The Mechanics of a Reasonably Fitted Quarterly New Keynesian Macro Model

Eric Mayer
University of Würzburg
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Abstract

The last years have witnessed a sharp increase of interest in monetary policy rules (see Taylor [1999]). This normative branch of monetary policy tries to evaluate the performance of alternative monetary policy rules in terms of associated monetary policy outcomes. Nevertheless this exercise is crucially based on the assumption that key parameters of the model are realistically specified. This holds in particular true for the preference vector of the central bank which trades off the individual goal variables of monetary policy and the degree of forward lookingness in the Phillips curve and the IS equation. Based on matching moments and the implied autocorrelations and cross correlations we present evidence for the USA covering the term of Allan Greenspan (1987:4-2002:2) that hybrid specifications of the Phillips curve and the IS-curve are characterized by approximately 60% of backward looking economic agents. The predominant goal of monetary policy is price stability and financial market stability. Output gap stabilization only seems to play a minor role as an independent goal for the conduct of monetary policy.

Keywords: New Keynesian Macro Model, hybrid Phillips curve, hybrid IS curve, forward looking behaviour, rule-of-thumb behaviour, calibration

JEL Classification: C51, E52, E58.

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1 Introduction

The last years have witnessed a sharp increase of interest in monetary policy rules (see Taylor [1999]). This normative branch of monetary economics tries to evaluate the performance of alternative monetary policy rules in terms of associated monetary policy outcomes as measured by the attached variances of the goal variables. This exercise is crucially based on the assumption that key parameters of the model, in particular the degree of forward-lookingness in the IS-equation and the Phillips curve, as well as the preference vector of monetary policy which trades off the different goal variables, hence the stabilization of the inflation rate, the stabilization of the output gap and the stabilization of the change in interest rates are correctly specified. This paper targets at proposing a strategy that simultaneously identifies these key parameters by matching moments. We construct for each variable (the inflation rate, the output gap, the interest rate and the first difference of each variable) a separate criterion, which measures the sum of squared percentage deviations of the individual moments from its historical counterparts. We choose to identify that vector of key parameters which minimizes the equally weighted sum over all constructed criteria six.

As a toolkit to analyse the plausibility of the identified key parameters we proceed along the following lines. First, we compute the impulse response function as a cross cheque for the embedded transmission structure. Second, we test for stability and uniqueness along the lines as proposed by Blanchard et. al. [1980]. Third and most importantly we evaluate the mechanics of the model in detail by performing a battery of baseline evaluations. In particular the sensitivity of the implied cross correlations with respect to changes in the identified key parameters will be useful to make inferences on the plausible ranges of the identified vector.

The following results stand out: Approximately 60% of all pricing and consumption decisions are made by rule of thumb setters which simply centre their pricing decisions on heuristics. Additionally we present evidence in line with earlier studies that the dominant goal of monetary policy besides stabilizing the inflation rate is interest rate smoothing. The stabilization of the output gap only seems to play a minor role for the conduct of monetary policy as an independent goal variable.
2 The Structure of New Keynesian Macro Models

Standard New Keynesian Macro Models share a simple structure that is centred around three building blocs:

- A new Keynesian Phillips curve as price determining relationship.
- An Euler-equation mapping aggregate demand.
- An optimal or simple monetary policy rule depicting the way according to which monetary policy is conducted, and

We will shortly discuss each of these building blocs in term.

2.1 The New Keynesian Phillips Curve

A cornerstone of New Keynesian Macro-Models is the New Keynesian Phillips Curve (NKPC). The NKPC curve relates the inflation rate to some measure of economic activity. Hence it gives a description of the supply side of the economy. The standard NKPC can be summarized as follows\(^1\):

\[
\pi_t = \beta \pi_{t+1} + \gamma y_t + \varepsilon_t,
\]

where \(\beta\) depicts the discount factor of households. Note that \(\gamma\) is a function of the underlying deeper structural parameters of a New Keynesian Macro model. The concrete relationship depends in particular on the assumptions made on the production technology of firms.

In applied work it has prevailed that purely forward-looking NKPCs are unable to fit the facts which means in particular that they do not replicate the hump-shaped response embedded in impulse response functions. Therefore, as we will see from related studies some degree of backward lookingness is necessary to introduce persistence in the inflation rate. A popular approach to endogenize persistence was proposed by Altig et al. [2002]. They introduce rule of thumb behaviour on some part of price setters. Hence besides Calvo pricing some price setters update their prices following a rule of thumb. In particular one may assume that some price setters simply update their prices by yesterday's aggregate price level. This rule of thumb behaviour can be rationalized as follows (see Amato and Laubach [2003]):

- Rule of thumb behaviour does not produce any computational costs.

\(^1\) For an overview on Phillips curves see Roberts [1995], Jondeau et al [2001].
• The fraction of price setters that updates by rule of thumb implicitly learns as \(\pi_{t-1}\) incorporates the pricing decisions of those agents that have optimised in period \(t-1\).
• In steady state rule of thumb setters will set prices equal to those who do Calvo [1983] pricing.

Based on these this notion the NKPC in its most general form at a quarterly frequency can be written as follows:

\[
\pi_t = \mu_t E_{t-1} \pi_{t+k} + (1 - \mu_t) \sum_{j=1}^n \beta_{xj} \pi_{t-j} + \sum_{l=1}^n \beta_{yj} y_{t-j} + \epsilon_t
\]  

The current rate of inflation is explained by a weighted average of past and future inflation rates as well as the current and lagged variables of the output gap. No consensus has yet emerged up to which degree the price and wage setting behaviour of economic agents is governed by forward-looking behaviour. Table A.1 (Appendix A.1) presents some evidence from estimated and calibrated ‘baseline’ versions. The presented baseline estimates of the degree of forward-lookingness vary from 0.1 to 0.75. As we will see in section 4 the degree of forward lookingness in the Phillips curve is crucial for the ‘mechanics’ of the model.

2.2 The New Keynesian IS-Curve: Euler Equation

The second building bloc of New Keynesian Macro Models is the IS-equation. It gives a description of the demand side of the economy. The New Keynesian IS-curve is a relationship that relates the output gap negatively to the expected real interest rate and future to tommorrows output.

\[
y_t = \beta_0 - \beta_1 [i_t - E_t \pi_{t+1}] + E_t y_{t+1} + \eta_t
\]  

Unfortunately this relation is like the New Keynesian Phillips curve at odds with the data as it is unable to display the inertia nested in aggregate output. The standard Euler-equation predicts that a shock to aggregate demand will generate only a single jump in output which stands in contrast to the hump shaped response documented in VAR studies\(^2\). One remedy to this problem was offered by Fuhrer [2000]. He reintroduced persistence into the aggregate

\(^2\) For an overview on minimum requirements for reasonable impulse response patterns see Christiano J. et al. [1998].
spending relationship by introducing habit formation in the utility function of households. Hence households centre their optimal consumption choice around a targeted consumption level (‘habit stock’). A second approach to generate an inertial IS-relationship is to introduce rule of thumb consumers. Rule of thumb consumers simply set today’s desired consumption level equal to last periods consumption level. Thus, some fraction of households optimises while another fraction \((1 - \zeta)\) of households simply centres its consumption decisions around last periods consumption level. Both approaches can lead to the following (hybrid) specification of an IS-curve at a quarterly frequency:

\[
y_t = \mu_t E_{t-1} \sum_{j=1}^{n} \beta_j y_{t-j} + (1 - \mu_t) \sum_{j=1}^{n} \beta_j y_{t-j} - \beta_t \sigma_t \left[ \bar{y}_{t-1} - E_{t-1} \pi_{t+3} \right] \\
+ \beta_t (1 - \mu_t) \left[ \bar{y}_{t-1} - \pi_{t-3} \right] + \eta_t
\]

(4)

Table 2 (Appendix 1) presents some baseline estimates for the IS-curve. Reviewing these studies there seems to crystallise a consensus that a substantial degree of backward lookingness is needed to fit the actual data. At least 20\% of the economic agents are assumed to be backward looking according to the reviewed studies.

2.3 Specifying the Conduct of Monetary Policy

The third building bloc of New Keynesian Macro Models is a relationship depicting the way according to which monetary policy is conducted. The overall goal of monetary policy is to promote economic welfare. Given the legal mandate of most prominent central banks this is usually interpreted in terms of keeping the inflation rate close to the inflation target while equally stabilizing the output gap around its potential. As it is our aim to take the New Keynesian Model to the data we additionally introduce interest rate smoothing as an independent goal of monetary policy. It is an observable fact that monetary policy is implemented gradually. Typically short-term rates are not changed by more but 25 or 50 basis points. In other words monetary authorities do not implement their desired interest rate target cold turkey but perform a gradual adjustment to the desired target level. This observable interest rate setting behaviour can be rationalized among others by the following argument: Policymaker’s are confronted with three major types of uncertainties. Model uncertainty, parameter uncertainty and data uncertainty. It is well documented that each of these

\[3\] Note we will not consider interest rate smoothing around a long run nominal equilibrium rate as alternative goal of monetary policy. As noted by Woodford [2002] this notion of smoothing might be rationalized by a desire to avoid the zero lower bound, hence a deflationary trap.
uncertainties tends to reduce the aggressiveness with which policymakers react with their instrument to the set of predetermined variables. In other words the coefficients in the optimal monetary policy rules are smaller in absolute values. This automatically translates into a smoother interest rate setting behaviour. One straightforward way to introduce interest rate smoothing in the model is to put a higher weight on interest rate smoothing in the loss function (Martin et al. [1999]).

Given these goals of monetary policy we can state the loss function as follows:

\[ L = \pi^2 + \lambda y^2 + \nu \Delta i^2 \]  

There are only a few studies available that try to pin down the true preferences \((\lambda; \nu)\) of monetary policy makers for the US. Reviewing these studies (Table 3; Appendix 1) there seems to emerge the following consensus: Central banks seem to put a higher weight on stabilizing the inflation rate around the inflation target than stabilizing output at its capacity level. Additionally a high weight is put on interest rate smoothing. Output stabilization only seems to play a minor role for the conduct of monetary policy.

3 Rewriting the Model in State Space Form

A convenient representation of the New Keynesian Macro Model can be given by the following set of difference equations:

\[ X_{t+1} = AX_t + Bi_t + v_{t+1} \]  

\( X_t \) is the state vector, which defines each period the state space. The matrix \( A \) is the so-called companion matrix. It captures the structural or reduced form coefficients of the economy. \( B \) is a vector consisting of the interest rate impact multipliers. Since the economy is subject to shocks that drive the system these are captured by the vector \( v_t \). Due to the specific model set up the variance covariance matrix \( \Omega \) is diagonal. Therefore we interpret the individual shocks

4 Note that this way of stating the central bank’s loss function makes use of the fact that after scaling the intertemporal loss function by \((1-\beta)\) it converges to the weighted sum of the unconditional variances of the goal variables in the limit (Svensson, 2003).
as structural shocks. As some of our variables will not be predetermined we can conveniently partition the system as follows:\footnote{For an in depth discussion of state space systems containing forward looking variables see Söderlind [1999], Svensson [1999].}

\[
\begin{bmatrix}
X_{t+1} \\
E_t X_{2t+1}
\end{bmatrix} = 
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
X_t \\
X_{2t}
\end{bmatrix} + 
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} i_t + 
\begin{bmatrix}
\varepsilon_{t+1} \\
0_{n_2 \times 1}
\end{bmatrix}
\]  

(7)

n_1 Predetermined variables: \(X_{1t}\)
n_2 Forward-Looking Variables: \(X_{2t}\)

At the start of period \(t\) \(X_{1t}\), driven by the shock terms \(\varepsilon_t\) is realized. Then the central bank, conditional on the available information set \((\varepsilon_t, X_{1t}, \varepsilon_{t-1}, X_{2t-1}, i_{t-1}, \ldots)\) chooses \(i_t\). At the end of period \(t\) \(X_{2t}\) results. Finally rational expectations on \(E_t X_{2t+1}\) are formed on the available information at the end of period \(t\). Assume for the moment that we can represent the instrument of monetary policy as a linear function of the state variables.

\[r_t = -FX_t\]

(8)

Given our system we can then evaluate the closed loop dynamics (the economy in conjunction with the policy rule) if we insert \(r_t = FX_t\) in the dynamic law of motion:

\[X_{t+1} = (A - BF)X_t + v_{t+1}\]

(9)

Hence we arrive at the following modified system:

\[X_{t+1} = MX_t + v_{t+1} \quad \text{With } M = (A - BF)\]

(10)

Equation (10) gives a complete description of the closed loop dynamics. Therefore it represents our basic equation when analysing the properties of monetary policy rules. The variance of the state vector is given by the expression:

\[\text{vec}(\Sigma_{XX}) = \left[I - M \otimes M\right]^{-1} \text{vec}(\Omega)\]

(11)
The second equation, which completes the description of our state space system, is the so-called measurement equation. It can be stated as:

\[ Y_t = C_X X_t + C_i F \] (12)

The measurement equation defines the vector of variables as a function of the state variables and the specified monetary policy rule. If we partition \( C_X \) in two blocks associated with the backward and forward looking state variables we arrive at the following equation:

\[ Y_t = (C_{X1} + C_{X2})X_t + C_i F \] (13)

which can equally be expressed only in terms of predetermined variables:

\[ Y_t = (C_{X1} + C_{X2}C + C_i F)X_{tr} \] (14)

Therefore we can write the variance of the variables in which we are interested as follows:

\[ \Sigma_{YY} = E[Y_t Y_t'] = C^* X_{tr}(C^* X_{tr})' = C^* \Sigma_{XX} C^{**} \]

with \( C^* = C_{X1} + C_{X2}C + C_i F \)

4 Calibrating a New Keynesian Macro Model: Calibration Method

In the following section we will propose a novel calibration scheme that simultaneously calibrates the degree of forward-lookingness in the Phillips curve and the IS-curve as well as the preference vector of monetary policy. For the remaining parameters we use estimates as provided by Rudebusch [2000]. Compared to related literature (See Castelnuovo [2003], Söderlind et. al. [2002]) we do not rely on simulation techniques but on a analytically constructed variance-covariance matrix which allows us to compute diverse moments.

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6 Note that one might equally speak of a target equation. We prefer the expression measurement equation as not all variables included in the vector \( Y_t \) need necessarily to be a goal variable of monetary policy.

7 For a definition of the individual matrices see appendix A.3 and A.4.

8 For an interpretation of the C matrix see A.4.
4.1 Calibration Method

Let us make (as untested apriority) the assumption that a New Keynesian Macro Model describes the true data generating process at a quarterly frequency. Taking this apriority we calibrate the model in order to meet simultaneously a set of well-defined criteria. We calibrate simultaneously the vector \( \psi = [\mu_x, \mu_y, \mu_r, \lambda, \nu] \) with:

- \( \mu_x \) the degree of forward lookingness in the Phillips curve.
- \( \mu_y \) the degree of forward lookingness in the IS-curve.
- \( \mu_r \) the degree of forward lookingness with respect to real interest rates in the IS-curve.
- \( \lambda \) weight monetary policy puts on output stabilization relative to stabilizing the inflation rate.
- \( \nu \) weight the central bank puts on interest rate smoothing relative to inflation rate stabilization.

Let us in particular define the following measurement vector.

\[
Y_t = [\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, i_t, i_{t-1}, i_{t-2}, \Delta i_t, \Delta i_{t-1}, \Delta i_{t-2}, \Delta \pi_t, \Delta \pi_{t-1}, \Delta \pi_{t-2}, \Delta y_t]
\]

with variance covariance matrix as defined by equation (15). The calibration technique proceeds along the following lines:

We iterate the individual parameters over the following ranges:
- \( \mu_x \) in the interval from 0 to 1 with step size 0.1.
- \( \mu_y \) from 0 to 1 with step size 0.1.
- \( \mu_r \) was set alternatively equal to null or one.
- \( \lambda \) in the interval from 0 to 4 with step size 0.05.
- \( \nu \) in the interval from 0 to 6 with step size 0.1.

This results in a total of 960,000 parameter constellations over which the model is evaluated. For each preference vector \( \psi = [1, \lambda, \nu, \mu_x, \mu_y, \mu_r] \) we simultaneously compute the implied
variances, covariances and autocorrelations of the measured variables. In the next step we compute the following criterion:

$$W^\theta$$

(16)

Where $\theta$ is a vector consisting of the chosen criteria. For a detailed definition of the individual criteria see Appendix A5. Each individual element of each criterion consists of the absolute squared percentage difference between the individual variances, covariances, and auto correlations of each variable (the inflation rate, the output gap, the interest rate and the respective differences) implied by $\psi = \begin{bmatrix} 1 & \lambda & \mu_x & \mu_y & \mu_r \end{bmatrix}$ minus the corresponding empirically observed values in the historical data (1987:4-2002:2). This sample period covers the term of the chairmen Alan Greenspan. As weightening matrix $W$ we propose an equal weightening of all constructed six criteria. Accordingly $W$ is specified as:

$$W = I_{6,1}$$

(17)

Among this the set of 960,000 parameter constellations we choose to identify those ten combinations $\psi$ that minimize the following criteria:

$$\min_W \left( W^\theta \right)$$

s.t: $\Delta \%\sigma_x \leq c; \Delta \%\sigma_y \leq c; \Delta \%\sigma_{\pi} \leq c;$

(18)

As can be seen from equation (18) we additionally impose the restriction that the individual standard deviations of the goal variables should not display a greater percentage deviation but $c$ from historical counterparts. Hence we implicitly give a dominant role to the individual variances of the time series while calibrating the model. We set $c=0.5$.

4.2 Calibrating the Remaining Parameters

The backward looking inflation polynomial in the Phillips curve $\alpha_\pi$, the impact of economic activity on inflation $\alpha_y$, the interest rate sensitivity of economic activity in the IS-curve $\beta_r$, and the autoregressive part in the output gap equation $\beta_{y_t}$ was specified by estimates as reported by Rudebusch [2000] which are displayed in Table 1. Rudebusch [2000] used the following specifications: $\pi_t$ was specified as the quarterly inflation rate in the GDP chain-weighted price index $p_t$ seasonally adjusted and calculated at an annual rate $4(\ln p_t - \ln p_{t-1})$;
\( \bar{\pi} \) is the four quarter moving average constructed as \( (1/4) \sum_{t=0}^{3} \pi_{t-j} \); \( \bar{t} \) is the four quarter average federal funds rate, hence \( \frac{1}{4} \sum_{t=0}^{3} \bar{t}_{t-j} \); \( y_t \) is the output gap constructed as the percentage deviation of the output \( Y_t \) from trend output \( Y^*_t \), where \( Y^*_t \) was taken from the Congresional Budget Office. All variables were demeaned prior to estimation. Note in particular that the specification as proposed by Rudebusch [2000] implies that the sum over the inflation polynomial (\( \sum_{i=1}^{4} \beta_{\pi i} = 1 \)) is equal to one, so that the long run neutrality of money holds. This means in steady state (\( \pi^T = \pi^T_{t=1} = \pi^T_{t=2} = \pi^T_{t=3} = \ldots \)) it holds that:

\[
y = \frac{1 - [\beta_{\pi 1} + \beta_{\pi 2} + \beta_{\pi 3} + \beta_{\pi 4}]}{\alpha} \pi^T
\]

(19)

Obviously the property of long run neutrality is violated as long as \( [\beta_{\pi 1} + \beta_{\pi 2} + \beta_{\pi 3} + \beta_{\pi 4}] \neq 1 \). Higher inflation targets \( \pi^T \) could boost output permanently, which would violate the long run neutrality of money. Thus, it is desirable to set the slope coefficient equal to one \( \beta=1 \), which translates into \( \frac{1 - \beta}{\alpha} = 0 \). This is from an economic point of view somewhat problematic as \( \beta \) should be interpreted as a discount factor.

Table 1: Calibrating the New Keynesian Model

<table>
<thead>
<tr>
<th>Quarterly New Keynesian Model (Rudebusch 2000)</th>
<th>Inflation Equation</th>
<th>Output Gap Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha_{s1} = 0.67 )</td>
<td>( \beta_{s1} = 1.15 )</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{s2} = -0.14 )</td>
<td>( \beta_{s2} = -0.27 )</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{s3} = 0.4 )</td>
<td>( \beta_{s3} = 0.09 )</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{s4} = 0.07 )</td>
<td>( \sigma_y = 0.833 )</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{\gamma} = 0.13 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sigma_{\gamma} = 1.012 )</td>
<td></td>
</tr>
</tbody>
</table>

Note however that some might argue that changes in the equilibrium inflation rate induce new steady states, so that the argument is somewhat flawed.
5 Calibration Results: An Evaluation of the Proposed Baseline Calibration\textsuperscript{10}

In the following section we will present and evaluate the outcomes of the proposed evaluation method. The proposed toolkit to analyse the identified baseline specification proceeds along the following lines. In a first step we will analyse the basic plausibility of the identified baseline calibration by computing the impulse response patterns following a supply, demand and interest rate shock. The impulse response function serves as a cross cheque for the implied correlation structure embedded in our preferred calibration vector \( \psi = [1 \ \lambda \ \nu \ \mu_x \ \mu_y \ \mu_r] \). As a second crude tool we will evaluate the stability and the uniqueness properties by testing whether the number of unstable eigenvalues is equal to the number of forward looking variables as proposed by Blanchard et al. [1980]. Following these preliminary examinations we will systematically analyse the mechanics of the model by a battery of baseline evaluations.

5.1 Top Ranked Calibration Vectors: Uniqueness and Stability

- The top ranked vector combinations \( \psi \) identified by the proposed calibration method are displayed in Table 2.

\begin{table}[h]
\centering
\begin{tabular}{lllll}
\hline
RANKING & \( \lambda \) & \( \nu \) & \( \mu_x \) & \( \mu_y \) \\
\hline
1 & 0.05 & 1.65 & 0.6 & 0.2 \\
2 & 0 & 1.65 & 0.6 & 0.2 \\
3 & 0 & 1.55 & 0.6 & 0.2 \\
4 & 0 & 1.45 & 0.6 & 0.2 \\
5 & 0 & 1.35 & 0.6 & 0.2 \\
6 & 0 & 1.25 & 0.6 & 0.2 \\
7 & 0 & 1.15 & 0.6 & 0.2 \\
8 & 0 & 1.05 & 0.6 & 0.2 \\
9 & 0 & 0.95 & 0.6 & 0.2 \\
10 & 0.15 & 1.85 & 0.4 & 0.4 \\
\hline
\end{tabular}
\caption{Calibration Output}
\end{table}

\textsuperscript{10} All codes for basic computations were taken from Paul Söderlind homepage: http://www.hhs.se/personal/PSoderlind/Research/MonEEAMatLab.zip
Following Blanchard et al. [1980] we test for uniqueness and stability by computing the eigenvalues. It has to hold that the number of unstable eigenvalues is equal to the number of forward-looking variables. A look at the partitioned state vector tells us that the number of predetermined variables is equal to nine. The number of forward-looking variables is equal to four:

- \( X_1_t = \{ \pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}, i_{t-1}, i_{t-2}, i_{t-3}\} \)
- \( X_2_t = \{ E_t\pi_{t+3}, E_t\pi_{t+2}, E_t\pi_{t+1}, E_t y_{t+1}\} \)

Table 3 confirms that for \( \psi_{10} = [1 \ 0.15 \ 1.85 \ 0.4 \ 0.4 \ 1] \) the number of forward-looking variables satisfies the proposition as stated by Blanchard et al. [1980]. Hence we conclude that the identified baseline configuration \( \psi_{10} \) generates a stable and unique solution. We have equally tested for stability and uniqueness for the combinations \( \psi_{1-9} \). All identified vectors were stable, but not unique. In other words the number of stable eigenvalues was larger than the value of predetermined variables. The identified vector combination \( \psi_{10} = [1 \ 0.15 \ 1.85 \ 0.4 \ 0.4 \ 1] \) is described by the following characteristics:

- The weight \( \lambda \) of stabilizing squared deviations of the output gap around zero is rather small compared to the weight put on the other two goal variables of monetary policy. It is well known that this does not mean that monetary policy does not care on the output gap. This is quickly confirmed if one takes a look at the optimal monetary policy rule which is given by:

\[
\begin{align*}
    i_t &= 0.2947\pi_t + 0.1140\pi_{t-1} + 0.1169\pi_{t-2} + 0.0166\pi_{t-3} + 0.2348y_t + 0.0701y_{t-1} + 0.6391i_{t-1} \quad (20)
\end{align*}
\]

Monetary policy reacts c.p. with an increase of 0.2348 of its instrument to current changes in the output gap and with a coefficient of 0.0701 to changes in last periods output gap. This can be explained as follows: Even a central bank that only puts a modest weight on output stabilization opts to react on movements in economic activity in order not to lose control over the inflation rate as the output gap is the driving variable of the inflation process (see Svensson [2003]). The finding that output gap

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11 Note that \( i_{t-2} \) and \( i_{t-3} \) dropped out of the optimal discretionary policy rule as the coefficients are equal to null. This result was equally retrieved by Castelnuovo [2003].
stabilization only seems to be of minor importance as an independent goal of monetary policy is well in line with related studies that coherently come to the same result.

Table 3: Stability and Uniqueness of the Identified Solution

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Stable</td>
</tr>
<tr>
<td>0</td>
<td>Stable</td>
</tr>
<tr>
<td>3.97</td>
<td>Unstable</td>
</tr>
<tr>
<td>-1.27 + 1.67i</td>
<td>Unstable</td>
</tr>
<tr>
<td>-1.27 - 1.67i</td>
<td>Unstable</td>
</tr>
<tr>
<td>-0.122 + 0.53i</td>
<td>Stable</td>
</tr>
<tr>
<td>-0.122 - 0.53i</td>
<td>Stable</td>
</tr>
<tr>
<td>-0.157</td>
<td>Stable</td>
</tr>
<tr>
<td>1.05</td>
<td>Unstable</td>
</tr>
<tr>
<td>0.80 + 0.30i</td>
<td>Stable</td>
</tr>
<tr>
<td>0.80 - 0.31i</td>
<td>Stable</td>
</tr>
<tr>
<td>0.61</td>
<td>Stable</td>
</tr>
<tr>
<td>0.34</td>
<td>Stable</td>
</tr>
</tbody>
</table>

For the identified baseline calibration: \( \psi = \begin{bmatrix} 1 & 0.15 & 1.85 & 0.4 & 0.4 & 1 \end{bmatrix} \).

- The relatively high weight on financial market stability as an independent goal of monetary policy confirms earlier results by Dennis [2001] and Söderlind et al. [2002]. The high weight on interest rate smoothing is reflected in the optimal discretionary monetary policy rule as the coefficient on \( i_{t-1} \) is equal to 0.6391.
- The degree of forward-lookingness in the Phillips curve is identified to be equal to 0.4. Hence 40% of economic agents seem to build rational expectations on the inflation rate whereas 60% set their prices based on rule of thumbs. This result lies in the midst of the estimates presented by related studies. Accordingly the calibration results give further evidence that purely forward-looking Phillips curves do not fit the facts.
- The degree of forward lookingness in the IS equation is identified to be equal to 0.4. Hence only a modest degree of forward lookingness seems to be present in the data, which confirms earlier results by Fuhrer [2000]\(^{12}\). In other words a purely forward looking IS equation is not able to describe the optimal consumption plan of households. Consumption decisions seem to be mainly driven by rule of thumb behaviour and habit formation. Households centre their current and future-spending

\(^{12}\) Linde [2002] comes to the same conclusion: “I have not been able to find any estimates of \( \beta_f \) [degree of forward lookingness, the author] and \( \beta_b \) [degree of backward lookingness, the author], but the results in the literature Fuhrer [2000] suggests that \( \beta_f \) is consistently less that one and that \( \beta_b \) is positive”. 

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decisions around yesterday's consumption level or alternatively around some targeted level of consumption.

Table 4: Time Series Properties: Simulated and Actual Data: (1987:4-2002:1)

<table>
<thead>
<tr>
<th>RANK</th>
<th>LEVELS</th>
<th>ONE-QUARTER-CHANGES</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>STANDARD DEVIATION</td>
<td>AC(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Inflation Data</td>
<td>0.9794</td>
<td>0.649</td>
</tr>
<tr>
<td>1</td>
<td>1.4678</td>
<td>0.6325</td>
</tr>
<tr>
<td>2</td>
<td>1.4688</td>
<td>0.6323</td>
</tr>
<tr>
<td>3</td>
<td>1.4588</td>
<td>0.627</td>
</tr>
<tr>
<td>4</td>
<td>1.4485</td>
<td>0.6217</td>
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<tr>
<td>5</td>
<td>1.4377</td>
<td>0.6161</td>
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<tr>
<td>10</td>
<td>1.4668</td>
<td>0.67328</td>
</tr>
<tr>
<td>Output Gap Data</td>
<td>1.6953</td>
<td>0.945</td>
</tr>
<tr>
<td>1</td>
<td>1.6026</td>
<td>0.8256</td>
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<tr>
<td>2</td>
<td>1.613</td>
<td>0.8278</td>
</tr>
<tr>
<td>3</td>
<td>1.6085</td>
<td>0.8265</td>
</tr>
<tr>
<td>4</td>
<td>1.6038</td>
<td>0.8252</td>
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<tr>
<td>5</td>
<td>1.5988</td>
<td>0.8237</td>
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<tr>
<td>10</td>
<td>1.4906</td>
<td>0.78226</td>
</tr>
<tr>
<td>Federal Funds Rate Data</td>
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<td>0.930</td>
</tr>
<tr>
<td>1</td>
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<td>0.58</td>
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<tr>
<td>2</td>
<td>1.7262</td>
<td>0.58</td>
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<tr>
<td>3</td>
<td>1.7332</td>
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<tr>
<td>4</td>
<td>1.7411</td>
<td>0.58</td>
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<tr>
<td>5</td>
<td>1.7501</td>
<td>0.58</td>
</tr>
<tr>
<td>10</td>
<td>1.5113</td>
<td>0.58</td>
</tr>
</tbody>
</table>

As Table 4 indicates the identified vector $\psi_{10} = [1 \ 0.15 \ 1.85 \ 0.4 \ 0.4 \ 1]$ captures the correct signs of the autocorrelation functions over all relevant variables. Nevertheless the model has some problems in displaying the low variance in the inflation rate and the low variance in the first difference of the output gap.
5.2 Impulse Response Functions

The main characteristics of our identified baseline configuration are depicted in Figure 1. The high degree of interest rate smoothing and the lags in the Phillips-curve and the IS-equation translate into hump shaped impulse response functions that can be considered in line with conventional New Keynesian Macro Models (see Walsh [2003], chapter 11). We will shortly discuss each impulse response function in term.

Quite remarkably the impulse response function of the inflation rate with respect to an interest rate shock does not exhibit a prize puzzle Figure 1(b)\(^{13}\). Following an interest rate shock the impulse response function of the interest rate starts to decline and reaches its peak response after three quarters. Due to the drop in economic activity the inflation rate equally starts to decline and reaches its peak response with a lag of six quarters. After approximately 20 periods all series are back at their baseline values. Hence long run neutrality holds. The impulse response function nicely depicts the transmission structure encapsulated within this particular specification of a New Keynesian Macro Model. The peak response in the output gap leads the peak response in the inflation rate which can be explained by the backward-looking inflation dynamics in the hybrid Phillips curve. This reflects that the output gap is the driving variable of the inflation process within a hybrid specification and that monetary policy can only disinflate by deeds.

Given the identified parameter constellation \( \psi \), monetary policy largely accommodates supply shocks (see Figure 1(a)). The initial unit supply shock leads to a pronounced increase in the interest rate response, which goes hand in hand with a drop in the output gap induced by a tighter stance in monetary policy (peak response after 3 quarters). Consequently the inflation rate starts to decline and returns to its baseline after 13 quarters. The output gap exhibits a pronounced reaction, which reaches its peak response after 6 quarters.

Following a positive unit demand shock (see Figure 1(b)), monetary policy reacts by raising real interest rates (peak response after 3 quarters). Due to the stronger economic activity the inflation rate equally starts to rise. It reaches its peak response after 3 quarters. All depicted time series return to their baseline values around 13 quarters. This somewhat pronounced response compared to supply shocks might reflect that monetary policy only puts a modest weight on output gap stabilization (\( \lambda = 0.15 \)).

\(^{13}\) Note that we simulated the interest rate shock within our control theoretic framework by shocking the instrument itself. We did not model the shock as recently proposed by Söderlind et. al [2002].
Figure 1 Impulse Response Function of the Baseline Configuration

(a) Supply Shock

(b) Interest Rate Shock

(c) Demand Shock

Impulse Response Function for the baseline configuration: \( \psi = [1 \ 0.15 \ 1.85 \ 0.4 \ 0.4 \ 1] \).

Table 5: Peak Response in Impulse Response Patterns

<table>
<thead>
<tr>
<th>Shock</th>
<th>Peak Response</th>
<th>Back to Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inflation</td>
<td>Output gap</td>
</tr>
<tr>
<td>Supply</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Demand</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

For the identified baseline specification \( \psi = [1 \ 0.15 \ 1.85 \ 0.4 \ 0.4 \ 1] \).

5.3 Baseline Evaluation: Robustness of the Identified Solution

In the following section we will perform a battery of baseline evaluations to get a deeper understanding on the mechanics of the model. In particular we will take a look at the sensitivity of the variances, covariances and the implied autocorrelations and cross
correlations with respect to changes in the individual elements of the identified vector 
\[ \psi = \begin{bmatrix} 1 & \lambda & \nu & \mu_x & \mu_y & \mu_v \end{bmatrix}. \]

Figure 2 shows how the goal variables of monetary policy respond ceteris paribus to a change in the individual elements of \( \psi \). The results are largely in line with expectations. An increasing weight on the individual goal variables, hence the inflation rate, the output gap and the interest rate respectively lead to a drop in the variances of each of these variables. E.g. if monetary policy puts an increasing weight on interest rate smoothing (increasing \( \nu \)) the variance of the interest rate starts to decline. The same holds true for the other target variables of monetary policy. But of course reducing the variance of one goal variable is no free lunch. Let us assume that monetary policy puts a higher weight on stabilizing the inflation rate (increasing \( \phi \)). As side effect the variance of the interest rate increases. In other words the central bank needs to make a more rigorous use of its instrument in response to supply and demand shocks. This point is in particular obvious if we take a look at Figure 2(b). Figure 2(b) depicts what happens ceteris paribus if monetary policy has a greater concern for economic activity. As we see the variance of the output gap drops with an increasing \( \lambda \). Nevertheless this can only be realized at the cost of an increase in the variance of the inflation rate. This means in particular that central banker’s take a less vigorous stance on supply shocks thereby increasing the fluctuation in inflation. With respect to the degree of forward lookingness the following seems to hold true. An increasing degree of forward lookingness in the Phillips curve \( \mu_x \) and in the IS-curve \( \mu_y \) result in a sharp drop of the variance of the interest rate. Hence if we keep the preference vector corresponding to the period loss function \( L_y = \pi^2 + 0,15y^2 + 1,85\Delta\pi^2 \) fixed an increasing degree of forward-lookingness serves as a substitute for a more aggressive monetary policy stance. Therefore one might say that an increasing degree of forward lookingness implies that monetary policy does not need to “lean against strong persistence” in the data. Hence the results presented by purely backward looking models stating that estimated response coefficients in monetary policy rules are smaller than optimal coefficients retrieved by control methods my be spurious. In the light of the results shown in Figure 2 these studies might simply neglect the degree of forward-lookingness \( \mu_y = \mu_x = 0.4 \) present in the data.\footnote{Influential backward looking studies are for instance Ball [1997] or Rudebusch et al [1999].}
Figure 2: Variances with changing $\psi = [\phi \lambda \nu \mu_x \mu_y]^*$

(a) Standard Deviations with changing $\phi$

(b) Standard Deviations with changing $\lambda$

(c) Standard Deviations with changing $\nu$

(d) Standard Deviations with changing $\mu_x$

(e) Standard Deviations with changing $\mu_y$

*setting $\mu_x = 1$ dominated setting $\mu_x = 0$ in terms of the chosen criterion, therefore we kept $\mu_x = 1$ for all possible specifications.

Figure 3 evaluates the impact of changes in the identified vector $\psi = [1 \lambda \nu \mu_x \mu_y]$ on the autocorrelation patterns of the inflation rate. As one would expect an increasing weight on stabilizing the inflation rate around the inflation target leads to a drop in the persistence of the inflation process (see Figure 3(a)).
In other words if monetary policy uses its instrument more rigorously to keep the inflation rate close to the inflation target the degree of persistence in the inflation process declines. This underlines that the degree of persistence is endogenous to the monetary policy regime. Nevertheless the ‘beneficial’ reduction of persistence in one variable comes at a cost. E.g., an increasing weight on stabilizing the output gap around zero leads to an increase in the persistence of the inflation process. One likely explanation can be given as follows: As
monetary policy tends to react stronger to movements in the output gap it will tend to ‘overlook’ supply shocks leading to a higher degree of persistence in the inflation rate. An increasing weight on stabilizing the change in interest rates leads to an increase in the inflation persistence, which can be quite naturally explained by the fact that monetary policy uses its instrument less vigorously to keep the inflation rate on track. As expected an increasing degree of forward lookingness leads to a drop in the degree of persistence of the inflation rate, as shocks are more self-stabilizing if monetary policy is conducted according to a stable and unique policy rule. Note that in the limit with \( \mu_s \) converging to one, hence when we approximate the purely forward looking New Keynesian Phillips curve the inflation process converges towards white noise.

Figure 4 shows the sensitivity of the autocorrelations of the interest rate with respect to the individual elements in \( \psi \). Increasing weights on interest rate stabilization raises the persistence in interest rates as monetary policy uses its instrument more cautious and gradual (Figure 4(c)). Hence the interest rate reaction in response to shocks will be more sustained. This automatically leads to a higher degree of persistence. Varying weights on stabilizing the output gap do not have a significant impact on the autocorrelation structure (Figure 4(b)).

**Figure 4: Autocorrelations of the interest rate** \( \psi = [\phi \lambda v \mu_z \mu_f]^\ast \)
Figure 5 depicts some cross correlations inherently nested in the chosen baseline calibration $\psi$. Figure 5(a) depicts the cross correlation of the inflation rate $\pi_t$ with the lagged differences of the interest rate $\Delta i_t, \Delta i_{t-1}$ and $\Delta i_{t-2}$. An increasing degree of forward lookingness in the IS-curve strengthens the correlation between past changes in the interest rate and today’s inflation rate. This result can be interpreted as a faster ‘path-through’ effect running from interest rates to the inflation rate. Figure 5(b) depicts the cross correlation between the current output gap $y_t$ and past changes in the interest rate $\Delta i_t, \Delta i_{t-1}$ and $\Delta i_{t-2}$. It can be seen that an increasing degree of forward lookingness in the IS-curve weakens the link between changes in yesterdays interest rate and changes in the output gap. This reflects that relatively modest movements in the change of interest rates are sufficient to control the output gap. Interestingly for values of $\mu_y$ larger than 0.5 $\text{Corr}(y_t, \Delta i_{t-1})$ and $\text{Corr}(y_t, \Delta i_{t-2})$ become negative which seems to be at odds with both, intuition and the data. Hence given the preference vector $\psi = \begin{bmatrix} 1 & 0.15 & 1.85 & 0.4 & 0.4 \end{bmatrix}$ there is a restriction on the set of reasonable parameter constellations $\mu_y$. Values larger than approximately 0.6 do not seem to be realistic. Figure
5(d) depicts the cross correlations between the interest rate $i_t$ and the output gap $y_t$ and $y_{t-1}$.

The identified preference vector puts again restrictions on the set of reasonable parameter constellations. The model needs at least 40% of economic agents that are forward looking.

**Figure 5: Selected Cross correlations with changing** $\psi = [\phi \; \lambda \; \nu \; \mu_\pi \; \mu_y]$

* setting $\mu_\pi = 1$ dominated setting $\mu_\pi = 0$ in terms of the chosen criterion, therefore we kept $\mu_\pi = 1$ for all possible specifications.

An increasing degree of forward lookingness in the Phillips curve tightens the link between the interest rate and past movements of the output gap. Past movements in the output gap are faster transmitted into changes in the interest rate. Figure 5(e) reflects that an increasing weight on interest rate smoothing leads to a modest increase in cross correlations of the interest rate with past movements in the output gap. Figure 5(f) shows that an increasing weight on stabilizing the inflation rate around the inflation target loosens the link between
lagged inflation rates and the current interest rate $\text{Cor}(i_t; \pi_{t-1})$. An increasing degree of aggressiveness with which monetary policy reacts on inflation breaks the persistence in the inflation process which automatically weakens the link between past changes in the inflation rate and the current use of the interest rate.

6 Concluding Remarks

It has become standardized practice to evaluate the performance of alternative simple and optimal monetary policy rules based on the associated monetary policy outcomes. We take the point of view that this exercise necessarily rests on the apriority that key parameters of the New Keynesian Macro Model are correctly specified. Within this paper we proposed a calibration technique, which explicitly takes the variances, covariances, autocorrelations and cross correlations into account. Based on this technique we present evidence that around 60% of the pricing and consumption decisions are not made by optimising agents but by rule of thumb setters. This result is in line with earlier studies and underlines that purely forward-looking Phillips curves and IS-equations are unable to match the persistence present in the data. The finding that a majority of households and firms do not seem to optimise but base their decisions on heuristics may be a fruitful area for future research.

We have indicated at the point that some ‘conventional wisdom’ stating that estimated coefficients are smaller than those retrieved by means of optimal control may be spurious. The analysis of the level of variances present in the data as well as the evaluation of selected cross correlations clearly indicate that some degree of forward lookingness is necessary to fit the facts. If monetary policy opts for a stable and unique rule, the job of monetary policy makers is much easier at it would be in a purely backward looking system, due to the implied self stabilizing properties of forward looking systems grounded on peoples expectations on stabilizing monetary policy itself (self-fulfilling expectations). The evaluation of some selected crosscorrelations served as a useful benchmark to put restrictions on the degree of forward and backward lookingness in the data in the Phillips curve and the IS-equation. The identified preference vector of monetary policy indicates that the dominant goal of monetary policy is the stabilization of the inflation rate around the inflation target. Output gap stabilization as an independent goal of monetary policy only seems to play a minor role for the conduct of monetary policy.


Castelnuevo, Efrem (2003), Squeezing the Interest rate Smoothing Weight with a Hybrid New-Keynesian Model, mimeo.


Fuhrer, Jeff (2000), Habit Formation in Consumption and Its Implications for Monetary Policy, Working paper, Federal Reserve bank of San Francisco.


Goodhart, Charles (1996), Why Do the Monetary Authorities Smooth Interest Rates?.


Woodford, Michael (2002), Interest and Prices, Chapter 1.
Appendix

A.1 Tables

Table 1: Hybrid: Phillips Curves

<table>
<thead>
<tr>
<th>Study</th>
<th>Phillips-curve</th>
<th>Period</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Castelnuovo [2003]</td>
<td>[\pi_t = 0.1 \cdot E_{t-1} \pi_{t+3} + 0.9 \cdot [0.282 \cdot \pi_{t-1} - 0.025 \cdot \pi_{t-2} + 0.292 \cdot \pi_{t-3} + 0.385 \cdot \pi_{t-4}] + 0.141 \cdot y_{t-1}]</td>
<td>1987Q3-2001 Q1</td>
<td>Calibration based on a distance criterion</td>
</tr>
<tr>
<td>Linde [2002]</td>
<td>[\pi_t = 0.463 E_{t} \pi_{t+1} + 0.72 \pi_{t-1} + 0.032 y_t + \varepsilon_t]</td>
<td>1960Q1-1997Q4</td>
<td>FIML</td>
</tr>
<tr>
<td>Söderlind et al. [2002]</td>
<td>[\pi_t = 0.1 \cdot E_{t-1} \pi_{t+3} + 0.9 \cdot [0.67 \cdot \pi_{t-1} - 0.14 \cdot \pi_{t-2} + 0.4 \cdot \pi_{t-3} + 0.07 \cdot \pi_{t-4}] + 0.13 \cdot y_{t-1}]</td>
<td>1987Q4-1999Q4</td>
<td>Calibration, based on matching moments</td>
</tr>
<tr>
<td>Domenech et al. [2001]</td>
<td>[\pi_t = 0.537 E_{t} \pi_{t+1} + 0.463 \pi_{t-1} + 0.063 y_{t-1}]</td>
<td>1986Q1-2000Q4</td>
<td>GMM</td>
</tr>
<tr>
<td>Jondeau et al [2001]</td>
<td>[\pi_t = 0.747 E_{t} \pi_{t+1} + 0.462 \pi_{t-1} + 0.037 m_c t]</td>
<td>1996Q1</td>
<td>GMM</td>
</tr>
<tr>
<td>Gali et al. [2001]</td>
<td>[\pi_t = 0.364 E_{t} \pi_{t+1} + 0.599 \pi_{t-1} + 0.02 m_c t]</td>
<td>1996Q4</td>
<td>GMM</td>
</tr>
<tr>
<td>Rudd et al [2001]</td>
<td>[\pi_t = 0.605 E_{t} \pi_{t+1} + 0.393 \pi_{t-1} - 0.000 y_t]</td>
<td>1997Q4</td>
<td>GMM</td>
</tr>
<tr>
<td>Rudebusch [2000]</td>
<td>[\pi_t = 0.29 E_{t} \pi_{t+1} + 0.71 \pi_{t-1} + 0.13 y_t]</td>
<td>1996Q4</td>
<td>OLS</td>
</tr>
<tr>
<td>Gali et al. [1999]</td>
<td>[\pi_t = 0.682 E_{t} \pi_{t+1} + 0.252 \pi_{t-1} + 0.037 m_c t]</td>
<td>1994Q3</td>
<td>GMM</td>
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Table 2: Hybrid IS-Equations

<table>
<thead>
<tr>
<th>Study</th>
<th>Phillips-Curve</th>
<th>Period</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Castelnuovo [2003]</td>
<td>[y_t = 0.2 \cdot E_{t-1} y_{t+1} + 0.8 \cdot [1.229 \cdot y_{t-1} - 0.244 y_{t-2} - 0.073 (i_{t-1} - E_{t-1} \pi_{t+3})] + \eta_t]</td>
<td>1987Q3-2001 Q1</td>
<td>Calibration based on a distance criterion</td>
</tr>
<tr>
<td>Söderlind et al. [2002]</td>
<td>[y_t = 0.5 \cdot E_{t-1} y_{t+1} + 0.5 \cdot [1.15 \cdot y_{t-1} - 0.27 y_{t-2} - 0.09 (i_{t-1} - E_{t-1} \pi_{t+3})] + \eta_t]</td>
<td>1987Q4-1999Q4</td>
<td>Calibration based on matching moments</td>
</tr>
<tr>
<td>Domenech et al. [2001]</td>
<td>[y_t = 0.499 E_{t} y_{t+1} + 0.488 y_{t-1} + 0.047 y_{t-2} - 1.09 y_{t-3} + 0.161 y_{t-4} - 0.08181 r_{t-2} - 0.00819 r_{t-3}]</td>
<td>1986Q1-2000Q4</td>
<td>GMM</td>
</tr>
<tr>
<td>Study</td>
<td>Identified Loss Function</td>
<td>Period</td>
<td>Method</td>
</tr>
<tr>
<td>----------------------------</td>
<td>---------------------------------------------------------------</td>
<td>------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Castelnuovo [2003]</td>
<td>$L_t = \pi^2 + 0.5y^2 + 0.5\Delta i^2$</td>
<td>1987-2001</td>
<td>Calibration based on a distance criterion</td>
</tr>
<tr>
<td>Söderlind et al. [2002]</td>
<td>$L_t = \pi^2 + 0.1y^2 + 1.5\Delta i^2$</td>
<td>1987-1999</td>
<td>Calibration, based on matching moments</td>
</tr>
<tr>
<td>Dennis [2001]</td>
<td>$L_t = \pi^2 + 0.23y^2 + 12.3\Delta i^2$</td>
<td>1979-2000</td>
<td>FIML</td>
</tr>
<tr>
<td>Favero and Rovelli [2002]</td>
<td>$L_t = \pi^2 + 0.00125y^2 + 0.0085\Delta i^2$</td>
<td>1980-1998</td>
<td>GMM, Euler Equation</td>
</tr>
<tr>
<td>Cecchetti and Ehrmann [1999]</td>
<td>$L_t = \pi^2 + 0.25y^2$</td>
<td>1987-1999</td>
<td>Slope of the Aggregate Supply Relationship</td>
</tr>
</tbody>
</table>
A.2 The General Model Setup

Closely following Söderlind et al [2002] we can rewrite our basic equation in state space form as follows. In a first step we lead our model one period ahead and solve for the rational expectations variables $E_t\pi_{t+4}$ and $E_t\pi_{t+2}$ with the highest time index:

$$\frac{\mu_t}{4} E_t\pi_{t+4} = \left(1 - \frac{\mu_t}{4}\right) E_{t+1}\pi_{t+1} - \frac{\mu_t}{4} E_{t+2}\pi_{t+2} - \frac{\mu_t}{4} E_{t+3}\pi_{t+3} - \left(1 - \mu_t\right) \left[\alpha_2\pi_t + \alpha_3\pi_{t-1} + \alpha_4\pi_{t-3} + \alpha_4\pi_{t-4}\right] - \alpha_t\pi_t$$

(A1)

$$\mu_t E_t\pi_{t+2} + \frac{\beta_t\mu_t}{4} E_t\pi_{t+4} = E_{t+1}\pi_{t+1} - \left(1 - \mu_t\right) \left[\beta_t\pi_t + \beta_t\pi_{t-1}\right]$$

$$+ \beta_t\mu_t \left[i_t - \frac{1}{4}E_t\pi_{t+1} + \pi_{t+2} + \pi_{t+3}\right]$$

$$+ \frac{\beta_t}{4} \left[i_t + i_{t-1} + i_{t-2} + i_{t-3} - \pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right]$$

(A2)

Hence we can rewrite the general model in state space form as:

$$A_0 \begin{bmatrix} X_{t+1} \\ E_tX_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ E_tX_{t} \end{bmatrix} + B \begin{bmatrix} i_t \\ 0_{2\times1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{2\times1} \end{bmatrix}$$

(A3)

$$X_t = \begin{bmatrix} \pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t+1}, \pi_{t+2}, \pi_{t+3}, y_t, y_{t-1}, i_t, i_{t-1}, i_{t-2}, i_{t-3} \end{bmatrix}$$

$$X_{t+1} = \begin{bmatrix} E_t\pi_{t+1}, E_t\pi_{t+2}, E_t\pi_{t+3}, E_t\pi_{t+4}, E_t\pi_{t+4} \end{bmatrix}$$

$$\nu_t = \begin{bmatrix} \varepsilon_t, 0, 0, 0, \eta_t, 0, 0, 0, 0 \end{bmatrix}$$

Where $X_t$ is a 9×1 vector of predetermined state variables? $X_{t+1}$ is a 4×1 vector of forward looking variables and $\nu_t$ is a vector of shocks. Following Söderlind et al. (2002) we have made use of the fact that $\pi_{t+1} = E_t\pi_{t+1} + \varepsilon_{t+1}$ and that $y_{t+1} = E_ty_{t+1} + \eta_{t+1}$. 
$$A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}$$

$$A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$B_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$
A.3 The Linear Quadratic Control Problem

The starting point of the linear quadratic control problem is the following value function:

\[
V(X_t) = \min_{i} \left\{ \sum_{i=0}^{T} \beta^i \left( \frac{1}{2} X_i'Q_i X_i + i'R_i i' + \frac{1}{2} X_{i+1}'W_{i+1} X_{i+1} \right) \right\}
\]  

(A1)

Subject to the constraint:

\[
\begin{bmatrix}
X_{t+1} \\
E_t X_{2t+1}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
X_t \\
X_{2t}
\end{bmatrix} + \begin{bmatrix}
B_t \\
B_{2t}
\end{bmatrix} i_t + \begin{bmatrix}
v_{t+1} \\
0_{n+1}
\end{bmatrix}
\]  

(A2)

Premultiplying by \( A_0 \) yields the standard state space form: We get:

\[
\begin{bmatrix}
X_{t+1} \\
E_t X_{2t+1}
\end{bmatrix} = A \begin{bmatrix}
X_t \\
X_{2t}
\end{bmatrix} + B_i + v_{t+1}
\]  

(A3)

With \( A = A_0^{-1} A_t \) and \( B = A_0^{-1} B_t \). Given the specific structure of the matrix \( A_0 \) it holds that: \( A_0^{-1} v_{t+1} = v_{t+1} \). The variance covariance matrix will be given by:

\[
\Sigma_{v+1} = A_0^{-1} v_{t} \left( A_0^{-1} v_{t} \right)' = A_0^{-1} v_{t} v_{t}' A_0^{-1}
\]  

(A4)

Consequently it holds that the variance-covariance matrix stays a diagonal matrix with the following diagonal elements: \( \text{diag} \{ \sigma_x^2, 0, 0, \sigma_\eta^2, 0, 0, 0 \} \)

The value function has to satisfy in each period the following Bellman equation:

\[
V(X_t) = \min_{i} \left\{ X_t'Q_t X_t + i'R_i i' + \beta V(X_{t+1}) \right\}
\]  

(A5)

A cornerstone assumption in order to solve the model is to postulate a (linear) way according to which expectations are formed. We make the fundamental assumption that expectations are built as follows:
\[ E_t X_{2,t+1} = C_{t+1} E_t X_{1,t+1} \]  
(A6)

As every distinct policy rule is linked to a different \( C \) matrix the approach takes care of the well-known Lucas critique. The policy maker cannot take expectations as given when changing the policy rule. With this assumption at hand one can arrive at a value function, which is only expressed in terms of predetermined variables:

\[ V(t) = X_t^\prime Q_t X_t + r_t R_t r_t + \beta E_t(V_{t+1}) \]  
(A7)

Taking the F.O.C we arrive again at expressions for the optimal feedback rule as well as for the Riccati-matrix \( V \). Nevertheless contrasting the backward looking case our solution algorithm is quite different, as we do not only lack the matrix \( V \) but also the matrix \( C \). Therefore the algorithm functions as follows. With an initial guess for \( V_0 \) and \( C_0 \) at hand we can iterate on the respective matrix equation until some matrix norm \( \|C_{t+1} - C\| < \varepsilon \) and \( \|V_{t+1} - V_t\| < \varepsilon \) has converged.

The (converged) time invariant solution can be written as:

<table>
<thead>
<tr>
<th>TIME INVARIANT SOLUTIONS IN THE BACKWARD LOOKING MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ X_{1,t+1} = (A_{11} + A_{12} C - B_1 F) X_{1,t} ]</td>
</tr>
<tr>
<td>[ X_{2,t} = C X_{1,t} ]</td>
</tr>
<tr>
<td>[ F = - \left( R_t^* + \beta B_t^* V_{t+1} B_t^* \right)^{-1} \left( U_t^{<strong>} + \beta B_t^{</strong>} V_{t+1} A_t^* \right) X_{1,t} ]</td>
</tr>
</tbody>
</table>

The solution nicely depicts the expectational feedback, as the variable \( C \) does not only determine the forward looking variables \( X_{2,t} \) but also influences the predetermined variables \( X_{1,t} \).
A.4 Defining the Measurement Equation

Let us define a vector $Y_t$ as target variables in which the monetary policy maker is interested in. We assume that the goal variables are given by:

$$Y_t = \left[ \pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3} y_t y_{t-1} i_t i_{t-1} i_{t-2} i_{t-3} \Delta i_t \Delta i_{t-1} \Delta i_{t-2} \Delta \pi_t \Delta \pi_{t-1} \Delta \pi_{t-2} \Delta y \right] \quad (A.1)$$

We can define the target variables as a function of the state variables and the interest rate.

$$Y_t = \left( C_{X_1} C_{X_2} \right) X_t + C_i i_t$$ \quad (A.2)

$$C_{X_1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0,25 & 0,25 & 0,25 & 0,25 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (A.3)$$

$$C_{X_2} = 0_{18x4} \quad (A.4)$$

$$C_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^\top \quad (A.5)$$
A.5 The Individual Calibration Criterion were Specified as Follows

Criterion 1:

\[
\text{Crit1} = (1/4) \left[ \frac{\text{abs}(\text{stdv}(\pi) - \text{stdv}(\pi_{87-02}))}{\text{stdv}(\pi_{87-02})} + \frac{\text{abs}(\text{AC}(\pi) - \text{AC}(\pi_{87-02}))}{\text{AC}(\pi_{87-02})} + \frac{\text{abs}(\text{AC}(\pi) - \text{AC}(\pi_{87-02}))}{\text{AC}(\pi_{87-02})} \right]^2
\]

Criterion 2:

\[
\text{Crit2} = (1/2) \left[ \frac{\text{abs}(\text{stdv}(y) - \text{stdv}(y)_{87-02})}{\text{stdv}(y)_{87-02}} + \frac{\text{abs}(\text{AC}(y) - \text{AC}(y)_{87-02})}{\text{AC}(y)_{87-02}} \right]^2
\]

Criterion 3:

\[
\text{Crit3} = (1/4) \left[ \frac{\text{abs}(\text{stdv}(i) - \text{stdv}(i)_{87-02})}{\text{stdv}(i)_{87-02}} + \frac{\text{abs}(\text{AC}(i) - \text{AC}(i)_{87-02})}{\text{AC}(i)_{87-02}} + \frac{\text{abs}(\text{AC}(i) - \text{AC}(i)_{87-02})}{\text{AC}(i)_{87-02}} \right]^2
\]

Criterion 4:

\[
\text{Crit4} = (1/3) \left[ \frac{\text{abs}(\text{stdv}(d(\pi)) - \text{stdv}(d(\pi)_{87-02}))}{\text{stdv}(d(\pi)_{87-02})} + \frac{\text{abs}(\text{AC}(d(\pi)) - \text{AC}(d(\pi)_{87-02}))}{\text{AC}(d(\pi)_{87-02})} + \frac{\text{abs}(\text{AC}(d(\pi)) - \text{AC}(d(\pi)_{87-02}))}{\text{AC}(d(\pi)_{87-02})} \right]^2
\]

Criterion 5:

\[
\text{Crit5} = \left[ \frac{\text{abs}(\text{stdv}(d(y)) - \text{stdv}(d(y)_{87-02}))}{\text{stdv}(d(y)_{87-02})} \right]^2
\]

Criterion 6:

\[
\text{Crit6} = (1/3) \left[ \frac{\text{abs}(\text{stdv}(d(i)) - \text{stdv}(d(i)_{87-02}))}{\text{stdv}(d(i)_{87-02})} + \frac{\text{abs}(\text{AC}(d(i)) - \text{AC}(d(i)_{87-02}))}{\text{AC}(d(i)_{87-02})} + \frac{\text{abs}(\text{AC}(d(i)) - \text{AC}(d(i)_{87-02}))}{\text{AC}(d(i)_{87-02})} \right]^2
\]

We have proposed an equal weightening of all constructed six criteria.

The constructed criterion is given by:

\[
\text{Kriterium} = \sum_{i=1}^{6} \text{Crit}_i
\]
A.6 Stylised Time Series Properties: Levels and Differences

All data were taken from: http://research.stlouisfed.org/fred/