^{cr}}{\sqrt{\varepsilon}} \right) - 1 \right] \\
\equiv \tau \iota(\theta^{cr}; x^p, \sigma_u^2, \varepsilon) + R(\theta^{cr}; x^p, \varepsilon)
\end{align*}
\]

The term in square brackets, standing for the outflow of reserves, introduces a second strategic link between the private sector and the central authority. Importantly, the two strategic links are mutually offsetting (note that the two RHS terms of equation (19) move in opposite directions if \( \theta^{cr} \) changes): a higher interest differential increases the attractiveness of the domestic asset, which leads to a lower outflow of reserves. \( \tau \) and \( \sigma_u^2 \) enter (explicitly) only the first link and, as a result, their impact on the likelihood of equilibrium multiplicity is the same as in equation (18): this is what Proposition 1 states.

The parameter \( \varepsilon \) plays a special role for the strategic uncertainty in the system because it influences both links between the private sector and the central authority. As in equation (18), an increase in the precision of private information, attained by a decrease in \( \varepsilon \), enhances the impact of changes in private agents’ beliefs on the interest differential and, thus, on the incentives of the central authority. The more precise are private signals, however, the more similar they are. When, due to a small value of \( \varepsilon \), there is little dispersion of agents’ beliefs and actions, a given rise (drop) of the interest differential is more likely to be associated with a large rise (drop) in the net demand for the domestic asset.

An observed rise in the interest differential could thus coexist with higher aggregate devaluation expectations and an unchanged flow of reserves or with unchanged devaluation expectations and a higher inflow of reserves. (Symmetrically for a drop in the interest differential.) Further, the two scenarios, or combinations of them, have different overall implications for the authority’s incentives but cannot be disentangled by the agents as long as noise in their private signals produces heterogeneity of beliefs. The resulting strategic uncertainty is non-trivial even if there is virtually perfect common knowledge of the fundamentals and thus leads to the statement in Proposition 1 regarding \( \varepsilon \). Since it is based on the endogeneity of the interest differential, the just-described strategic uncertainty is of a type that is absent in the MS setup.

I conclude this section by noting that, as implied by equations (11) and (12), the equilibrium outflow of official reserves is fully determined by the realisation of the ex-
ogenous shock $U$. The reserve flow, however, possesses two key characteristics due to which it generates strategic uncertainty among agents and, as a result, plays a crucial role for the type of equilibrium in the EPS setup. The reserve flow both enters the authority’s reaction function and is set by the publicly unknown aggregate investment of informationally heterogeneous agents. Eliminating the first characteristic would produce an equilibrium condition as in (18). Eliminating the second characteristic results in the homogeneous-agent version of the EPS model whose equilibrium properties were reported at the end of Section 4.1.

4.4 Properties and Policy Implications of the Unique Equilibrium in the EPS Model

Unless explicitly stated otherwise, I assume in this section that the values of $\varepsilon$, $\sigma_u^2$ and $\tau$ are such that equilibrium uniqueness prevails in the EPS framework. The equilibrium interest differential is denoted by $\iota_{\text{eq}}$ and some of its arguments will be suppressed in order to reduce the notational burden. As already noted, $\iota_{\text{eq}}$ plays a dual role: it is a measure of the speculative pressure on the currency and, by being a monotone function of $x^p$, provides a public signal about the fundamentals. The present section considers $\iota$ only in its former role: the public signal is represented exclusively by $x^p$.

Equilibrium uniqueness allows comparative statics to deliver unambiguous conclusions regarding the implications of the EPS model. First, I examine the properties of the equilibrium interest differential as a function of the public signal about the fundamentals. It is seen, inter alia, that the state of the economy, as captured by the latter signal, has a stronger impact on the interest differential when the central authority’s commitment to the peg is weaker. Then, I turn to the model’s policy implications and establish that the introduction of small frictions in foreign-exchange markets might ward off violent speculative attacks. Further, the responsiveness of the interest differential to changes in the state of the economy is seen to depend on the precision of common knowledge in the system. This implies that an authority, which is in a position to influence the informational content of market prices, possesses a rather unconventional tool for managing currency crises.\footnote{The comparative statics results, stated in the section, are derived in Appendix 3.}

$\iota_{\text{eq}}$ decreases with the public signal because, by (13), the latter increases (on average) with the fundamentals, which strengthen the authority’s incentive to preserve the peg. Specifically:

$\iota_{\text{eq}}$ decreases with the public signal because, by (13), the latter increases (on average) with the fundamentals, which strengthen the authority’s incentive to preserve the peg. Specifically:
\[ \lim_{\varepsilon \to 0} \iota_{eq} \begin{cases} = 0 & \text{for } x^p > a^{-1}(1) \\ \in (0,1) & \text{for } x^p \in [a^{-1}(\tau - 1), a^{-1}(1)] \\ = 1 & \text{for } x^p < a^{-1}(\tau - 1) \end{cases} \]  \tag{20}

and, for \( x^p \in [a^{-1}(\tau - 1), a^{-1}(1)] \):

\[ \lim_{\varepsilon \to 0} \frac{\partial \iota_{eq}}{\partial x^p} = G(x^p) \left. \frac{d a(\theta^r)}{d \theta^r} \right|_{\theta^r = \theta^r_{eq}} < 0 \]  \tag{21}

where \( \theta^r_{eq} \) is the equilibrium critical value of the fundamentals corresponding to \( \iota_{eq} \) and \( G(\cdot) \) is defined in Appendix 3.

Consider the first line in equation \( (20) \) and note that, when \( x^p > a^{-1}(1) \), there is common knowledge that \( \theta > a^{-1}(1) \). On the basis of the authority’s reaction function, inequality \( (5) \), and the fact that \( R \in [-1,1] \), it is then commonly known that the flow of reserves cannot topple the peg by itself. Consequently, an equilibrium, in which \( \iota_{eq} = 0 \), emerges. As long as the assumptions of Proposition 1 are satisfied, this is the only equilibrium. Analogous intuition underlies the third line in expression \( (20) \).

Note that the authority’s resistance to speculative attacks would depend strongly on the fundamentals (ie \( \frac{d a(\theta)}{d \theta} \) would be large) if the commitment to peg preservation is weak. The equality in expression \( (21) \) would then stand for an intuitive equilibrium implication: a weaker commitment by the authority is built into agents’ expectations and intensifies the public signal’s effect on the interest differential.

The incorporation of \( \iota \) into the authority’s reaction function, inequality \( (5) \), follows an implicit assumption that a high interest differential hurts the domestic economy. In this respect, the following results reveal welfare implications of the policy options available to the central authority in the EPS model.

Consider changes in \( \tau \), the weight of the interest differential in the authority’s reaction function:\(^{31}\)

\[ \lim_{\sigma^2 \to 2 \times^{-\tau}} \frac{\partial \iota_{eq}}{\partial \tau} \bigg|_{x^p = a^{-1}(\tau/2)} = +\infty \text{ where } a^{-1}(\tau/2) \in (a^{-1}(\tau - 1), a^{-1}(1)) \]  \tag{22}

As implied by equations \( (4) \), \( (15) \) and \( (16) \), a drop in \( \tau \) would have the same

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\(^{31}\) The symbol “\( \downarrow \)” stands for “approaching the limit from above” and, recalling Proposition 1, \( \frac{\sigma^2}{\tau^2} \) stands for the lowest value of \( \sigma^2_a \) associated with equilibrium uniqueness.
equilibrium impact as the introduction (or an increase) of transaction costs, which would increase pivotal agents’ devaluation expectations relative to the interest differential. In the light of such an interpretation of \( \tau \), more sand in the wheels of foreign exchange markets would strengthen the peg and, by the second line of expression (22), might even prevent the materialisation of a violent speculative attack.

Next, I keep \( \tau \) fixed and examine the impact of changes in the value of the public signal \( x^p \):

\[
\lim_{\sigma_u^2 \to \sigma^2} \left. \frac{\partial \iota_{eq}}{\partial x^p} \right|_{x^p=a^{-1}(\tau/2)} = -\infty \tag{23}
\]

Further, in the case that \( \frac{da(\theta)}{d\theta} = 1 \):

\[
\frac{\partial \iota_{eq}}{\partial \sigma_u^2} \begin{cases} 
> 0 & \text{for } x^p > \tau/2 \\
< 0 & \text{for } x^p < \tau/2
\end{cases} \tag{24}
\]

where \( x^p = \frac{\tau}{2} \) is the middle point of the interval recorded on the second line of (20). On that interval of public signal values, it is commonly known that the aggregate holdings of the domestic asset can influence the authority’s decision, irrespective of the value of the interest differential. Relaxing the assumption \( \frac{da(\theta)}{d\theta} = 1 \) does not change the message of inequalities (24).

Equation (23), in conjunction with equation (17), demonstrates formally that the EPS model is in a position to account for sudden and violent speculative attacks: a drastic surge of the interest differential can be triggered by an infinitesimal change in the fundamentals. The potential for a strongly non-linear relationship between \( x^p \) and \( \iota_{eq} \) is rooted in agents’ strategic interaction and in their capacity to synchronise their behaviour by conditioning on the public signal. That capacity increases with the signal’s relative precision, which, in turn, improves as \( \sigma_u^2 \) decreases. If \( x^p \) is high, a lower \( \sigma_u^2 \) implies a stronger common belief that the fundamentals are high. Such a belief weakens agents’ intention to attack, as suggested by the first inequality in expression (24). A symmetric reasoning explains the second inequality. Finally, considering the two inequalities simultaneously, a decrease of the value of the public signal leads to a more drastic transition from low to high values of the interest differential when the precision of public information is higher: this leads to the result in equation (23).

The equilibrium implications recorded in (23) and (24) suggest that there is scope for managing speculative attacks by influencing prices’ informational content. Suppose that, contrary to the assumptions maintained in Section 4, the authority, or any other large player, steps into the domestic asset market in order to (partially) offset or amplify the supply shock. This would alter the variance of the equilibrium quantity demanded.
(equivalently, supplied) of that asset by the private sector. The random variable driving the latter quantity generates the noise in the endogenous public signal and thus affects the strategic uncertainty in the system. Consequently, the authority is able to influence foreign exchange market developments via purely market means, by sacrificing funds carrying opportunity costs. This is not possible in the MS setup.

The EPS setup thus provides a new point of view for interpreting the frequently observed sterilised interventions on foreign exchange markets, interventions that are deemed futile by analyses based on representative agent models. For example, the first line of expression (24) suggests that, if the fundamentals are sound and public information is rather noisy, increasing its relative precision up to a certain point (via systematic intervention) increases agents’ common awareness of the economy’s strength and is unambiguously beneficial for the peg. The model also highlights the limits of this policy. Proposition 1 implies that increasing the informational content of prices too much could strengthen agents’ coordination capacity to an extent that renders the currency regime vulnerable to sunspots.

5 Conclusion and a Look Ahead

The paper develops a currency crisis model in which beliefs can be self-fulfilling. A small perturbation of informational homogeneity in the private sector is shown to be in a position to account for abrupt and violent speculative attacks within equilibrium uniqueness. Consequently, policy analysis can be carried out via well-defined comparative statics.

There are two major, mutually related differences between the EPS model developed here and a model in the spirit of Morris and Shin (1998, 1999, 2000). First, public information is disseminated endogenously, via the equilibrium interest differential, in the former but exogenously in the latter model. Second, the interest differential and the flow of official reserves create two strategic links between the private sector and the central authority in the EPS setup. The informationally heterogeneous agents perceive the two links as having opposite effects on the central authority’s incentives to abandon the exchange rate regime. This allows strategic uncertainty to coexist with virtually

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32 In the extreme case, where the authority exactly offsets the supply shock, equation (13) becomes $X_p = 0$: the private sector net demand for the domestic asset would always be zero and market clearing would perfectly reveal the fundamental.

33 See, for example, Obstfeld and Rogoff (1995).

34 The discussion implicitly assumes that the authority can credibly commit to a particular intervention rule. The incentive compatibility of such a commitment must be verified within the model (a generalisation of the EPS setup) used for analysing the informational impact of interventions on the foreign exchange market.
perfect common knowledge of the fundamentals. Such coexistence is impossible in the MS setup, in which there is only one strategic link between the two types of players. As a result, equilibrium uniqueness emerges in the latter setup under a stronger requirement with respect to the relative precision of private and public information about the fundamentals.

Despite being highly stylized, the EPS model provides an insight of an empirical nature. The model relates to a series of speculative attacks, staged on foreign exchange markets in the 1990s, that have been described in informal accounts as not fully warranted by economic fundamentals. When attempting to determine the type of equilibrium leading to such events, structural empirical analysis has relied on the second-generation approach to currency crises. The conclusions of that analysis are potentially biased because, under standard assumptions, its underlying homogeneous-agent framework captures sudden and violent attacks only within equilibrium multiplicity.

To my knowledge, the EPS model is the first one to (i) account for seemingly unwarranted speculative attacks within both equilibrium multiplicity and uniqueness and (ii) allow for endogenous interest rates. The first feature suggests that it is not possible to reach an unambiguous conclusion, regarding the type of a currency attack’s underlying equilibrium, by examining only the attack itself. The intertemporal evolution of market devaluation expectations needs to be examined during times of speculative tranquility as well. Due to its second feature, the model is consistent with extracting these expectations from interest rate data that are available readily and at high frequencies. In sum, the EPS model warrants a shift away from the approach of the above-mentioned structural empirical analysis and provides the foundations for carrying out that shift. Delving into the issue is left for future research.

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6 Appendix 1

This appendix analyses the space of equilibrium-implementable investment strategies in the EPS model. Expressions (2) and (3) imply that, before any action has taken place in the first period, an $x_j$-agent perceives the following prior:

$$
\begin{bmatrix}
\Theta \\
U
\end{bmatrix} \sim \mathcal{N}
\left(
\begin{bmatrix}
x_j \\
\mu_u
\end{bmatrix},
\begin{bmatrix}
\varepsilon & 0 \\
0 & \sigma_u^2
\end{bmatrix}
\right)
$$

(25)

where $x_j$ is a realisation of $X_j = \Theta + Z_j$, $Z_j \sim \mathcal{N}(0, 1)$ and $Z_j$ is independent of $\Theta$, $U$ and all $Z_k$ for $k \neq j$.

Let the set of candidate equilibrium strategies be denoted by $\Pi$. Strategies within $\Pi$ are denoted by $\pi$: $\pi(x_j) \in [0, 1]$ sets the amount of currency that an $x_j$-agent invests in the foreign asset. Let $\Pi$ consist of strategy classes with the following properties. For any $\pi_a$ and $\pi_b$ in the same class, there exists a real $\lambda$ such that $\pi_a(x_j + \lambda) = \pi_b(x_j)$ for all $x_j$. Conversely, for any real $\lambda$ and $\pi_a$, there exists a strategy $\pi_b$ in $\pi_a$’s class such that $\pi_a(x_j + \lambda) = \pi_b(x_j)$ for all $x_j$.

The net outflow of official reserves is denoted by $R(\theta, \pi)$, where:

$$
R(\theta, \pi) = 2 \int \phi \left( \frac{n - \theta}{\sqrt{\varepsilon}} \right) \pi(n) \, dn - 1
$$

(26)

is a generalisation of equation (7). In turn, $A(\iota, \pi) \equiv \{ \Theta = \theta | R(\theta, \pi) \geq a(\theta) - \tau_\iota \}$ denotes the event in which the authority abandons the peg when agents follow strategy $\pi$ and the interest differential is $\iota$. Finally, market clearing implies a generalisation of equation (12):

$$
-R(\theta, \pi) = s(u) \equiv 2\Phi(u) - 1
$$

(27)

I adopt the following two assumptions:

**Assumption A.1**

$\Pi$ consists of strategy classes containing only monotone functions of the private signal.

**Assumption A.2**

There is some well-defined prior distribution of $\Theta$, $U$ and $\pi(\cdot)$. $\pi(\cdot)$ is independent of $Z_j$ for all $j$.

**Lemma A.1**
Any $\pi \in \Pi$ is a decreasing function. Moreover, there exist private-signal values, $x_j$ and $x_k$, such that $\pi (x_j) = 1$ and $\pi (x_k) = 0$.

**Proof**

Let us consider an arbitrary equilibrium pair $\{ \iota, \pi \}$, where $\iota \in (0, 1)$, and fix $\varepsilon > 0$ at an arbitrary finite value. Assumption A.2, in conjunction with the existence of dominance regions defined in equation (10), implies that, if an agent’s private signal is sufficiently small, that agent would choose to invest in the foreign asset irrespective of his/her perception of the measure of agents investing in either asset. A symmetric argument applies to an agent with a sufficiently high private signal. Thus, there exist two values of the private signal, $x_j$ and $x_k$, such that $x_j < x_k$ and $1 = \pi (x_j) > \pi (x_k) = 0$. Assumption A.1 then implies that $\pi$ is a monotonically decreasing function. QED

**Lemma A.2**

The joint prior distribution of $\Theta$, $U$ and $\pi$ is fully characterised by expressions (25)-(27) and the distribution of a sunspot variable that is independent of $\Theta$, $U$ and $Z_j$ for all $j$.

**Proof**

Consider an arbitrary pair $(\theta, u)$. Lemma A.1 implies that, within each class in $\Pi$, there exists exactly one $\pi$ that satisfies equation (27). This has two implications. First, there exists a sunspot variable that sets the equilibrium class of $\pi$’s and is independent of $\Theta$ and $U$ and $Z_j$ for all $j$. Second, conditional on the class of $\pi$’s, the joint prior distribution of $\Theta$, $U$ and $\pi$ is deduced exclusively on the basis of (25)-(27): note that any function within a given class is fully defined by an arbitrary function in that class and a real scaler. QED

**Proposition A.1**

If $\pi \in \Pi$, then there exists a real number $x^p$ such that $\pi (x_j) = \begin{cases} 1 & \text{for } x_j < x^p \\ 0 & \text{for } x_j > x^p \end{cases}$

**Proof**

Let us consider an arbitrary equilibrium pair $(\iota, \pi)$, where $\iota \in (0, 1)$. Consider an $x^p$-agent from whose point of view equation (4) is satisfied. The existence of such a zero-measure agent is consistent with Lemmas A.1 and A.2 and can be assumed without loss of generality. Consider next an $x_m$-agent, where $x^p < x_m$. Denote by $H (\cdot | Y_p)$ and $H (\cdot | Y_m)$ the posterior distribution functions of $\Theta$ when the private signal is $x^p$, respectively, $x_m$. ($Y$ thus stands for a posterior information set.) Equations (25)-(27) and Lemmas A.1 and A.2 imply that $H (\cdot | Y_m)$ first-order stochastically dominates $H (\cdot | Y_p)$. Further, Lemma A.1 implies that $R (\theta, \pi)$ decreases strictly in $\theta$. As a result, $A (\iota, \pi)$ is defined by a single cutoff value of the fundamentals: for $\theta$ below (above) this
value, the peg is abandoned (sustained). Thus, an $x_m$-agent attributes a strictly lower probability to devaluation than an $x_p$-agent and, thus, invests only in the domestic currency asset: $\pi(x_m) = 0$. Analogously, $\pi(x_n) = 1$ for $x_n < x^p$. QED

7 Appendix 2

In this appendix I first prove Proposition 1. Then I substantiate a statement, made at the end of Section 4.1, about the equilibrium properties of a homogeneous-agent version of the EPS model.

In order to prove Proposition 1 note first that expressions (6) and (15) imply that $\theta^{cr}$ is an increasing unbounded function of $\iota$. Thus, as $\iota$ increases (decreases), the LHS of equation (16), asymptotes gradually to unity (zero). As a result, an equilibrium pair $\{\iota, \theta^{cr}\}$, where $\iota \in (0, 1)$, exists irrespective of the values of $\varepsilon$, $\sigma^2_u$ and $\tau$.

Expressions (6) and (19) imply that an equilibrium pair $\{\iota, \theta^{cr}\}$ is unique if and only if it leads to the following inequality:

$$
\frac{2\sqrt{\sigma^2_u}}{\tau} \phi\left(\frac{\theta^{cr} - x^p}{\sqrt{\varepsilon} \sigma^2_u} \right) > \phi\left(\frac{\theta^{cr} - x^p}{\sqrt{\varepsilon}} \right)
$$

($O(\sqrt{\varepsilon})$ terms are eliminated in inequality (28), which implies that $a(\theta^{cr})$ should increases in $\theta^{cr}$ faster than the RHS of equation (19).

Fix an arbitrary $x^p$. Since $\frac{\sigma^2_u}{1 + \sigma^2_u} < 1$, the RHS of inequality (28) is weakly smaller than 1 at all values of $\theta^{cr}$ and equals 1 at $\theta^{cr} = x^p$. Further, the LHS is bigger than or equal to 1 only if $\tau \leq 2$ and $\sigma^2_u \geq \frac{\tau^2}{4-\tau^2}$. This proves the “if” part of the proposition’s statement.

For the “only if” part, observe that $\theta^{cr} = x^p$ is a solution to equation (19) when $x^p = a^{-1}(\tau/2)$. By continuity then, if the LHS of inequality (28) is smaller than 1, there is an interval on the real line, $\Omega$, with the following properties: (i) $a^{-1}(\tau/2) \in \Omega$; (ii) if $x^p \in \Omega$, then there is a value of $\theta^{cr}$, solving equation (19), at which inequality (28) does not hold. This completes the proof of Proposition 1.

For the remainder of the appendix, I consider a version of the EPS model in which all private agents hold the beliefs of an arbitrary $x_j$-agent in the original model: ie beliefs consistent with the posterior distributions of $\Theta$ and $U$ recorded in (14). In this homogeneous-agent case, the limit $\varepsilon \to 0$ can be replaced by $\varepsilon = 0$ without loss of
generality: $\theta$ is common knowledge. Observe that $\frac{x_j - x_p}{\sqrt{\varepsilon}}$ on the second line of (14) is a realisation of $(Z_j + U)$. In the homogeneous-agent version of the EPS model, the perceived mean of $U$ is then $\tilde{\mu}$, which is a realisation of $\tilde{U} \equiv (Z + U) \frac{\sigma^2_u}{1 + \sigma^2_u}$, where $Z \sim N(0, 1)$ and is independent of $U$.

If the identical agents are not indifferent between the two assets, their optimum investment position would generate aggregate net demand for the domestic asset equal to either 1 or $-1$. Such demand, however, would not match $S$ almost surely (recall (11)). In contrast, if agents are indifferent between the two assets, a market-clearing equilibrium would be implemented when (i) agents follow the strategy “invest domestic if called upon, invest foreign otherwise”, (ii) the measure of called-upon agents equals $\Phi(u)$ and (iii) calls are random and private. Agents perceive only one random variable, $U$, and indifference is attained when the interest differential is equal to $\Pr(U < u^{cr}) = \Phi \left( \frac{u^{cr} - \tilde{\mu}}{\sqrt{\sigma^2_u/(1 + \sigma^2_u)}} \right)$. Thus, for any pair $(\theta, \tilde{\mu})$, an equilibrium is defined by a critical value of $u$, solving

$$\tau \Phi \left( \frac{u^{cr} - \tilde{\mu}}{\sqrt{\sigma^2_u/(1 + \sigma^2_u)}} \right) = a(\theta) + 2 \Phi(u^{cr}) - 1 \quad (29)$$

Recalling (5), (7) and (12), equation (29) states that, when $u = u^{cr}$, the central authority is indifferent between preserving and abandoning the peg.

For a given pair $(\theta, \tilde{\mu})$ and out of equilibrium, each side of (29) is an increasing continuous function of $u^{cr}$ with the entire real line as its domain. These functions change curvature at $u^{cr} = \tilde{\mu}$ and $u^{cr} = 0$, respectively, and their range is bounded from below and above. Further, any relative position of the schedules of the two functions can be obtained because $\Theta$ and $\tilde{U}$ have an infinite support and the range of $a(\cdot)$ is unbounded. For any finite values of $\tau$ and $\sigma^2_u$, it is then possible to obtain a pair $(\theta, \tilde{\mu})$ such that the schedule of the two functions intersect more than once. This is equivalent to equilibrium multiplicity and, by continuity, occurs on a positive measure of the state space, which is in terms of $\Theta$ and $\tilde{U}$. This justifies the statement, made in Section 4.3, about the equilibrium properties of the homogeneous-agent version of the EPS model.

8 Appendix 3

In this appendix, I assume that $\{\tau, \sigma^2_u\} \in (0, 2) \times \left[ \frac{\tau^2}{4 - \tau^2}, \infty \right)$ and prove expressions (20)-(22). Using this parameter restriction, differentiation of (19) implies:

$$\lim_{\varepsilon \to 0} \frac{d}{dx_p} \left. \frac{d}{dx_p} \right|_{x_p = x^{cr}} \begin{cases} < 0 \end{cases} \quad (30)$$

30
where $\theta_{eq}^{cr}$ solves (19).

In order to prove expression (20), let $x^p$ belong to the interior of $(a^{-1}(\tau - 1), a^{-1}(1))$ and assume that $\lim_{\varepsilon \to 0} \left( \frac{\theta_{eq}^{cr} - x^p}{\sqrt{\varepsilon}} \right) = -\infty$: thus, a fortiori, $x^p > \theta_{eq}^{cr}$. This would imply that, when $\varepsilon \to 0$, the RHS of equation (19) converges to 1, which then means that $\lim_{\varepsilon \to 0} \theta_{eq}^{cr} = a^{-1}(1)$. The latter equality, however, contradicts the inequality $x^p > \theta_{eq}^{cr}$ when $x^p \in (a^{-1}(\tau - 1), a^{-1}(1))$. A symmetric argument applies to $\lim_{\varepsilon \to 0} \left( \frac{\theta_{eq}^{cr} - x^p}{\sqrt{\varepsilon}} \right) = +\infty$. This proves the middle line of expression (20).

Let now $x^p > a^{-1}(1)$ and assume that $\theta_{eq}^{cr} > a^{-1}(1)$. Since $\tau < 2$ and $\theta_{eq}^{cr} = x^p$ when $x^p = a^{-1}(\tau/2)$, expression (6), inequality (30) and $x^p > a^{-1}(1)$ imply that $x^p > \theta_{eq}^{cr}$. In turn, $x^p > \theta_{eq}^{cr}$ implies that the RHS of equation (19) is smaller than unity, contradicting the assumption that $\theta_{eq}^{cr} > a^{-1}(1)$. Thus, $\theta_{eq}^{cr} < a^{-1}(1)$ for any $x^p > a^{-1}(1)$. Equation (16) then implies the first line of expression (20). The third line of that expression is derived with a symmetric argument.

Defining $G(x^p) \equiv \left( \tau - \frac{\sigma_u^2}{1 + \sigma_u^2} \frac{\partial \pi^{cr}}{\partial \pi^{cr}} \right)^{-1}$, expression (21) is an immediate implication of equations (15) and (16), inequality (30) and Proposition 1. Trivial algebra leads to $\lim_{\sigma_u^2 \to \sigma_u^2} \frac{\partial \pi^{cr}}{\partial \pi^{cr}} = \psi(x^p) G(x^p)$, where $\psi$ is a function of $x^p$ such that $\lim_{\sigma_u^2 \to \sigma_u^2} \psi(x^p) |_{x^p = a^{-1}(\tau/2)} < 0$. The second line in (22) then holds because $\lim_{\sigma_u^2 \to \sigma_u^2} G(x^p) |_{x^p = a^{-1}(\tau/2)} = -\infty$. The first line in (22) is obtained after totally differentiating equation (19) with respect to $\theta_{eq}^{cr}$ and $\tau$ and then using equation (16) and the conditions for equilibrium uniqueness.

Equation (23) is derived analogously to the second line in (22). To prove expression (24), I assume that $\frac{\partial a(\theta)}{\partial \theta} = 1$. Using equations (16) and (19), the sign of $\frac{\partial \theta_{eq}^{cr}}{\partial \theta}$ is found to be the same as the sign of $(x^p - \theta_{eq}^{cr})$. The latter result, in conjunction with inequality (30) and the fact that $\theta_{eq}^{cr} = x^p$ when $x^p = \tau/2$, implies expression (24).
9 Bibliography


