The productivity slowdown puzzle: Technological and non technological shocks in the labor market

Enrico Saltari* and Giuseppe Travaglini†

Abstract

In this paper we argue that the slowdown in labor productivity and not the low contribution of labor is at the root of the slowdown of the European economic growth in the decade 1995-2004. Using a simple dynamic model of the labor market, we show that the poor performance of the European economies can only be accounted for by a combination of two shocks: a negative supply shock to the labor demand due to the deceleration of technological progress and a positive supply shock resulting from the labor market reforms. We use a structural VAR model to estimate the contribution of these two shocks to the dynamics of employment and labor productivity. Our conclusions are that the technological shock is able to explain the decrease of the growth rate of labor productivity but not the increase in employment. In turn, the non technological shock can capture the dynamics of employment but not the slowdown of labor productivity. Thus, both shocks are necessary to provide a complete picture of the employment-productivity tradeoff in Europe during the last ten years.

Key words: Productivity slowdown, Labor market, SVAR

\textit{JEL} classification codes: E32, J60, E29

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1 Introduction

From the beginning of the 1990s to the first half of this new century European countries have been recording the worst economic performance since the end of the Second World War. This disappointing evolution has been the subject of increasing scrutiny over recent years. The main issue tackled in the analysis is the deterioration of the European productivity and the simultaneous rise in employment relative to the USA in the last decade.

Table 1 – containing some growth indicators for USA, the average of European countries (EU15) and Italy – illustrates what happened; specifically, it shows that since mid-1990s in EU15 labor productivity fell, while employment increased. This dynamics witnessed a reversal of the traditional roles of employment and productivity in contributing to European growth: for the first time since the 60s, the output growth in EU15 mainly stems from an increasing employment rather than productivity. Yet, this strong recovery of labour utilization was accompanied by a corresponding negative trend which emerged for labour productivity. In contrast, in USA labour productivity increased strongly.

To explain these differences in economic performance, the literature almost exclusively focuses on shocks to labour supply, such as changes in real-wage aspirations and labour market reforms. In this paper we show that models tilted towards supply-side factors are not able to explain differences between European countries and USA during 1990s.

We begin describing some “stylized facts” of the European and USA economies during the last decade and focus on three specific issues. How does the EU15 compare to the USA in terms of economy-wide productivity trends, and how big is the role of labour participation, capital accumulation and technological progress in the EU15 economy? Is labor participation or labor productivity responsible for EU15’s decline during the last ten years? Is the decline in productivity growth structural in nature or is it cyclical?

We first ask whether the supply-side approach is able to capture both the evidence discussed above and the stylized facts presented below. The answer is largely negative. Indeed, those models do not give enough importance to the driving force of the technological progress in affecting the labor demand position in the short and long run. In this paper we focus instead on shocks both to labour supply and to labour demand to explain the slowdown of European economies. Our analysis has three main objectives:

1. We argue that multiple sources of technological and non technological shocks match the short and long run characteristics of the variables, but that the slowdown in labor productivity, and not the low contribution of labor, explains mainly the sluggish performance of the European economy. We show that this negative productivity pattern is structural, depending strongly on the deceleration of the technological progress that encompasses European industries in almost every sectors.
Table 1
Decomposition of the GDP growth rate

<table>
<thead>
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<tr>
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</tr>
<tr>
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<td>3.8</td>
<td>2.3</td>
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<td>1.5</td>
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<tr>
<td>ITALY</td>
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<td></td>
<td></td>
</tr>
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<td>2.2</td>
<td>1.0</td>
<td>1.7</td>
</tr>
<tr>
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<td>0.1</td>
<td>0.3</td>
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<td>0.9</td>
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<td>Hours worked per worker</td>
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<tr>
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<td>0.6</td>
<td>-0.2</td>
<td>1.2</td>
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<tr>
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<td>3.5</td>
<td>1.9</td>
<td>1.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>

2. We show how a dynamic labor market model with labour augmenting technological progress can potentially explain the negative correlation between employment and productivity in the long run. Under plausible assumptions the model predicts that technological shocks generate a positive comovement between these two variables in the long run, partly offset by the negative comovement arising from non-technological shocks in the short run (such as labor market reforms).

3. We use a structural vector autoregressive (SVAR) model to exploit the different responses of labor productivity and employment to each type of shock, and as a result their conditional correlation. The SVAR model is used to identify the effects of both the shocks on productivity and employment. Our aim is to derive measures on the relative magnitude of the responses of variables to the shocks and their changes in the long run.

The main result of our analysis is that labor productivity responds positively to labor-demand shock in the long run, while supply-demand shock induces only short run move-
ments in productivity. The empirical responses of employment to labor demand shocks is weak and unable to explain the increase in employment. Thus, both shocks are necessary to provide a complete picture of the employment-productivity trade-off in Europe during the last ten years.

The next section describes the four stylized facts which in our view mainly characterized the European economic dynamics in the last decade. Section 2.2 gives a simple graphical illustration of the model used to interpret these facts. Section 3 contains some simple growth accounting exercises, applied to the ICT and non ICT sectors. Next, section 4 presents and solves the dynamic model of the labor market. The results of the theoretical model are used in section 5 to run the structural vector autoregressive analysis. The last section concludes.

2 Four stylized facts

Before going any further, it is worth looking at the next four figures. They show that from the mid-1990s European economies were characterized by four seemingly conflicting stylized facts. The first is well known: the EU15 economies have experienced important changes in labor market institutions over the course of the 1990s with an increase in the employment rate (figure 1).

However, during the same period the strong recovery in labor utilization was accompanied by a correspondingly negative trend in labor productivity. Figure 2 illustrates this second fact showing that EU15 and Italy labor productivity growth rates have been declining from the mid-1990s by a 1 percentage point while the USA’s has been accelerating by
a roughly similar amount.

Figure 2
Labour productivity (growth rate). Ratio between real GDP (1995 prices) and total employment (employees and self-employed)

![Figure 2](image_url)

The others two basic facts are less well known but equally important. First, the post 1995 deterioration in labor productivity is associated with an increase in the profit share. Nowadays, in Italy this is higher than in EU15 and USA. In contrast, profit share in USA remained quite stable (figure 3).

Figure 3
Profit share dynamics. Ratio between profits (ratio between national income and wages of employees) and national income

![Figure 3](image_url)

While it can be questioned whether this change in income distribution can be useful for new investment and growth, what is more puzzling is the significant fall in EU15 capital deepening growth during the same last ten years (figure 4): during the decade 1995-2004
in EU15 and Italy the growth rate of capital deepening has decreased, signaling that firms switched to more labor-intensive forms of production. As a result, a growing gap in terms of labor productivity, technology and capital deepening emerged over the second half of the 1990s in favor of the USA.

Figure 4
Capital deepening (growth rate)

2.1 Related literature

There are innumerable papers dealing with the European economic performance. For reasons quite obvious almost all the papers, with the exception of Gordon and Drew-Becker (2005), study the opposite issue of the European unemployment increase during the previous two decades (Layard, Nickell, and Jackman 1991; Phelps 1994; Blanchard 1997; Caballero and Hammour 1998; Bertola, Blau, and Kahn 2002) when Europe had a high productivity growth.

Most papers focus on the supply of labor, emphasizing the effects of non technological shocks on the level of (un)employment. Basically, they give three different kinds of explanations of the unemployment evolution. A part of the research focuses on the interactions between shocks and institutions. The central idea here is that the rigidity in labor market institutions amplifies the initial shock (interest rate, aggregate demand, input price) with a negative and persistent effect on the equilibrium level of unemployment. The influential paper by Blanchard and Wolfers (2000) is perhaps the most notable example of this approach. They show that while adverse shocks can explain much of the rise in unemployment, only the interactions between shocks and institutions can capture the heterogeneity of unemployment across the European countries from the 1960s to the beginning of the 1990s. In such a view, labor demand is irrelevant to the determination of the employment rate because productivity adjusts to a constant level in the long run. Others argue that the large movements in unemployment across the European and OECD
countries can be explained exclusively by changes in labour market institutions with its effects on the supply curve (Belot and Van Ours 2000, 2001; Nickell and Layard 1999; Nickell, Nunziata, Ochel, and Quintini 2002; Nickell 2003). This strand of analysis is well represented by Nickell, Nunziata, and Ochel (2005) which presents evidence where the evolution of unemployment from the 1960s to the 1990s is explained by changes in labor market institutions over the same period.

Another line of research focuses on the labor demand side. According to this view, an adverse technological shock hit the European economies shifting the labor demand curve along the supply curve, thus explaining how a productivity-unemployment trade-off can emerge in the long run. Many empirical studies have been conducted showing that the EU15’s productivity growth rate declined dramatically throughout the 1990s (Denis, K., and Roger 2004; Estevao 2004; Nardozzi 2004; Faini and Sapir 2005; O’Mahony and van Ark (2003)). This suggests that the slowdown in labor productivity, and not the participation rate, is able to account for the decline of the European economic growth of the last decade.

We now combine both types of shocks to explain the dynamics of employment and productivity. This is the subject of the next section.

2.2 A simple graphical analysis

A key hypothesis is at the center of our model. We argue that shifts of labor supply should be combined with shifts of labor demand to explain the negative employment-productivity correlation. We offer the following explanation. During the last ten years European economies were affected by a shift in labor supply. There is a wide consensus that this shift comes from rising flexibility and wage moderation in the labor market. The main consequence of these changes was a rise in profit share in the short run.

According to the traditional view, over time, firms should react to this wage moderation investing in labor intensive technologies, thus reducing capital deepening. Moreover, the rise in profits should also lead firms to increase capital accumulation leading eventually to an increase in employment and wages. This dynamic adjustments would imply that capital accumulation should eliminate the trade-off between employment and labor productivity eventually.

However, European economies were also hit in the same period by an adverse technological shock that shifted to the left the labor demand. We claim that the long run component of labor productivity depends on the technological progress, so that the current productivity slowdown mainly reflect adverse technological shocks. In such a scenario, the capacity of the economy to compensate the initial reduction of labor productivity growth in the long run is much bleaker.

Figure 5 illustrates the interaction between the technological and non technological
shocks. The long run equilibrium of the labor market moves from $E_0$ to $E_1$, where employment is higher but labor productivity has a permanent reduction. Labor supply position is affected by non technological shocks, whose effect is summarized by a parameter $\theta$. Consider a positive labor supply shock leading to a decrease in $\theta$. The labor supply shifts down along the demand curve. Thus, in the short run, labor productivity decreases and so does unemployment moving equilibrium to point $A$. However, as we will see below, this cannot be a long run equilibrium.

On the other hand, technological shocks impinge on labor demand. Consider a negative shift in technological progress. This will decrease the labor marginal product, shifting permanently the labor demand down.

Figure 5
The joint effect of a greater labor flexibility and a productivity slowdown

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1 Speaking of a “negative” technological shock is just for sake of exposition. Formally, in fact, the labor market graph can be read directly in terms of growth rates – renaming the variables on the axes and suitably reinterpreting supply and demand curves. Thus, the variable on the horizontal axis is the rate of change of employment $N$, that is $\dot{N}$, where the dot stands for the time derivative. Likewise, on the vertical axis there is the rate of change of the marginal productivity of labor. With a Cobb-Douglas production function, such as $AN^{1-\alpha}$, where $A$ the labor-augmenting technical progress and $1-\alpha$ is the income share of labor, the marginal productivity of labor is $A(1-\alpha)N^{-\alpha}$. In terms of rates of change, this latter becomes $\frac{\dot{A}}{A} - \alpha \frac{\dot{N}}{N}$: ie, the “labor demand” is negatively sloped and shifts downwards every time there is a technical progress slowdown. Similarly, “labor supply” can be expressed in terms of rates of changes – say $\frac{\dot{\theta}}{\theta} + \frac{\dot{N}}{N}$, where $\theta$ represents institutional factors (such as EPL) and the curve shifts downwards when labor flexibility increases.
We conclude that only a combination of technological and non technological shocks can give a complete picture of the employment and productivity correlation in EU15 during the period 1995-2004. Basically, our explanation stresses the idea that in the short run labor market reforms increase profits, making rational for firms to keep unchanged their investment strategy: they remained in the traditional and less productive sectors, avoiding to adopt technological innovations. The failure in reorientating economy toward the higher productivity sectors (such as ICT) explains the aggregate slowdown of both labor productivity and technological progress in the EU15.

This interpretation poses the analytical challenge to evaluate at the aggregate level the relative magnitude of the technological and non technological shocks for overall employment and productivity. In our empirical analysis we discriminate between these shocks evaluating how the permanent shifts in employment and productivity are caused by technological and non technological shocks.

3 Some growth accounting

The recent poor economic performance in Europe and in Italy is due to the declining performance of labor productivity which in turn is the result of a slowdown of TFP. To justify and clarify this claim, we look at the following data for USA, EU15 and Italy. Data are from Ameco database (Eurostat) and OECD.

Simple growth accounting analysis shows that in the last ten years the EU15 and Italy economies recorded a slowing in capital deepening (see table 2). Note that in the same period USA economy (almost) doubled the growth rate of capital deepening, rising labour productivity and TFP as well. From a EU15 perspective potentially the most concerning aspect is the negative evolution of the TFP. For the first time the TFP growth in USA is higher than in EU15. This significant slowdown affected negatively labour productivity and growth.

\footnote{This weak investment pattern suggests that European firms preferred a capital-saving strategy, as a result of the wage moderation and reforms in labour market during 1990s. As we will see, in the short and medium run the main consequence of this policy has been to increase profit share, reducing the incentive of firms to invest in the ICT sector.}
Table 2
Decomposition of the GDP growth rate

<table>
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<tbody>
<tr>
<td>USA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labour productivity (hourly)</td>
<td>1.9</td>
<td>1.6</td>
<td>1.5</td>
<td>1.5</td>
<td>2.3</td>
</tr>
<tr>
<td>TFP</td>
<td>1.2</td>
<td>1.1</td>
<td>0.6</td>
<td>0.7</td>
<td>1.2</td>
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<tr>
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<td>0.5</td>
<td>0.9</td>
<td>0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>EU15</td>
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</tr>
<tr>
<td>Labour productivity (hourly)</td>
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<td>3.8</td>
<td>2.3</td>
<td>2.4</td>
<td>1.5</td>
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<tr>
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<td>1.1</td>
<td>1.1</td>
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<td>2.4</td>
<td>1.2</td>
<td>1.3</td>
<td>0.8</td>
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<td>ITALY</td>
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<td></td>
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<tr>
<td>Labour productivity (hourly)</td>
<td>5.9</td>
<td>3.5</td>
<td>1.9</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
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</tr>
<tr>
<td>Capital deepening</td>
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<td>1.1</td>
<td>1.0</td>
<td>1.2</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Hence, the poor performance of the European economy is a consequence of the productivity slowdown, not of labor participation.

It can be shown, analyzing the trends (using the H-P filter) of labor productivity, capital deepening and TFP in Italy since the beginning of the 1980s, that the downward evolution of productivity and TFP trends are positively correlated, while capital deepening remains substantially constant. This implies that the slowdown of productivity is structural in nature, depending on the technological progress.

3.1 Sectorial growth accounting

In the previous section we have examined some of the aggregate comparative evidence across countries on the measurement of the growth rates of employment, labor productivity, technological progress, profit share and capital deepening. From those data we concluded that there is strong evidence that the adverse changes in TFP can explain the productivity slowdown in the European countries during 1990s. This section extends the previous results taking a closer look at sectorial/industry level productivity developments. Indeed, it is necessary to dig deeper and to lay out more explicitly the adverse factors explaining both the structure and the changes of the labor demand schedule in European countries during the last decade. So, we ask whether the EU15 has a structural productivity problem with respect to the USA economy.

Table 3 shows that in EU15 and in Italy the non ICT (information communication and
technologies) sector is broadly similar in size (measured in percentage of value added), but greater than the one of the ICT sector. In turn, in USA the ICT sector has a higher share of the total value added equal to 55%. Then, and more importantly, looking at the evolution of the sectorial investment in these three areas we observe that the USA economy changed dramatically its industrial structure during the last ten years, reducing the weight of the traditional sector and increasing the one of the ICT. This reorientation of its industrial structure in post 1995 allowed the recovery of USA economy.

Table 3
Value added shares in the ICT and non ICT sectors in 2002 (1995 prices)

<table>
<thead>
<tr>
<th></th>
<th>ICT</th>
<th>Non ICT</th>
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<tr>
<td>EU15</td>
<td>41</td>
<td>59</td>
</tr>
<tr>
<td>Italy</td>
<td>36</td>
<td>63</td>
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</table>

Finally, tables 4 5 shows a growth accounting exercise using the ICT-non ICT taxonomy. This decomposition confirms our conclusions drawn from the aggregate analysis presented in Table 1: USA economy achieved a significant turnaround in its labour productivity performance since the mid 1990s, whereas the EU15 and Italy productivity differential relative to US widened. The higher productivity performance of the US economy is mostly a reflection of the different productivity performance of these two sectors. In turn, this stems from the different TFP dynamics. In USA the TFP in the ICT sector grows from 1.5% of the period 1990-1994 to the 7.7% in the 1995-2002 period. Over the same period in the European countries, and especially in Italy, the growth of the TFP in the ICT sector is much weaker. The aggregate productivity performance derives from the faster restructuring of industry towards the ICT sector in USA relative to EU15 and Italy. In 2002, the ICT value added share in USA was 55%, in EU15 40% and 36% in Italy. In 1995 it was more or less equal to 66% in all the three areas.

4 The model

The link between the technological progress and the change of the demand for labor is a key element in our hypothesis that adverse technical shocks have played an important role in explaining the employment-productivity correlation during 90s. The model we present in this section is a version of Blanchard (1997). The main results of the model are: (1) a change in the technological progress permanently modifies the steady state;

3Data are from Stan database (OECD) and GGCD database.
(2) only a combination of a technological shock to labor demand and a non technological shock to labor supply is able to account the negative correlation between employment and productivity in European countries in the last decade.

Assume that every firm uses one unit of capital so that \( k = 1 \). As a result, when the single firm decides how much labor to use it also decides the corresponding labor-capital ratio. Formally, the demand for labor is the solution of a dynamic programming problem expressed by the following Bellman’s equation for the single firm

\[
\rho v(n) = \max_n \left[ An^{1-\alpha} - wn - \frac{c}{2} (\dot{n})^2 + \frac{d}{dt} v(n) \right]
\]  

(1)

In this equation \( v(n) \) is the value of the firm as a function of the labor-capital ratio, \( n \). As usual, the left-hand side of this equation represents the cost of financing the firm, since \( \rho \) is the user cost of capital. The right-hand side is the value produced by the firms. The first three terms are in fact the operating profits. In turn, these are given by the
Table 5
Growth accounting in the non ICT sector

<table>
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<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Labour productivity</td>
<td>1.2%</td>
<td>0.5%</td>
<td>-2.7%</td>
</tr>
<tr>
<td>Capital deepening contribution</td>
<td>0.2%</td>
<td>0.3%</td>
<td>0.2%</td>
</tr>
<tr>
<td>capital share</td>
<td>24%</td>
<td>24%</td>
<td>24%</td>
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<tr>
<td>capital intensity</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>TFP</td>
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<td>-2.9%</td>
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<tr>
<td>Sector value added share</td>
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<td>66%</td>
<td>56%</td>
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<td>EU15</td>
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<tr>
<td>Labour productivity</td>
<td>2.2%</td>
<td>2.2%</td>
<td>0.0%</td>
</tr>
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<td>Capital deepening contribution:</td>
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<td>0.4%</td>
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<tr>
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<td>capital intensity</td>
<td>3%</td>
<td>3%</td>
<td>1%</td>
</tr>
<tr>
<td>TFP</td>
<td>1.5%</td>
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<td>-0.4%</td>
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<td>Sector value added share</td>
<td>70%</td>
<td>68%</td>
<td>64%</td>
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</tr>
<tr>
<td>Labour productivity</td>
<td>3.1%</td>
<td>2.2%</td>
<td>0.0%</td>
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<tr>
<td>Capital deepening contribution</td>
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<td>0.9%</td>
<td>0.5%</td>
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<td>-0.5%</td>
</tr>
<tr>
<td>Sector value added share</td>
<td>67%</td>
<td>66%</td>
<td>66%</td>
</tr>
</tbody>
</table>

real output – assuming a a Cobb-Douglas production function, \( y = An^{1-\alpha} \), to get an analytical solution – less the real wages paid \( wn \) and the quadratic costs the firm has to bear to adjust employment, \( \frac{c}{2} \dot{n}^2 \).

The first-order and the envelope conditions imply:

\[
\begin{align*}
    c \dot{n} &= v_n \\
    \rho v_n &= A (1 - \alpha) n^{-\alpha} - w + v_{nn} (n) \dot{n}
\end{align*}
\]

Putting \( v_n = q \), the previous conditions become:

\[
\begin{align*}
    c \dot{n} &= v_n \\
    \rho v_n &= A (1 - \alpha) n^{-\alpha} - w + v_{nn} \dot{n}
\end{align*}
\] (2)

That is, the first equation says that the firm will increase employment up to the point where its marginal value \( q \) exceeds the cost of adjustment, \( c \dot{n} \). Second, the marginal cost
of financing the firm must be equal to the marginal value produced by the firm. Note that this is a system of differential equations with the usual characteristics.

Now, substitute the two differential equations (2) in the Bellman equation (1):

\[ v(n) = \frac{1}{\rho} \left\{ An^{1-\alpha} - wn + \frac{q^2}{2c} \right\} \quad (3) \]

At any instant \( t \) the firm decides if it is convenient to keep producing or exit the market by comparing its value \( v(n) \) with the cost of one unit of capital, \( p_k \). In turn, this depends on the aggregate demand for capital. This is because the unit cost of capital \( p_k \) includes, beyond the direct price normalized to 1, a (linear) adjustment cost with coefficient \( h \). That is,

\[ p_k = 1 + h\dot{K} \quad (4) \]

From the adjustment cost for net investment just specified and assuming free entry, we get:

\[ v(n) = 1 + h\dot{K} \quad (5) \]

from which we obtain the aggregate investment demand

\[ \dot{K} = \frac{1}{h} [v(n) - 1] \quad (6) \]

Finally, we have the labor market equations. Labor demand is simply the sum of the single firm individual demand:

\[ N = nK \]

Labor supply is a simple linear increasing function of real wage

\[ w = \theta N \quad (7) \]

where \( \theta \) is the parameter whose variations give account of institutional changes, and where we normalized to 1 the labor force. The labor market equilibrium is then:

\[ w = \theta nK \quad (8) \]

We suppose that the user cost of capital is an exogenous variable – fixed for instance on the international capital markets. Note that, by the hypothesis of constant returns to scale, this implies that the long run real wage is also fixed exogenously, say to the steady state level \( w^* \).
From the above description of the model economy, we have that a system of three differential equations characterizes the dynamics:

$$\begin{align*}
\dot{n} &= \frac{1}{c} q \\
\dot{q} &= \rho q - [A (1 - \alpha) n^{-\alpha} - w] \\
\dot{K} &= \frac{1}{n} (v - 1)
\end{align*}$$

(9a)

We first determine the steady state values of the variables. Using the fact that $$\dot{q} = q = 0$$ in the second equation of the system (9a), we get that the steady state real wage is equal to the marginal product of capital:

$$w^* = A (1 - \alpha) n^{\star^{-\alpha}}$$

(10)

Since in steady state the user cost of capital is equal to the marginal product of capital (which is what the Bellman equation states), we have

$$\rho = y - wn = \alpha An^{1-\alpha}$$

so that the steady state labor-capital ratio is:

$$n^* = \left( \frac{\rho}{aA} \right)^{\frac{1}{1-\alpha}}$$

(11)

It follows that in steady state real wage is

$$w^* = A^{\frac{1}{1-\alpha}} (1 - \alpha) \left( \frac{\rho}{a} \right)^{\frac{1}{1-\alpha}}$$

(12)

Substituting these values in the labor supply equation, the employment and capital steady state values are:

$$K^* = \frac{1}{\theta} A^{\frac{2}{1-\alpha}} (1 - \alpha) \left( \frac{\rho}{a} \right)^{-\frac{1 + \alpha}{1 - \alpha}}$$

$$N^* = \frac{1}{\theta} A^{\frac{2}{1-\alpha}} (1 - \alpha) \left( \frac{\rho}{a} \right)^{-\frac{-\alpha}{1 - \alpha}}$$

Using the steady state solutions to linearize the differential equations in the neighborhood of the steady state, it can be shown that in the system there are two state variables ($K$ and $N$) and one jump variable ($q$) (see Appendix A for details). That is, the dynamics of the system is characterized by a saddle path: there is only one path converging to the steady state. We can now examine what are the effects of the two shocks on the steady state.
4.1 The labor supply shock

To see what are the effects of a greater labor flexibility, suppose that the system is initially in steady state. Then, an employment shock reduces the value of \( \theta \) modifying the labour supply slope and bringing about a wage reduction. Given the capital stock, a lower wage increases the labor demand and the labor-capital intensity. These effects are shown in figure 6 when equilibrium moves from \( E \) to \( A \).

In the long run, however, the reduction in wage increases the firm value making attractive to enter the market. As a result capital and employment increase, the labor demand curve shifts out moving the economy in the new steady state \( E' \). Notice that in the long run this shock does not change neither the labor intensity nor the labor productivity (and the real wage), even if it does increase the use of both capital and labor.

The dynamics of the system main variables is described in figure 7. The parameters values used are: \( \rho = 0.15, \alpha = 0.3, h = 10, c = 4, A = 0.5 \). The initial value of \( \theta \) is \( \theta_0 = 0.385 \); for analytical simplicity, the initial value of the labor–capital ratio is \( n_0 = 1 \), so that \( K_0 = \frac{w_0}{\sigma_0 n_0} = 0.909 \), and employment is \( N_0 = n_0 K_0 = 0.909 \). We then suppose a reduction of \( \theta \) such that is reduced to \( \theta_1 = 0.35 \).
4.2 The labor demand shock

Now, let us consider the effects of a negative productivity shock, that is the effects of a lower value of $A$. These are shown in figure 8 where the reduction of $A$ moves the labor demand and changes the steady state equilibrium.

Figure 8
The effects of a technical progress slowdown
In the short run, a slowdown in the rate of technical progress decreases the profit rate, the wage and the labor intensity. The lower productivity leads the firm to invest less, thus modifying the steady state. Given that $\rho$ is unchanged, a lower productivity can be offset only by a lower wage in the long run. In turn, this also leads to an increase of the labor-capital ratio. Note that in the long run one of the main result of the lower productivity is to increase the unemployment. This conflicts with one of a stylized facts seen above, i.e. the employment increase. The slowdown of technical progress is thus a potential, even if incomplete, explanation of the European economic dynamics of the last decade.

The dynamics of the system main variables is described in figure 9. The parameters values used are now: $\rho = 0.15$, $\alpha = 0.3$, $h = 10$, $c = 4$, $\theta = 0.35$. The initial value of $A$ is $A_0 = 0.5$; we then have a negative shock to $A$ such that $A_1 = 0.49$.

Figure 9
The dynamic effects of a technical progress slowdown

4.3 A combination of the two shocks
In the last experiment we combine the two previous shocks, i.e. we examine the effects of a lower value of $\theta$ and a lower value of $A$. As can be seen from figure 10, in this case the steady state moves from $E$ to $E'$. Comparing the two steady state equilibria, we note that two effects arise: employment increases while labor productivity decreases. The increase in employment is originated by the greater labor flexibility; in turn, the wage
and productivity reductions are caused by the slowdown of technical progress. Second, since the user cost of capital is exogenously given, the lower capital productivity must be compensated by a decrease of capital accumulation, and thus by an increase in capital deepening.

The dynamics of the system main variables is described in figure 11. The parameters values used are now: $\rho = 0.15$, $\alpha = 0.3$, $h = 10$, $c = 4$, $\theta = 0.385$, $A = 0.5$. Then, both $A$ and $\theta$ are reduced to $A_1 = 0.48$ and $\theta_1 = 0.35$. 
In summary, two lessons can be drawn from this last simulation. Firstly, technological shock can potentially explain the slowdown in labor productivity since the early 1990s. Secondly, we need two separate shocks to capture the joint behavior of employment and productivity. Technological shocks, shifting permanently the labor demand, can capture the evolution of labor productivity; non technological shocks, shifting labor supply, rises the level of employment in equilibrium.

5 The structural VAR

Now, the next natural step is to quantify the relative importance of the separate shocks in explaining the actual employment-productivity trade-off. Our theoretical model shows that only a combination of shocks is able to explain the “four stylized” facts seen above. But, the problem is that these shocks are expected to produce opposite movements in productivity and employment: labor demand responses to technological shocks imply a procyclical relation between productivity and employment; while, labor supply responses to non technological shocks predict countercyclical productivity. Thus, to identify the nature (and the size) of these changes we have to measure the response of both labor demand and supply to each shock separately.

To this end, we employ the structural VAR methodology (SVAR) suggested by Blanchard and Quah (1989). We impose two identifying restrictions to decompose both labor
demand and supply into technological and non technological shocks. From the theoretical model we get the following two restrictions:

1. technological shocks have short and long run effects on labor productivity and employment;
2. non technological shocks have short and long run effects on employment, but do not affect labor productivity in the long run;

Further, in the empirical specification an additional identifying restriction is imposed: we include the aggregate demand to capture the effects of its variation on the short-run comovement of employment and productivity.

Basically, we find that labor productivity responds positively to technological shocks in the short and long run, while it is unaffected by the non technological shocks in the long run. This evidence is consistent with our theoretical model. In addition, the employment response to both shocks is positive in the long run. This latter outcome casts doubt on growth models where changes in technological progress do not affect employment in steady state.

To decompose labor demand and supply into three orthogonal shocks we impose restrictions on the long run multipliers of the estimated vector autoregressive model (VAR). These assumptions allow us to identify the SVAR. The estimated VAR in the growth rates is:

\[ X_t = \Pi X_{t-1} + u_t, \quad t = 1, \ldots, T \]

with \( X_t = [\Delta p_t, \Delta l_t, \Delta d_t]' \), where \( p_t = \log(y_t) - \log(l_t) \) is the log of productivity, \( l_t = \log(l_t) \) is the log of employment, and \( d_t = \log(da_t) \) is the log of aggregate demand. The stationary tests (not reported here) show that it is possible a moving-average representation of the original VAR. Since the residuals of the original VAR are, in general, contemporaneously correlated, we cannot interpret them as structural shocks. But, they can be orthogonalized by imposing restrictions on the long run multipliers of the system. Using the assumption that only technological shocks have a permanent effect on labor productivity, and the orthogonality between shocks, one can recover the estimates of the dynamic responses to a structural shock, as well as the components of the variation in each time series associated with those shocks and, as a residual, the value of the components driven by the remaining two non technological shocks. The alternative representation of \( X_t \) can thus be written as:

\[ X_t = C(L)\epsilon_t \]

where, the contemporaneous effect of \( \epsilon \) on \( X \) is given by \( C(0) \), while \( C(L) \) gives the
subsequent lag effects. Assuming that the structural shocks $\epsilon$ are ordered labor demand, labor supply, and aggregate demand our long-run restrictions imply:

$$
\begin{bmatrix}
\Delta p_t \\
\Delta l_t \\
\Delta d_t
\end{bmatrix} =
\begin{bmatrix}
C_{11}(L) & 0 & 0 \\
C_{21}(L) & C_{22}(L) & 0 \\
C_{31}(L) & C_{32}(L) & C_{33}(L)
\end{bmatrix}
\begin{bmatrix}
\epsilon_p \\
\epsilon_l \\
\epsilon_d
\end{bmatrix} = C(L)\epsilon_t
$$

(13)

The empirical model interprets the observed variations in productivity, employment and aggregate demand as originating in three types of orthogonal shocks, whose impact is propagated over time through various unspecified mechanisms. The orthogonality assumption implies that $E\epsilon\epsilon' = I$, and for convenience, the structural shocks are normalized so that $var(\epsilon_p) = 1, var(\epsilon_l) = 1, var(\epsilon_d) = 1$.

Matrix (13) imposes that the secular component in productivity originates in the technological shocks, and the coefficient $C_{11}(L)$ identifies the long run multiplier of this initial shock. In turn, the zeros in the matrix imply that both employment and demand shocks do not affect productivity in the long run (however, they may well have short and medium run effects on it), but that the secular component of employment depends on both technological and non technological shocks.

In order to use the SVAR technique, variables must be in a stationary form. In the preliminary analysis of data, this condition is motivated by the outcomes of the Levin, Lin Chu, and Breitung for common unit roots, and by the ADF and PP tests for individual unit root process. All these tests (not reported here) do reject the null of unit root when applied to the growth rates of variables (at the 5% significance level). Nonetheless, these tests cannot discriminate between stationary process fluctuating around a shifting trend, and differenced stationary process. Thus, in our empirical analysis we estimate several specifications with alternative generating process.

---

4In the matrix $C(L)$, we find the polynomials $C_{ij}(L)$ with individual coefficients denoted by $c_{ij}(k)$. The innovations $\epsilon_p$, $\epsilon_l$ and $\epsilon_d$ represent the structural shocks of productivity, employment and aggregate demand on labor supply and demand. The restriction $\sum_{k=0}^{\infty} c_{ij}(k) = C_{ij}(L) = 0$ implies that in the long-run a specific structural shock has no effect on the level of variables.

5As Blanchard and Quah (1989) clarify “this orthogonality assumption does not eliminate for example the possibility that supply disturbances directly affect aggregate demand. Put another way the assumption that the two disturbances are uncorrelated does not restrict the channels through which demand and supply disturbances affect output and unemployment.” (p.659) We can use the same assumption in our context: we agree that non technological shocks can have long run effects on productivity. However, we also believe that if so, those effects are small in the long run compared to those of the technological shocks. In the long run, labor productivity growth depends on technological progress (as it is in the neoclassical growth model, and how confirm the “classical” styled facts of Kaldor). Thus, our identifying scheme accepts this stylized fact.
5.1 Empirical results: evidence from a trivariate model

The trivariate model (13) is estimated using annual data covering the period 1960-2004. The employment rate is the first difference of the (log) of total employment of the economy. Productivity rate is constructed by subtracting the log of employment from the log of real GDP.

Time series for EU15 and Italy exhibit an apparent time trend, and there is a slowdown in both real growth productivity and aggregate demand beginning in the mid-1990s. Since there is no obvious way to address these difficult issues we estimate different VARs. They also include two dummies allowing for the change in the rate of growth of employment and output in 1974 and 1993 for EU15, and in 1975 and 1993 for Italy.

Three different empirical models for the variables in VAR are run to test the robustness of the results. In the basic estimation presented in the paper we use variables expressed in growth rates, with a linear trend, a constant, and two dummies. For the remaining two specifications we remove means and trends at the outset. In one case we employ a linear trend, in the second one we take the acceleration rate of the variables. The VARs contain one or two lags of each variable. Based upon these specification we find the residuals to be white noise.

The qualitative results are similar across the alternative treatments of deterministic components, breaks and lags. So, we report here only the results of the basic specification. The impulse response functions for the VARs are quite similar. The only significant difference appears in the short run response of the employment rate to the initial technological shock: with a positive technological shock, for some specifications it is initially negative (short run) to become positive with a permanent effect in the long run. This initial conditional negative correlation between technological shock and employment response can be the consequence of nominal and real rigidities in the labor market: when technological progress rises (decreases) employment does not rise (decrease) enough in the initial periods to maintain labor market in equilibrium. However, these rigidities disappear in the long run, and the adjustment process assures a stable increase in employment. 6

In order to understand the source of the relationship between employment and productivity, we look at the estimated dynamic responses of productivity, employment and aggregate demand to each type of shock.

Labor demand shifts Figure (12) displays the impulse response function (IRF) of labor productivity and employment to a positive structural technological shock of a size

---

6 For similar consideration on both the optimal lag structure of the VAR model and the short run variations of the impulse response function see for example Blanchard and Quah (1989), Galí (1999), Galí (2004).

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equal to one standard deviation. This shock has a permanent cumulative effect on productivity and employment. We interpret the response to this shock as a shift of the labor demand curve along the supply curve. For EU15 and Italy the initial positive (negative) technological shock have a persistent positive (negative) effect on productivity and employment. Interestingly, it has in the medium and long run a permanent effect on productivity and employment. However, this comovement does not explain entirely the evolution of employment: the order of magnitude of the employment response is only less then one tenth of the size of the productivity variation in the period. Thus, technological shocks appear as the main source of the productivity variation, while it does not capture the actual changes in employment. This is fully consistent with the growth accounting results given in table 2 of a decline in TFP growth of the order of 0.5 of a percentage point in EU15 and of 0.8 in Italy, with technological progress considered to be the structural component of the productivity trend.

**Labor supply shifts** We interpret the non technological shock as a shift of the labor supply curve induced by labor market reforms. It has a permanent effect only on employment. The time paths of these institutional shocks are hump-shaped. In the short run, the impulse responses are mirror images of each other; initially, employment increases and productivity decreases. For Italy, the effects peak after 3 periods; afterwards, productivity converges to its original level while employment adjusts to the new steady state. The effect on productivity is however negative in the short and medium run – as it is in the theoretical model – signaling that an higher employment level, given technology and capital stock, is initially associated with a decrease in labor quality.

It is worth noting that, while the effect on productivity vanishes over time (by as-
Assumption), the non technological shock in labor market has a permanent impact on employment. In the long run the size is similar across European countries: a shock leads to a permanent increase in the level of employment of about 1.2%. More precisely, after ten periods (years) the institutional shocks explain respectively 76 and the 64 percent in EU15 and Italy of the total response of the growth rate of employment to the three different structural shocks. This implies that the huge preponderance of the trend in employment is due to institutional shocks.

**Aggregate demand shifts** Finally, the aggregate demand shock has only transitory effects on both productivity and employment. It is initially associated with a positive productivity effect. This is plausible since an expansion of aggregate demand rises the level of GDP and of capital utilization in the short run. But, as this initial impact fades away, productivity and employment returns to its original level, and the effect on productivity and employment vanishes over time.

### 5.1.1 Variance decomposition

We now use the forecast-error variance decomposition to quantify how much of the alternative shocks account for most of the variation in productivity and employment.

To run this test, we must preliminary evaluate the forecast-errors of the variable – for example of productivity – at different time horizons as the difference between the actual value of any variable and its forecast from the VAR model. Then, we can calculate at any time \( t \) the percentage value of the variance of forecast error due to variation in productivity,
Figure 14
Cumulated IRF of a demand shock on productivity and employment. European countries

employment and aggregate demand. Table 6 gives this variance decomposition for horizon 1, 5, 15. We find that the alternative methods of treating the slowdown in productivity growth, and the trend in employment do not affect the variance decomposition.

Table 6
Variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>Labour productivity</th>
<th></th>
<th>Employment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>periods</td>
<td>productivity</td>
<td>employment</td>
<td>demand</td>
</tr>
<tr>
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<td></td>
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</tr>
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<td></td>
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<td>1</td>
</tr>
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<td>75</td>
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<td>16</td>
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</tr>
<tr>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>71</td>
<td>1</td>
<td>28</td>
</tr>
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<td>83</td>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

Three main conclusions emerge from this table.
First, the technological shocks explain the main part of labour productivity variation (from about 65 to 85 percent in the two areas at all horizons). In turn, employment disturbances capture only a small part of productivity fluctuations: for Italy and EU15 the relative contribution of employment disturbances to productivity fluctuation, at a 1
horizon, is respectively equal to 35% and 1%. For EU15, this contribution reduces a bit in the long run (horizon 15) and aggregate demand have a residual role in explaining the variation of productivity. Interestingly, for Italy aggregate demand shocks accounts for 28 – 14 percent of the error variance for productivity. However, when trends and shifts are removed at the outset aggregate demand shocks have a little effect on the explained movements in employment, 5 – 8 percent.

All in all, we interpret these results as suggesting a negligible importance for non technological shocks in productivity fluctuations. This empirical evidence is coherent with the theoretical model and with the impulse response functions.

Similarly, estimates of the relative contributions of the different disturbances to employment fluctuation show that employment disturbances explain the large part of the employment variation (they vary from 64 to 94 percent in the two areas at all horizons), with a small role reserved to productivity, again confirming the results of the theoretical model.

Hence, only the contemporaneous shift of the demand and supply of labor can appropriately explain the employment-productivity trade off occurred in European countries during the decade 1995-2004. The contribution of the technological shock explains the main part of the slowdown in productivity growth, while it is unable to explain the evolution of employment. A similar interpretation holds for the change of employment which, generally, at longer horizon reflects variations in non technological shocks.

6 Conclusions

Over recent years, in European countries employment and productivity growth patterns have diverged sharply. These two facts, together with the increase of profits and the slowdown in capital deepening characterized the sluggish growth of the European economy in the last ten years. These facts are at least seemingly contradictory.

This paper has asked whether this evolution is consistent with a combination of shifts of labor supply and demand curves. The answer is a qualified yes. The sluggish European economic dynamics has been largely driven by shocks that did have permanent effects on labor productivity and employment. To the extent that technological shocks can account largely for the slowdown in productivity, this result seems to provide a picture of European economy in the last decade that is in stark contrast with the one based on the labor supply approach. The slowdown in productivity is structural, and the higher flexibility in labor market induced further deceleration in both capital accumulation and technological progress because firms find optimal to invest in capital saving technologies.

While there may well be alternative interpretations of the joint behavior of productivity and employment, we interpret their correlated process as resulting from the dynamic
responses of labor demand and supply to technological and non technological shocks through the channels characterized in the following way. During the decade 1995-2004 the increasing profits of European firms did not sustain accumulation, failing in reorientating investment towards industrial sectors with higher productivity growths. European firms, and specially Italian ones, preferred to remain in the traditional sector, investing in capital saving technologies with low productivity. Thus, the recent poor performance of European economy depends directly on the slowdown of labor productivity and not on labor participation. The paper documented and qualified this proposition.

We argued, however, that both technological shocks and non technological shocks are necessary to explain the dynamics of European economy in the last decade. Technological shocks are able to explain the structural decrease of the growth rate of labor productivity, but they cannot explain the increase in employment. In turn, the non technological shocks can capture the dynamics of employment, but they cannot explain the slowdown of labor productivity. Further, we found that the most of the variation in productivity is due to labor demand shift; while, the employment response to supply shift is positive and significantly different from zero in the long run.

Almost none of the literature on the European productivity slowdown relates it to the slide down of the labor demand curve. But the responses of productivity and employment are difficult to reconcile with theories based only on labor supply shift. We believe that the hypotheses employed in the paper are a plausible set of assumptions to use. The empirical results are consistent with our theoretical model and, needless to say, the policy to remedy to this situation appears much more complex than the ones drawn from the traditional labor supply explanation.

References


Appendix. Stability analysis

In this appendix we conduct the stability analysis of the dynamic part of the model in the main text to show that the steady state of the system is a saddle point so that there is a unique path converging to equilibrium.

For convenience of the reader, we here repeat the differential equations characterizing the economy

\[
\begin{align*}
\dot{n} &= \frac{1}{c} \bar{q} \\
\dot{q} &= \rho \bar{q} - [A (1 - \alpha) n^{-\alpha} - w] \\
\dot{K} &= \frac{1}{\rho} (v - 1)
\end{align*}
\]  
(A.1)

and the steady state values of the variables

\[
\begin{align*}
q^* &= 0 \\
K^* &= \frac{1}{\theta} A^{\frac{2}{1-\alpha}} (1 - \alpha) \left( \frac{\rho}{\alpha} \right) \frac{1 - \alpha}{1 - \alpha} \\
N^* &= \frac{1}{\theta} A^{\frac{1}{1-\alpha}} (1 - \alpha) \left( \frac{\rho}{\alpha} \right) \frac{-\alpha}{1 - \alpha}
\end{align*}
\]  
(A.2)

Using the steady state values to linearize the system (A.1), we get:

\[
\begin{align*}
\begin{cases}
\dot{\bar{n}} &= \frac{1}{c} \bar{q} \\
\dot{\bar{q}} &= \rho \bar{q} - \left[ d - g\bar{n} - \theta (n^* K^* + n^* \bar{K} + K^* \bar{n}) \right] \\
\dot{\bar{K}} &= \frac{1}{\rho} \left\{ \frac{1}{\rho} [l + d\bar{n} - \theta (n^2 \bar{K} + 2K^* n^* \bar{n} + (n^* K^*)^2)] - 1 \right\}
\end{cases}
\end{align*}
\]

where the variables with bar indicate deviations from the steady state values – for instance, \( \bar{n} = n - n^* \) – and \( d \) is the labor marginal product in steady state, \( d = A (1 - \alpha) n^{*-\alpha} \), \( g \) its derivative in absolute value, \( g = Aa (1 - \alpha) n^{*-\alpha-1} \), \( l \) the output in steady state, \( l = An^{*1-\alpha} \).

We now look at the homogenous part of the differential equation system to analyze its stability.

\[
\begin{pmatrix}
\dot{\bar{n}} \\
\dot{\bar{q}} \\
\dot{\bar{K}}
\end{pmatrix} =
\begin{pmatrix}
0 & 1/c \rho & 0 \\
g + \theta K* & \theta n* & 0 \\
-\theta n* K* & 0 & \theta n*^2
\end{pmatrix}
\begin{pmatrix}
\bar{n} \\
\bar{q} \\
\bar{K}
\end{pmatrix}
= M
\begin{pmatrix}
\bar{n} \\
\bar{q} \\
\bar{K}
\end{pmatrix}
\]

31
where account has been taken of the fact that in equilibrium the demand wage is equal to the supply wage, so that $d\bar{n} = \theta K^*n^*\bar{n}$.

From the previous system we get the characteristic equation

$$M - \lambda I = 0 \implies -\lambda^3 + \lambda^2 \left( \rho - \frac{\theta (n^*)^2}{h\rho} \right) + \lambda \left( \frac{g}{c} + \frac{\theta K^*}{c} + \frac{\theta n^*}{h} \right) + \frac{g\theta n^*}{ch\rho} = 0$$

Since the sequence of the signs is in any case $-?+\ldots$, it is straightforward to conclude that there are two negative roots and a positive one. Thus, the equilibrium is a saddle: that is, there is only one path to the steady state.\(^7\) Intuitively, this conclusion derives from the presence of two state variables ($n$ and $K$), whose number must equal the number of negative roots, and one jump variable, which is the marginal value of the firm $q$ (Gandolfo (1997), pp.401-3).

We now eliminate the positive root to position the system on the saddle path. If we denote the two negative roots with $\lambda_1$ and $\lambda_2$ and the eigenvectors with con $v$ and $w$, we can write the solution of the system (A.1) as

$$n_t = \gamma v_1 \exp(\lambda_1 t) + \beta w_1 \exp(\lambda_2 t) + n^*$$

$$K_t = \gamma v_2 \exp(\lambda_1 t) + \beta w_2 \exp(\lambda_2 t) + K^*$$

$$q_t = \gamma v_3 \exp(\lambda_1 t) + \beta w_3 \exp(\lambda_2 t) + q^*$$

where $\gamma$ and $\beta$ are determined by the initial conditions of the state variables:

$$n_0 = \gamma v_1 + \beta w_1 + n^*$$

$$K_0 = \gamma v_2 + \beta w_2 + K^*$$

In the main text we use this solution to simulate the model.

\(^7\)It can be shown that this is true even when there are complex roots, so that we cannot apply Cartesio rule.