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A bayesian estimation of a DSGE model with financial frictions  

Abstract  

Episodes of crises that have recently plagued many emerging market economies have lead to a wide-spread questioning of the two traditional generations of models of currency crises. Distressed banking system and adverse credit-markets conditions have been pointed as sources of serious macroeconomics contractions, so introducing these imperfections into standard economic models can help to explain the more recent crises.  

This effort introduces financial frictions à la Bernanke Gertler and Gilchrist in a two-sector small open economy. We analyze the impulse response functions to various structural shocks.  

The model is estimated on simulated data applying both Bayesian techniques and maximum likelihood method and comparing the results under the two different estimation procedures. First, we will analyze the influence of the prior on the estimation outcomes. Second, we will test the sensitivity of estimation outcomes to the sample size, showing how, for large samples, results under Bayesian estimation converges asymptotically to those obtained applying maximum likelihood.  

A further extension would be to perform the estimation on historical data for a “less” developed economy.  

Keywords: DSGE models, Bayesian estimation, financial accelerator  
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1 Introduction

This paper estimates a a two-sector small open economy model with financial accelerator mechanism, using both Maximum Likelihood and Bayesian methods. We develop a modeling framework in order to evaluate the role of financial frictions.

Recently, a line of research has proceeded to analyze micro-founded macroeconometric models that incorporate an expanded set of nominal and real rigidities and hence can be matched more closely to observed aggregate data. For example, Christiano, Eichenbaum, and Evans (2005) specified a dynamic general equilibrium model with a number of distinct structural features: staggered wage and price setting with partial indexation; habit persistence in consumption; endogenous capital accumulation with higher-order adjustment costs; and variable capacity utilization. Altig, Christiano, Eichenbaum, and Linde (2004) have extended the model to incorporate firm-specific capital accumulation, while Christiano, Motto and Rostagno (2004) have incorporated a banking system and capital market frictions in their study of the Great Depression.

The presence of financial market imperfections in capital inflows to emerging markets has received widespread attention in the last few years. An important theme in this literature is the moral hazard problems with financing investment in emerging markets. We focus on the financial accelerator mechanism developed by Bernanke, Gertler and Gilchrist (1998, hereafter BGG) in which information asymmetries between lenders and entrepreneurs introduce inefficiencies in financial markets, affecting the supply of credit and amplifying business cycles. The argument is that during booms (recessions), an increase (fall) in borrowers’ net worth decreases (increases) the borrowers’ cost of obtaining external funds and then stimulates (destimulates) investments amplifying the effects of the initial shock. This approach has become widely spread in the literature and many studies have introduced these types of frictions in SDGE models (Christiano, Motto and Rostagno, 2004; Aghion, Bacchetta and Banerjee, 2001; Gertler, Gilchrist and Natalucci, 2003; Devereux and Lane, 2001).

In particular, as emphasized by Aghion, Bacchetta and Banerjee (2001), emerging market borrowers may find that interest rate and exchange rate fluctuations have large effects on their real net worth position, and so, through balance sheet constraints affecting investment spending, have much more serious macroeconomic consequences than for richer industrial economies.

This credit-based approach better explains some empiric evidences observed in recent crises. First, countries with a not-well developed financial sector are more likely to experience an output fall during a crisis. Second, countries where most of firms’ debt is denominated in foreign currency are more likely to go into a crisis, because exchange rate fluctuations have a strong impact on the indebtedness and so the net worth position of firms. Obviously, fiscal variables still play an important role at least in facilitating the occurrence of a crisis. However in contrast with the first and the second generation models of currency crises, a deterioration of fiscal balances will lead to a crisis mainly through its impact on private firms’ balance sheet rather than
through money demand adjustment.

Despite the amount of theoretical papers using the financial accelerator framework, not so much has been done when it comes to the econometric estimation of these models. Smets and Wouters (2003) have applied full-information Bayesian methods to estimate a micro-founded macroeconometric model with rigidities and found that the model is competitive with an unrestricted Bayesian VAR in terms of goodness-of-fit and out-of-sample forecasting performance.

In this paper we extend the standard BGG model and we estimate it using both Bayesian and Maximum Likelihood methods. First, our aim is to determine if a model with financial frictions delivers a better estimation than a model without such frictions. Second, our purpose is to compare outcomes under two different estimation techniques. The benefits of using Bayesian methods is that we can include prior information about the parameters, especially information about structural parameters from microeconomic studies. Another advantage is related to the fact that some parameters have a specific economic interpretation and a bounded domain, which can be incorporated in the priors.

This paper is organized as follows: section 2 describes the model; in section 3 we discuss the estimation methodology. In section 4 we present the main results both for model with financial accelerator and for model without financial accelerator. First, we will analyze the influence of the prior on the estimation outcomes. Second, we will test the sensitivity of estimation outcomes to the sample size, showing how, for large samples, results under bayesian estimation converges asymptotically to those obtained applying maximum likelihood. Section 5 reviews some of the main conclusions. A technical note on convergence tests is reported in appendix.

2 Model Presentation

The structure is a standard two-sector "dependent economy" model. Two goods are produced: a domestic non-traded good, and an export good, the price of which is fixed on world markets.

Three central aspects are highlighted: a) the existence of nominal rigidities; b) the presence of lending constraints on investment financing; and c), the degree of exchange rate pass through in import prices.

The first feature is of course necessary to motivate a role for the exchange rate regime at all. The specific assumption made is that the prices of non-traded goods are set by individual firms, and adjust only over time, following the specification à la Calvo. On the contrary, we assume that exporters are price-takers so that the law of one price must hold for export goods.

Concerning the second feature, the lending mechanism outlined represents a transmission channel linking balance sheet conditions to real spending decisions. We follow the Bernanke, Gertler and Gilchrist (1998) approach, which assumes that entrepreneurs should take up external funds to undertake investment projects. As lenders should bear agency costs to observe the returns on
investments, entrepreneurs face higher costs of external financing of investment relative to internal financing. This leads investments to depend on entrepreneurial net worth. In particular these financial frictions can be summarized by two key parameters: the elasticity of the premium on external funds with respect to the leverage and the degree of leverage itself.

Finally, the third aspect that should be highlighted is the degree of exchange rate pass-through, because it may lead to different prescriptions of monetary policy, especially for emerging economies that are more vulnerable to external shocks. Devereux and Lane (2001) show that if the degree of pass-through is high, a policy of non-traded goods inflation targeting is better in terms of stabilization. Nevertheless, there is a trade-off between output stabilization and volatility in the exchange rate and hence in inflation. This trade-off is eliminated when exchange rate pass-through is low. In this case, the best policy is total inflation (CPI) inflation targeting. If the rate of pass-through is low, exchange rate does not affect strongly domestic price by affecting the domestic currency price of imports that exchange rate shocks. So it does not induce substitution effects between domestic and foreign goods and hence the influence of output is limited.

There are four sets of domestic actors in the model: consumers, production firms, entrepreneurs, and the monetary authority. In addition, there is a "rest of world" sector where foreign-currency prices of export and import goods are set, and where lending rates are determined.

2.1 Consumers

We assume that the economy is populated by a continuum of consumer/households of measure unity. We will describe the model in terms of the representative consumer. She has preferences given by:

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t, \frac{M_t}{P_t}) \]

where \( C_t \) is a composite consumption index, \( H_t \) is labor supply, and \( \frac{M_t}{P_t} \) represents real balances, with \( M_t \) being nominal money balances, and \( P_t \) being the consumer price index. Let the functional form of \( u \) in [1] be given by:

\[ u = \frac{1}{1-\sigma} (C_t - hC_{t-1})^{1-\sigma} + \frac{b_t}{1-\varepsilon} \left( \frac{M_t}{P_t} \right)^{1-\varepsilon} - \eta \frac{H_t^{1+\psi}}{1+\psi} \]

where \( h \) measures the coefficient of habit in consumption. When \( h = 0 \), the external habit stock is assumed not to be dependent on aggregate past consumption.

Composite consumption is a CES function of consumption of non-traded goods and an import good, where

\[ C_t = [a^{1/\rho} C_{Nt}^{1-1/\rho} + (1-a)^{1/\rho} C_{Mt}^{1-1/\rho}]^{\rho/\rho-1} \text{, } \rho > 0 \]

The implied consumer price index is then

\[ P_t = [aP_{Nt}^{1-\rho} + (1-a)P_{Mt}^{1-\rho}]^{1/1-\rho} \]

Since we wish to introduce nominal price setting in the non-traded goods
sector, we need to allow for imperfect competition in that sector. In order to do this, we assume that the consumption of non-tradable goods is differentiated as follows:

\[ C_{Nt} = \left[ \int_0^1 C_{Nt}(i)^{1-\theta} di \right]^{1/1-\theta} \quad \theta > 1 \]

Consumers maximizes their utility subject to the following budget constraint:

\[ P_tC_t = W_t H_t + \Pi_t + S_t D_{t+1} + B_{t+1} + M_t - M_{t-1} - (1 + i^*) S_t D_t - (1 + i) B_t + T_t \]

where \( W_t \) are wages, \( T_t \) are transfers from government, \( \Pi_t \) are profits from firms in the non traded sector, \( M_t \) is domestic money demand, \( B_t \) are domestic bonds, \( S_t \) is the nominal exchange rate, \( D_t \) is the outstanding amount of foreign-currency debt and \( (1 + i^*) S_t D_t \) is debt repayment from last period.

The household will choose non-traded and traded goods to minimize expenditure conditional on total composite demand \( C_t \). Demand for non-traded and imported goods is then:

\[ C_{Nt} = a \left( \frac{P_{Nt}}{P_t} \right)^{-\rho} C_t \]

and

\[ C_{Mt} = (1-a) \left( \frac{P_{Mt}}{P_t} \right)^{-\rho} C_t \]

The first order conditions are:

\[ E_t[\beta(C_{t+1} - hC_t)^{-\sigma}] = \lambda_{t+1} \]

\[ \lambda_t = R_t^\varepsilon \frac{P_t}{P_{t+1}} \lambda_{t+1} \]

\[ = C_t - hC_{t-1} \]

\[ b_t \left( \frac{M_t}{P_t} \right)^{-\varepsilon} = -\lambda_t \left( 1 - \frac{1}{R_t^\varepsilon} \frac{P_t}{P_{t+1}} \right) = -(C_t - hC_{t-1})^{-\sigma} \left( 1 - \frac{1}{R_t^\varepsilon} \frac{P_t}{P_{t+1}} \right) = \]

\[ -(C_t - hC_{t-1})^{-\sigma} \left( 1 - \beta C_t^{1-\sigma} \frac{P_t}{P_{t+1}} \right) \]

\[ \frac{W_t}{P_t} = \eta(C_t - hC_{t-1})^\sigma H_t^\psi \]

\[ 2.1.1 \text{ Labour supply decisions and the wage setting equation} \]

The latter equation is based on the assumption that prices are completely flexible. Nevertheless, Christiano, Eichenbaum and Evans (2001) argue that in this framework wage rigidity, and not price rigidity, seems to be a stronger effect on dynamics of inflation and output.

Moreover, the introduction of nominal-wage rigidity in this model limits the sensibility of wages and the marginal costs to output shocks, at least over the short term.

Households act as price-setters in the labour market. It is assumed that wages can only be optimally adjusted after some random signal that is received with probability \( (1 - \varphi_w) \). Whenever the household received this signal, wage are reset equal to \( w^*_t \); otherwise an household \( j \) will chose wages according to:
\[ W_t^* = \left( \frac{P_{t-1}}{P_{t-2}} \right)^\gamma_w W_{t-1}^* \]

where \( \gamma_w \) is the degree of wage indexation. When \( \gamma_w = 0 \), there is no indexation and the wages that cannot be re-optimized remain constant. When \( \gamma_w = 1 \), there is perfect indexation to past inflation.

Households supply differentiated labor services to the wholesale sector, where labor is aggregated according to the Dixit-Stiglitz form:

\[ H_t = \left[ f_t^1(H_t)^{1/1+\varphi_w dj} \right]^{1+\varphi_w} \]

The optimal demand for labor is

\[ H_t^* = \left( \frac{W_t}{W_{t-1}} \right)^{\varphi_w + 1} H_t \]

Integrating this equation and imposing the Dixit-Stiglitz aggregator type function, we can express the aggregate wage index as

\[ W_t = \left[ f_t^1(W_t)^{-1/\varphi_w dj} \right]^{-\varphi_w} \]

The law of motion of the aggregate wage index is given by

\[ (W_t)^{-1/\varphi_w} = \varphi_w W_t [W_{t-1} \left( \frac{P_{t-1}}{P_{t-2}} \right)^\gamma_w]^{-1/\varphi_w} + (1 - \varphi_w) (W_t^*)^{-1/\varphi_w} \]

The maximization problem results in the following mark-up equation for the re-optimized wage:

\[ \frac{W_t^*}{P_t} = (1 + \varphi_w) \frac{E_t\left[ \sum_{i=0}^{\gamma_w} \beta^i \varphi_w \gamma_w H_{t+i+w} U_{H,t+i+w} \right]}{E_t\left[ \sum_{i=0}^{\gamma_w} \beta^i \varphi_w \gamma_w \left( \frac{P_t}{P_{t-i+w}} \right)^{\gamma_w} H_{t+i+w} U_{C,t+i+w} \right]} \]

where \( U_H \) is the marginal disutility of labour, \( U_C \) is the marginal utility of consumption and \( lw = \frac{1}{1 - \varphi_w} \) is the average length of time that wages remain unchanged. When wages are perfectly flexible (\( \varphi_w = 0, lw = 1 \)), the real wage will be a mark-up (equal to \( 1 + \varphi_w \)) over the current ratio of the marginal disutility of labour and the marginal utility of an additional unit of consumption.

After log-linearizing,

\[ \frac{\pi^w_t}{\varphi_w} = (1 - \beta \varphi_w)(1 - \varphi_w) \left[ mrs_{C,H} - (w_t - p_t) \right] + \beta E_t \pi^w_{t+1} \]

where \( \pi^w_t \) is the rate of nominal wage inflation and \( mrs_{C,H} \) is the marginal rate of substitution between leisure and consumption. If real wage is lower than the marginal rate of substitution, \( (w_t - p_t) < mrs_{C,H} \), workers will want to raise their nominal wages when the opportunity to adjust will arises.

### 2.2 Production Firms

Production is carried out by firms in each sector. The two sectors, tradable and non-tradable, differ in their production technologies. Both types of goods are produced by combining labour and capital. We differ from both Bernanke, Gertler and Gilchrist (1999) and Devereux and Lane (2001) as labour comes only from consumer/households and not from entrepreneurs.

In the non-traded sector the overall production technology is

\[ Y_{Nt} = A_N K_{Nt}^\beta L_{Nt}^{1-\alpha} \]

where \( A_N \) is the productivity parameter.
Similarly, exporters (all domestically-produced tradable goods are exported) use the production function

\[ Y_{Xt} = A_X K_{Xt}^{\gamma} H_{Xt}^{1-\gamma} \]

Firms minimize production costs, so the first order conditions are

\[ W_{Nt} = MC_{Nt}(1 - \alpha) \frac{Y_{Nt}}{H_{Nt}} \]

\[ r_{Nt}^K = MC_{Nt} \frac{Y_{Nt}}{K_{Nt}} \]

\[ W_{Xt} = P_{Xt}(1 - \gamma) \frac{Y_{Xt}}{H_{Xt}} \]

\[ r_{Xt}^K = P_{Xt} \gamma \frac{Y_{Xt}}{K_{Xt}} \]

where \( MC_{Nt} \) denotes the marginal production cost for a firm in the non-traded sector (which is common across firms). Wage are equalized over the two sectors because of labour mobility, so \( W_{Nt} = W_{Xt} \).

Production of capital goods is also carried out by competitive firms. These firms combine imports and non-traded goods to produce unfinished capital goods. There are adjustment costs of investment, so that the marginal return to investment in terms of capital goods is declining in the amount of investment undertaken, relative to the current capital stock.

The produced capital goods replace depreciated capital and add to the capital stock. We assume that capital producers are subject to quadratic capital adjustment costs. Their optimization problem, in both export and non-traded sectors is

\[ \text{Max}_{I_{jt}} E_t[q_{jt} I_{jt} - I_{jt} - \frac{X}{2}(I_{jt} - \delta)^2 K_{jt}] \quad j = X, N \]

and the first order condition is

\[ E[q_{jt} - 1 - \chi(I_{jt} - \delta)] = 0 \quad j = X, N \]

which is the standard Tobin’s Q equation that relates the price of capital to the marginal adjustment costs. This equation gives the supply of capital.

Furthermore, capital stocks in the export and non-traded sectors evolve according to

\[ K_{jt} = I_{jt} + (1 - \delta)K_{jt-1} \quad j = X, N \]

### 2.3 Price setting and Local Currency Pricing

We introduce a monopolistic competition framework of Dixit and Stiglitz (1977):

\[ P_{Nt+t} = \left( \int P_{Nj+t}^{1-\theta} dj \right)^{1/1-\theta} \]

\[ Y_{Nt+t} = \left( \int Y_{Nj+t}^{\theta-1} dj \right)^{\theta/\theta-1} \]

The aggregate price is:
Following Calvo we are assuming that firms cannot change their selling prices unless they receive a random signal. The constant probability to receive such a signal is $(1- \varphi)$. Each firm $j$ sets the price $p^*_N$ that maximizes the expected profit for $l$ periods.

The maximization problem is:

$$\text{Max} E_0 \sum_{t=0}^{\infty} \left( (\beta \varphi)^t \lambda_{t+l} (p^*_N(j) - m_{ct+l}) \right) \frac{Y_{N_{t+1}}(j)}{P_{N_{t+1}}}$$

where $l = \frac{1}{1 - \varphi}$ is the average length of time that a price remains unchanged.

$$\text{s.t.}$$

$$Y_{N_{t+1}}(j) = (\frac{p^*_N(j)}{P_{N_{t+1}}})^{-\varphi} Y_{N_{t+1}}$$

The first order condition is:

$$p^*_N = \varphi \frac{E_0 \sum_{t=0}^{\infty} \left( (\beta \varphi)^t \lambda_{t+l} m_{ct+l} \right) \frac{Y_{N_{t+1}}(j)}{P_{N_{t+1}}} \frac{1}{1 - \varphi} \left( \frac{p^*_N(j)}{P_{N_{t+1}}} \right)^{\varphi} Y_{N_{t+1}}}{E_0 \sum_{t=0}^{\infty} \left( (\beta \varphi)^t \lambda_{t+l} \right) \frac{Y_{N_{t+1}}(j)}{P_{N_{t+1}}}}$$

These equations lead to the following New Keynesian Phillips curve specific to the non-tradable sector:

$$\pi_{N_t} = (1 - \beta \varphi)(1 - \varphi) m_{cN_t} + \beta E_t \pi_{N_{t+1}}$$

where $m_{cN_t} = MC_{N_t}$ and $m_{cN_t}$ is the log deviation of real marginal cost in the non-traded sector from its steady state level.

Without loss of generality, we may assume that imported goods prices are adjusted in the same manner as prices in the non-traded sector, where $(1 - \varphi^*)$ is the probability for foreign firms to adjust their prices. Moreover, for imported goods we allow for the possibility that there is some delay between movements in the exchange rate and the adjustment of imported goods prices. The coefficient $\varphi^*$ determines the delay in the "pass-through" of exchange rates to prices in the domestic market: if the pass-through is complete $\varphi^* = 0$, otherwise we set $\varphi^* = \varphi$.

Using the same approach described above, we can derive the familiar inflation equation for the import good inflation:

$$\pi_{M_t} = \frac{(1 - \beta \varphi^*)(1 - \varphi^*)}{\varphi^*} (\hat{s_t} + \hat{P}_{M_t} - \hat{P}_{M_{t+1}}) + \beta E_t \pi_{M_{t+1}}$$

where $\pi_{M_t}$ is the domestic-currency inflation rate for the imported good, and $\hat{s_t}$ and $\hat{P}_{M_t}$ represent respectively the log deviation of the exchange rate and the world price for import goods from steady state.

For export goods, we assume that the law of one price must hold, so that

$$\pi_{X_t} = S_t P^*_{X_t}$$

where $P^*_{X_t}$ is exogenously given.
2.4 Entrepreneurs

There are two groups of entrepreneurs. One group provides capital to the non-traded sector, while the other provides capital to the traded sector.

The entrepreneurs' behaviour is similar to that proposed by Bernanke, Gertler and Gilchrist (1998). First, in the basic model, for simplicity, we assume that entrepreneurs' debt is denominated only in domestic currency, then we introduce foreign currency denominated debt. The probability that an entrepreneur will survive until the next period is \( \nu \), so the expected lifetime horizon is \( \frac{1}{1-\nu} \). This assumption ensures that entrepreneurs' net worth (the firm equity) will never be enough to fully finance the new capital acquisition, so they issue debt contracts to finance their desired investment expenditures in excess of net worth.

The entrepreneurs’ demand for capital depends on the expected marginal return and the expected marginal external financing cost.

If debt is denominated in the foreign currency

\[
E_t f_{jt+1} = E_t \left[ \frac{r_{jt+1} + (1-\delta)q_{jt+1}}{q_{jt}} S_t \right] \quad j = X, N
\]

where \( f_{jt+1} \) is the external funds rate and and \( r_{jt+1} \) is the marginal productivity of capital, at \( t+1 \) in sector \( j \). Following BGG (1998), we assume the existence of an agency problem that makes external finance more expensive than internal funds. The entrepreneurs costlessly observe their output which is subject to a random outcome. The financial intermediaries incur an auditing cost to observe an entrepreneur’s output. After observing his project outcome, an entrepreneur decides whether to repay his debt or to default. If he defaults, the financial intermediary audits the loan and recovers the project outcome less monitoring costs. Accordingly, the marginal external financing cost is equal to a gross premium for external funds plus the gross real opportunity costs equivalent to the riskless interest rate. Thus, the demand for capital should satisfy the following optimality condition:

\[
E_t f_{jt+1} = E_t \left[ \left( \frac{N_{jt+1}}{K_{jt+1} q_{jt}} \right)^{-\omega_j} R_t^* \right] \quad j = X, N
\]

The parameter \( \omega_j \) is the elasticity of the external finance premium with respect to the leverage ratio \( \frac{K_{jt+1} q_{jt}}{N_{jt+1}} \) in sector \( j \). The gross external finance premium \( \left( \frac{N_{jt+1}}{K_{jt+1} q_{jt}} \right)^{-\omega_j} \) depends on the borrowers leverage ratio\(^1\).

Aggregate entrepreneurial net worth evolves according to

\[
N_{jt+1} = \nu [f_{jt} q_{jt-1} K_{jt} - R_t^* \left( \frac{N_{jt}}{Q_{jt-1} K_{jt}} \right)^{-\omega_j} S_t D_t^t] + (1-\nu)g_t
\]

\( ^1 \)In a model without financial accelerator mechanism, the elasticity of premia with respect to leverage ratio is equal to zero. The leverage ratio is equal to one because enterprises are able to fully finance the new capital acquisition. Therefore, \( \nu \) is set equal to one and the elasticity of premia to the leverage ratio is equal to zero in both sectors.
where $D_t^e$ denotes the share of total debt denominated in foreign currency hold by entrepreneurs; $1 - \nu$ is the share of new entrepreneurs entering the economy; $g_t$ is the transfer or "seed money" that newly entering entrepreneurs receive from entrepreneurs that die and depart from the scene; $f_{jt}$ is the ex-post real return on capital held in $t$, and $E_{t-1}f_{jt}$ is the ex-post cost of borrowing. Earnings from operations this period become next period's net worth.

2.5 Monetary Policy Rule

The general form of the interest rate rule used may be written as

$$R^n_t = \left( \frac{P_{Nt}}{P_{Nt-1}} \frac{1}{1 + \pi_N} \right)^{\mu_{xN}} \left( \frac{P_t}{P_{t-1}} \frac{1}{1 + \pi} \right)^{\mu_x} \left( \frac{S_t}{S} \right)^{\mu_s} \left( R^n_{t-1} \right)^{\mu_r}$$

where it is assumed that $\mu_x \geq 0$, $\mu_{xN} \geq 0$ and $\mu_s \geq 0$. The parameter $\mu_{xN}$ allows the monetary authority to control the inflation rate in the non-traded goods sector around a target rate of $\pi_N$. The parameter $\mu_x$ governs the degree to which the CPI inflation rate is targeted around the desired target of $\pi$. Finally, $\mu_s$ controls the degree to which interest rates attempt to control variations in the exchange rate, around a target level of $S$. We may consider the steady state value as target $R^n$ for nominal interest rate. However, if monetary authority does not react immediately in adjusting interest rate, an alternative interest rate-smoothing monetary rule should be written as

$$R^n_t = \left( \frac{P_{Nt}}{P_{Nt-1}} \frac{1}{1 + \pi_N} \right)^{\mu_{xN}} \left( \frac{P_t}{P_{t-1}} \frac{1}{1 + \pi} \right)^{\mu_x} \left( \frac{S_t}{S} \right)^{\mu_s} \left( R^n_{t-1} \right)^{\mu_r}$$

where the parameter $\mu_r$ controls the degree to which interest rates attempt to control variations around a target level that can be set equal to either the past value or the steady state value.

We choose the monetary rule described in equation [37.2].

2.6 Equilibrium

Domestic demand and total output are set to be equal to

$$DD_t = C_t + I_{Nt} + I_{Xt}$$
$$Y_t = DD_t + \left( \frac{P_{Xt}Y_{Xt}}{P_t} - \frac{P_{Mt}Y_{Mt}}{P_t} \right)$$

The evolution of net debt is determined only in households’ sector. It is set to be equal to

$$S_tD_{t+1} = R^n_t S_t D_t - P_{Xt}Y_{Xt} + P_{Mt}Y_{Mt}$$

where the evolution of demand of imported goods and non-traded domestic goods are determined by the following equations:

$$Y_{Mt} = (1 - a)DD_t \left( \frac{P_{Mt}}{P_t} \right)^{-\rho}$$
$Y_{Nt} = aDD_t\left(\frac{P_{Nt}}{P_t}\right)^{-\rho}$

Labour market clearing condition implies:

$H_t = H_{Nt}^1aH_{Nt}^a$

3 Estimation

3.1 Maximum Likelihood Methods

Following Sargent (1989), a number of authors have estimated SDGE models using classical maximum likelihood methods: using kalman filter to form the likelihood function, parameters are estimated by maximizing the likelihood function.

The Kalman filter is one of the most important instrument to estimate a log-linearized SDGE model, once it has been written in state space form. This filter can be used to optimally estimate the unobservable states and to update estimates when a new observation becomes available. The procedure involves the following steps: we first select initial conditions, then we predict variables and we construct the mean square of the forecasts using information available at the previous period. After observing data, we update state equation estimates. We predict state equation random variables next period and we repeat these steps until the final period.

Beside providing minimum Mean Square Error (MSE), forecasts of the endogenous variables and optimal recursive estimates of the unobserved states, it is also an important building block in the prediction error decomposition of the likelihood. Maximization of the likelihood function is problematic when observations are not independent. In this case, it turns out that there is a convenient format, called prediction error decomposition. The building blocks of this likelihood function conditional on the initial observations $L(y \mid \phi)$ are the forecast errors and their MSE. Maximization of the likelihood conditional on the initial observations $L(y \mid \phi)$ can be obtained applying the kalman filter procedure after choosing some initial value for $\phi$. At each step we save forecast errors and their MSE to construct maximum likelihood $L(y \mid \phi = \phi_0)$. Then we update initial estimates of $\phi$ using methods as simplex methods or gradient methods and we repeat all the previous steps until $|\phi^t - \phi^{t-1}| \leq \iota$, for $\iota$ small.

3.2 Bayesian estimation methodology

There are various ways of estimating the parameters of a linearized Stochastic Dynamic General Equilibrium model (SDGE). As pointed in recent papers, there are two main advantages of Bayesian estimation relative to maximum likelihood.

First, this approach allows one to formalize the use of prior information coming either from micro-econometric studies or previous macro-econometric studies. In such a way, it makes an explicit link with the previous calibration-based
literature. Second, the Bayesian approach provides a framework for evaluating fundamentally mis-specified models on the basis of the marginal likelihood of the model or the Bayes’ factor. For instance, as shown by Geweke (1998), the marginal likelihood of a model is directly related to the predictive density function. The prediction performance is a natural criterion for validating models for forecasting and policy analysis.

In order to estimate the parameters of the SDGE model presented in section 2, we simulate 1000 data points on six key macro-economic variables, selected at levels: output in the tradable and the non-tradable sector ($Y_N$ and $Y_X$), domestic and foreign nominal interest rates ($R^n$ and $R^m$), import price ($P_M$) and export price ($P_X$). Simulated data are based on the values that will be discussed below, when we treat the choice of the prior distribution.

Then we introduce six exogenous shocks: no-tradable sector technology shock (i.e. an increase in $A_N$), export sector shock (i.e. an increase in $A_X$), domestic nominal interest rate shock (i.e. an increase in domestic nominal interest rate, that is a tightened monetary policy), foreign nominal interest rate shock (i.e. an increase in foreign nominal interest rate), import price shock and export price shock (i.e. and increase in price level).

Domestic nominal interest rate shock is introduced in the linearized monetary rule, while all the other shocks follow first-order autoregressive processes:

\[
A_{Nt} = \rho_{AN}(A_{Nt-1}) + \varepsilon_{AN} \\
A_{Xt} = \rho_{AX}(A_{Xt-1}) + \varepsilon_{AX} \\
R^n_t = \rho^R(R^n_{t-1}) + \varepsilon^R \\
P^M_t = \rho_{PM}(P^M_{t-1}) + \varepsilon_{PM} \\
P^X_t = \rho_{PX}(P^X_{t-1}) + \varepsilon_{PX}
\]

We start by solving the model for an initial set of parameters. Then, after specifying the prior distributions for the parameters, we use the kalman Filter to calculate the likelihood function of the data (for given parameters). Combining prior distributions with the likelihood of the data, we obtain the posterior kernel which is proportional to the posterior density.

Since the posterior distribution is unknown, we use Monte Carlo Markov Chain (MCMC) simulation methods to conduct inference about the parameters.

The posterior output can be used to compute any posterior function of the parameters: impulse responses, moments, etc.

### 3.2.1 Prior distribution

Following the literature, we set the steady state rate of depreciation of capital ($\delta$) equal to 0.025 which corresponds to a rate of depreciation equal to 10 per cent annual, the discount factor $\beta$ equal to 0.99, which corresponds to an annual real rate in steady state of 4 per cent. The steady state share of capital in the non-tradable output ($\alpha$) is equal to 0.3, while the steady state share of capital in the tradable output, $g$, is set equal to 0.7. As suggested by Bernanke Gertler and Gilchrist (1998), the leverage ratio is set equal to 0.5 in both sectors; adjustment costs $\chi$ take a value between 0 and 0.5 (here 0.2), but there is not agreement in the literature on the value of this parameter. The
probability $\pi$ that an entrepreneur will survive for the next period is set equal to 0.9, therefore on average an entrepreneur may alive 36 years. Following Gertler Gilchrist and Natalucci (2003), the elasticity of substitution between domestic goods and imported goods in consumption ($\rho$) is set equal to 1 and the share of non tradable goods in CPI ($\alpha$), is set equal to 0.5 (very close to 0.55, the value chosen by Gertler, Gilchrist and Natalucci). Finally, $c$, the inverse of elasticity of substitution in real balance is set equal to 2; $\psi$, the elasticity of labor supply; and $\eta$, the coefficient of labor in utility, are both set equal to 1.

The other 14 parameters and the standard errors of the 6 shocks are estimated using the Bayesian procedure.

Regarding the shocks affecting the economy, all the autoregressive coefficients have a beta distribution with mode 0.85, while the standard deviations for the shocks follow a gamma distribution with mode 0.10 and degree of freedom equal to 2. This distribution guarantees a positive variance with a rather large domain. The distribution of the autoregressive parameters in the Taylor’s rule is assumed to follow a beta distribution because this distribution covers the range between 0 and 1. A rather strict standard error was used for the autoregressive coefficient of the exchange rate, in order to have a clear separation between persistent and non-persistent shocks. The consumption and price setting parameters are assumed to be either Normal distributed or Beta distributed (for the parameters that are restricted to the 0-1 range). The mean is set at values that are equal or very close to those estimated in other studies in the literature. The standard errors are set so that the domain covers a reasonable range of parameter values. For example, the mean of the Calvo parameters in the price and wage setting equations are respectively set equal to 0.75 and 0.70, so that the average duration is longer for price contracts than for wage contracts. Other priors come from the literature (Smets and Wouters, 2002): the relative risk aversion coefficient, $\sigma$, has a normal distribution with mean 1; the habit persistence parameter, $h$, has a beta distribution with mean 0.70. Finally, for both sectors the elasticity of risk premium to the leverage ratio is assumed to be normally distributed with mean 0.1.

### 3.2.2 Posterior distribution

We first estimate the mode of the posterior distribution maximizing the posterior density $p(\theta \mid Y)$ with respect to the parameters and given the data $Y$.

The objective is to maximize

$$
\log p(\theta \mid Y) = \log p(Y \mid \theta) + \log p(\theta) - \log p(Y)
$$

where $p(Y \mid k)$ is the sample density or likelihood function, $p(k)$ is the prior density of the parameters and $p(Y)$ is the marginal likelihood.

However, since $p(Y)$ does not depend on $\theta$, the posterior mode can be obtained maximizing

$$
\log p(\theta \mid Y) = \log p(Y \mid \theta) + \log p(\theta)
$$

We use Markov Chain Monte Carlo (MCMC) to obtain the posterior distribution. This is necessary when it is not possible to sample the parameters...
directly from the posterior distribution\textsuperscript{2}. MCMC is a method of sampling a target probability distribution by constructing a Markov chain such that the target distribution is the stationary distribution of the chain, and such that the chain converges in distribution to the stationary distribution. The idea behind MCMC is to sample from a given distribution by constructing a chain, i.e. a kernel, and then to run the chain until realizations come from the given target. In order to find an appropriate kernel, we used the Metropolis-Hasting algorithm. This algorithm uses an acceptance/rejection rule to converge to the posterior distribution.\textsuperscript{3} Convergence in Dynare is checked by performing Brooks and Gelman statistic (1998). It consists in running a number of chains simultaneously: in this model we have run two parallel chains for 30000 replications. Then the convergence is monitored by comparing variation between and within chains until the "within" variation approximates the "between" variation. The statistic they use is initially larger than one but falls toward one as the length of chains increases. Alternatively, convergence can be checked applying the Geweke’s test or the CUMSUM statistic.

### 3.2.3 Model comparison

To check the relevance of the financial accelerator mechanism, we compare the performance of two different models: model with financial accelerator mechanism (FA) and model without financial accelerator (NoFA). In the former model specification, for both sectors, the leverage ratio is set equal to 0.5 and the elasticity of premium to the leverage ratio is set equal to 0.1; in the latter model specification, the leverage ratio is set equal to 1 and the elasticity of premium to the leverage ratio is set equal to 0. We need to calculate the marginal data density of both models. Lets call $M_{\text{fa}}$ the model with financial frictions and $M_{\text{nofa}}$ an alternative specification of the model without financial frictions. The

\textsuperscript{2}Markov chain Monte Carlo (MCMC) methods produce dependent samples of the posterior, rather than independent samples produced by direct sampling. However, they are more general than direct sampling, since they can deal with unknown type of posterior.

\textsuperscript{3}Metropolis-Hastings (MH) sampling and Gibbs sampling are two general methods of MCMC algorithms.

MH uses an auxiliary density, called the candidate density, to generate $\theta$. It uses an acceptance/rejection mechanism to decide if a draw can/cannot be accepted as a draw of the posterior. Let $\theta^{(i)}$ be the last accepted draw.

The next draw $\theta^{(i+1)}$ is generated as follows:

1) generate $\theta^* \sim \iota(\theta)$

2) compute

$$p = \min\left[ \frac{\varphi(\theta^{(i)})}{\varphi(\theta^* \iota(\theta^*))} \frac{\iota(\theta^*)}{\iota(\theta^{(i)})}, 1 \right]$$

3) take $\theta^{(i+1)} = \theta^*$ with probability $p$

or

$\theta^{(i+1)} = \theta^{(i)}$ with probability $1 - p$

We use as the jumping function a random walk around the parameter space. In particular, we set $q(\theta^{(i+1)} | \theta^{(i)}) = N(\theta^{(i)}, c^{2}\Sigma)$ where $\Sigma$ is the inverse of the Hessian computed at the joint posterior mode, and $c$ is a scale factor set to obtain efficient algorithms. After we obtain the first round of simulations, we repeat the exercise setting $\Sigma$ equal to the estimated covariance matrix.
marginal data density for each model will be

\[ p(Y \mid M_i) = \int \theta p(Y \mid \theta, M_i)p(\theta \mid M_i) d\theta ; \quad i = f, nof \]

where \( \theta \) are the parameters of model \( i \); \( p(Y \mid \theta_i, M_i) \) is the sample density of model \( i \) and \( p(\theta_i \mid M_i) \) is the prior density of the parameters for model \( i \).

The posterior probability for each model will be

\[ p(M_i \mid Y) = \frac{p(Y \mid M_i)p(M_i)}{\sum p(Y \mid M_i)p(M_i)} \]

Bayesian model selection is done pairwise comparing the models through the posterior odds ratio:

\[ PO_{ij} = \frac{B_{ij}}{B_{ji}} \]

where the prior odds \( p(M_j) \) are updated by the Bayes factor, defined as

\[ B_{ij} = \frac{p(Y \mid M_i)}{p(Y \mid M_j)} \]

Geweke (1998) proposes different methods to calculate the marginal likelihood \( p(Y \mid M_i) \) necessary for model comparison. Generally, the most popular is the modified harmonic mean because it works for all sampling methods and it is not sensitive to the step size. Alternatively we can use the Laplace approximation that assumes that the posterior distribution is close to a normal distribution. The advantage is that, given the normality assumption and the estimated mode, it can generate an approximation of the marginal likelihood very quickly. It turns out that this approximation works very well in practice and it is often very close to the modified harmonic mean.

Bayes factor in [48] can be interpreted as follows:

- \( B_{ij} < 1 \) \( \Rightarrow \) support for \( M_j \)
- \( 1 < B_{ij} < 10 \) \( \Rightarrow \) slight evidence against \( M_j \)
- \( B_{ij} > 10 \) \( \Rightarrow \) support for \( M_i \)

Some authors (Canova, 2005) criticize Bayes factors because, despite their popularity, they may not be very informative about the quality of the approximation to the data, in particular, when the models one wishes to compare are mis-specified. 4

4 Estimation on simulated data

In this section we first estimate the model comparing two different methods, Bayesian approach and Maximum Likelihood (ML). In particular we will focus on the sensitivity of estimation results to the sample size.

In this case, Schorfheide (2000) proposes an alternative procedure to choose among mis-specified models, both of which are likely to have very low posterior probability.

The actual data is assumed to be a mixture of the competing structural models and of a reference one, which has two characteristics: (i) it is more densely parametrized than the SDGE models; (ii) it can be used to compute a vector of population functions \( h(\theta) \). One such model could be a VAR or a BVAR. Given this setup, model comparisons can be undertaken using loss functions.
Then we will check for the influence of prior on estimation outcomes under the bayesian approach.

This section is structured as follows: in subsection 4.1 we test the baseline model; we check for the robustness of these results to the length of the dataset in subsection 4.2 and to the prior specification in subsection 4.3. Finally, in 4.4 we present results for the model without financial accelerator mechanism.

4.1 Baseline model

As described in subsection 3.2.1, there is a total of 28 free parameters in this model. We first fix 14 steady state parameters, while the rest of the parameters are determined by estimating the model using Bayesian procedures. Sample consists of 1000 observations.

Most parameters are estimated quite precisely, as showed in Table 1, in the third and fifth column. All the shocks except the one affecting prices in import sector are more persistent in data than in priors. Prices are more sticky than wages both in data and in priors. In data, premia are less sensitive to leverage ratio than in priors. However, for all the parameters mentioned above, differences between priors and posteriors are negligible. Looking at Figures 1A-1C displaying Priors and Posteriors, we can observe that dispersion in posteriors is lower than in priors for all parameters. Moreover, posteriors are symmetric as values for posterior mode are close to values for posterior mean. The only parameter that is not correctly estimated is $\sigma$. The standard error for the estimation on $\sigma$ is relatively large. This parameter, as also $h$, is linked to consumption that is not included in the six macroeconomics variables on which we are simulating data. If we include consumption, results on these parameters improve, as showed in Figure 2A-2C.

In the eighth column of Table 1, are reported the same parameters estimated through ML method. As under the Bayesian approach, autoregressive coefficients are estimated to be higher than 0.85, the value set for the prior, pointing out that shocks are more persistent in data than in the prior. Again, prices are more sticky than wages and estimated values are very close to those obtained using Bayesian techniques. Autoregressive coefficients in Taylor rule are estimated to be very close to the values set for the prior; interest rate is a little bit more persistent under ML estimation than under Bayesian approach. The elasticity of premia with respect to the leverage ratio is estimated to be consistent with both the prior and the Bayesian estimation. For large samples ML converges to Bayesian approach.

4.2 Robustness to the length of the data set

If the sample is reduced to 150 observations, we can show that ML estimation is sensitive to sample size a little bit more than Bayesian approach. Normally, for larger samples, estimation displays lower standard error even if parameters are not estimated correctly. As showed in the last two columns of Table 2,
if we repeat the same ML estimation using a 150 observation sample, results worsen dramatically. Larger standard errors arise especially for autoregressive coefficients, wage stickiness parameter and the elasticity of premia with respect to the leverage ratio.

The forth and fifth columns of Table 2 reports results obtained through Bayesian estimation using 150 observations. For autoregressive coefficients and standard errors of the shocks, Bayesian estimates is less sensitive to sample size than ML. Nevertheless, also when we apply the Bayesian approach, results worsen when sample size is reduced.

4.3 Robustness to the prior specification

When Bayesian techniques are implemented, if the prior is mis-specified with respect to the data generated with the "true" parameters, estimation outcomes are not always correct. In this case, enlarging the sample can not improve the results.

Columns 4-6 in Table 3 show the outcomes for a mis-specified model, simulating 150 observations for variables YN, YX, PM, PX, RN and RNF that are respectively non-tradable output, tradable output, import price index, export price index, domestic nominal interest rate, foreign nominal interest rate. For autoregressive coefficients, the value in the model is greater than the mean chosen for the priors; while for the standard deviation of the shocks it is lower than the mean of the priors.

The impact of prior is generally very small: posterior mean and mode for autoregressive coefficients, coefficients in Taylor rule and standard deviation for the shocks are very close to the "true" parameter in the model. Only autoregressive coefficient $\rho_{AN}$ and $\rho_{PM}$ are most sensitive to the prior. For these two parameters data seem to be not very informative, as confirmed by the highest standard deviation.

If we enlarge the sample and we simulate 1000 observations, results improve, as showed in the last three columns of Table 3. Estimation is better performed both for the standard deviation of the shocks and for the autoregressive coefficients. Coefficients $\rho_{AN}$ and $\rho_{PM}$ are still the parameters most sensitive to the impact of the prior, even if, for larger sample, the mode is closest to the real value set in the model and the standard deviation is a little bit lower.

4.4 Model without financial accelerator mechanism

Table 4 reports estimation outcomes for the model without the Financial Accelerator mechanism. The financial accelerator mechanism can be removed setting, in equations [35] and [36], the elasticity of premia with respect to the leverage ratio equal to zero ($\omega_N = \omega_X = 0$), the leverage ratio equal to one ($N/K = 1$) and the share of surviving entrepreneurs at the end of each period equal to one ($\nu = 1$).
Shocks to prices and to production in the non-tradable sector are much more persistent in prior than in data. For all these parameters, except \( \rho_{AX} \), standard deviation is very low meaning that data are very informative. In the model without the FA both import and export prices are more sticky than in the data. On the contrary, wages stickiness is under-estimated, as well the autoregressive coefficient \( \rho_r \) in the Taylor rule.

To confirm this conclusion, we can look also at the plots of priors and posteriors represented in Figures 3A-3C. Data result to be very informative as, for most of the parameters, posterior distributions and posterior mode do not fit the priors. The only parameters consistent with the model specification are the autoregressive coefficient of the technological shock in the non-tradable sector and the autoregressive coefficient of the exchange rate in the monetary rule.

Therefore, we can conclude that for 150 observations, if the exercise is run on simulated data resulting from the FA model, the model with financial accelerator delivers a better estimation than the model without such frictions.

Finally, we proceed to compare model with the FA and model without the FA comparing the Laplace approximation for marginal likelihood under both models. The logarithm of data density of model with the FA and model without the FA are respectively -1051.0457 and -846.6866. As shown by Geweke (1998), the marginal likelihood of a model is directly related to the predictive density function and the prediction performance is a natural criterion for validating models for forecasting and policy analysis. In this case, data density is higher under model with the FA, supporting model with the FA. Moreover, if we consider the ratio of data density under the two different model specifications, the Bayes factor results to be greater than 1, supporting again model with financial accelerator mechanism.

Looking over again the 1000 observation model correctly specified, we have to check for the convergence of the Metropolis-Hasting sampler.

We run two chains of 30000 draws obtaining an acceptance rate equal to 0.5781 for the first block and 0.5724 for the second one. The first 3000 observation were dropped for all chains.

We check for convergence in terms of the variance of the sample using both the univariate statistic (Gelman and Rubin,1992) and the multivariate statistic (Brooks and Gelman,1998).

Figure 4, in the upper graph, plots the multivariate reduction factor, while, in the lower graph, it plots the pool and the within variance estimated. The two measures show the same pattern and converge around a constant value and the variance reduction factor is below 1.2, value suggested by Brooks and Gelman as critical value. Also the variance reduction factor for each parameter (x-axis) is higher than the critical value, as showed in Figure 5.

Moreover, the CUMSUM statistic \( \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{\vartheta} - E(\vartheta)}{\sqrt{\text{var}(\vartheta)}} \) displayed in Figures 6A-6B indicates that for most of the parameters the convergence is quickly achieved. A divergence of even 0.25 is not a bad result as it means that, after \( n \)
draws, the posterior expectation diverges from the final estimate by 25 per cent in units of the final estimate of the posterior standard deviation.

Figures 7-8 show the convergence for the model without financial accelerator mechanism, when the sample size consists of 150 observations. In this case it should have been better to drop the first 5000 draws. The Brooks and Gelman statistics, both univariate and multivariate, show that for model without financial accelerator convergence is more difficult to be achieved. In particular, there is one coefficient with a 1.4 value for the univariate test and the multivariate test do not completely convergence. Differently, the CUMSUM statistic displayed in Figures 9A-9B, is more optimistic.

5 Conclusions

We estimate a small open economy model with financial frictions using both Bayesian techniques and ML.

We have showed that for large samples ML asymptotically converges to Bayesian estimation. Moreover, if the estimation is performed through Bayesian techniques and the model is not correctly specified, outcomes for larger samples are less sensitive to the prior error. This last result seems to confirm that one of the main advantages of Bayesian approach is the ability of providing a framework for evaluating fundamentally mis-specified models on the basis of the marginal likelihood of the model or the Bayes’ factor. In this exercise, results from comparison of model with the FA and model without the FA support model with financial frictions.

Finally, the convergence is monitored by comparing variation between and within chains until "within" variation approximates "between" variation, following Brooks and Gelman (1998). The plot shows that within and pooled variance estimates follow the same pattern and converge after 30000 draws. The univariate statistic is lower than 1.2, the critical value chosen by Brooks and Gelman, meaning that convergence is achieved for all parameters. This result is confirmed if we check for convergence plotting the CUMSUM statistic.

References


A. The steady-state equilibrium

Consumers:
\( (1 - h)c = (\lambda)^{-1/\sigma} \) where \( c = C/P \)
\( w = \eta H^\rho e^{\delta} \) where \( w = W/P \)
\( bm^{-\varepsilon} = \lambda(R - 1) \) where \( m = M/P \)
\( \frac{1}{\beta} = R = R^n = R^{*n} \)
\( c_M = \frac{1 - a}{a} c_N \)
\( c = \frac{1}{a} c_N \) and \( c = \frac{1}{1 - a} c_M \)
\( P = 1 \)
\( P_M = 1 \)
\( P_X = 1 \)
\( P_N = 1 \)

Firms:
\( \frac{Y_N}{H_N} = A_N(\frac{K_N}{H_N})^\alpha \) or \( \frac{Y_N}{K_N} = A_N(\frac{H_N}{K_N})^{1-\alpha} \)
\( \frac{Y_X}{H_X} = A_X(\frac{K_X}{H_X})^\gamma \) or \( \frac{Y_X}{K_X} = A_X(\frac{H_X}{K_X})^{1-\gamma} \)
\( W_N = mc_N(1 - \alpha)(\frac{Y_N}{H_N}) \)
\( W_X = (1 - \gamma)(\frac{Y_X}{H_X}) \)
\( r_N^K = \alpha mc_N \frac{Y_N}{K_N} \)
\( r_X^K = \gamma \frac{Y_X}{K_X} \)
\( l_N = \delta K_N \)
\( l_X = \delta K_X \)

Entrepreneurs:
\( q_N = 1 \)
\( q_X = 1 \)
\( f_N = r_N^K + 1 - \delta \)
\( f_X = r_X^K + 1 - \delta \)
\( f_N = (\frac{N_N}{K_N})^{-\omega_n} R^{*n} \)
\( f_X = (\frac{N_X}{K_X})^{-\omega_x} R^{*n} \)

Monetary authority and exchange rate:
\( S = 1 \)
\( \mu = 1 \)
Equilibrium:

\[ DD = (c + I_N + I_X + c_{e_N} + c_{e_X}) \]
\[ Y_N = a(c + I_N + I_X + c_{e_N} + c_{e_X}) \]
\[ Y_M = (1 - a)(c + I_N + I_X + c_{e_N} + c_{e_X}) \Rightarrow Y_M = \frac{1 - a}{a} Y_N \]
\[ Y = (c + I_N + I_X + c_{e_N} + c_{e_X}) + (Y_X - Y_M) \]
\[ D = \frac{Y_X - Y_M}{R^{\alpha} - 1} \]
\[ H = H_X + H_N \]
B. The log-linearized equilibrium system

**Consumers:**
\[
\dot{c}_t = \frac{h}{1 + h} \dot{c}_{t-1} + \frac{1 - h}{1 + h} \dot{c}_{t+1} - \frac{1 - h}{(1 + h)\sigma} [\hat{R}_t - (\hat{P}_{t+1} - \hat{P}_t)] \\
\sigma(\dot{c}_{t+1} - \dot{c}_t) = \hat{R}_t^m - (\hat{P}_{t+1} - \hat{P}_t) \text{ if there is no habit in consumption, } h = 0
\]
\[
\dot{b}_t - \varepsilon \ddot{m}_t = \frac{1}{R - 1} \dot{R}_t - \sigma \dot{c}_t
\]
\[
\dot{W}_t - \dot{P}_t = \psi \dot{H}_t + \sigma \dot{c}_t
\]
\[
\lambda_{t+1} = \lambda_t - \hat{R}_t
\]
\[
\hat{P}_{t+1} - \hat{P}_t = \hat{R}_t^m - \hat{R}_t
\]
\[
\hat{S}_{t+1} - \hat{S}_t = \hat{R}_t^m - \hat{R}_t^{m*} \quad (\text{UIP})
\]
\[
\hat{P}_t = a\hat{P}_{Nt} + (1 - a)\hat{P}_{Mt}
\]
\[
\hat{c}_t = a \hat{c}_{Nt} + (1 - a) \hat{c}_{Mt}
\]
\[
\hat{c}_{Mt} = \hat{c}_{Nt} + \rho (\hat{P}_{Nt} - \hat{P}_{Mt})
\]
\[
\dot{W}_t - \dot{W}_{t-1} = \dot{W}_{t+1} - \dot{W}_t + \frac{(1 - \beta \varphi_w)(1 - \varphi_w)}{\varphi_w} (\psi \dot{H}_t + \sigma \dot{c}_t - \dot{W}_t - \dot{P}_t)
\]

**Firms:**
\[
\hat{Y}_{Nt} = \hat{A}_{Nt} + \alpha \hat{K}_{Nt} + (1 - \alpha)\hat{H}_{Nt}
\]
\[
\hat{Y}_{Xt} = \hat{A}_{Xt} + \gamma \hat{K}_{Xt} + (1 - \gamma)\hat{H}_{Xt}
\]
\[
\hat{W}_{Nt} = \hat{Y}_{Nt} + m \hat{c}_{Nt} - \hat{H}_{Nt}
\]
\[
\hat{r}_{Nt} = \hat{Y}_{Nt} + m \hat{c}_{Nt} - \hat{K}_{Nt}
\]
\[
\hat{W}_{Xt} = \hat{Y}_{Xt} + \hat{P}_{Xt} - \hat{H}_{Xt}
\]
\[
\hat{r}_{Xt} = \hat{Y}_{Xt} + \hat{P}_{Xt} - \hat{K}_{Xt}
\]
\[
\hat{K}_{Nt} = \delta \hat{I}_{Nt} + (1 - \delta)\hat{K}_{Nt-1}
\]
\[
\hat{K}_{Xt} = \delta \hat{I}_{Xt} + (1 - \delta)\hat{K}_{Xt-1} \Rightarrow \delta^* = \delta
\]
\[
\hat{q}_{Nt} = \chi (\hat{I}_{Nt} - \hat{K}_{Nt})
\]
\[
\hat{q}_{Xt} = \chi (\hat{I}_{Xt} - \hat{K}_{Xt}) \Rightarrow \chi_X = \chi_N = \chi
\]

**Entrepreneurs:**
\[
\hat{f}_{Nt} + \hat{q}_{Nt-1} = \frac{\hat{r}_{Nt}}{\hat{f}_N} \hat{r}_{Nt} + (1 - \delta) \hat{q}_{Nt}
\]
\[
\hat{f}_{Xt} + \hat{q}_{Xt-1} = \frac{\hat{r}_{Xt}}{\hat{f}_X} \hat{r}_{Xt} + (1 - \delta) \hat{q}_{Xt}
\]
\[
\hat{f}_{Nt+1} + \omega_n \hat{N}_{Nt} - \omega_n \hat{K}_{Nt+1} = \hat{R}_t + \omega_n \hat{q}_{Nt}
\]
\[
\hat{f}_{Xt+1} + \omega_x \hat{N}_{Xt} - \omega_x \hat{K}_{Xt+1} = \hat{R}_t + \omega_x \hat{q}_{Xt}
\]
\[
\frac{\hat{N}_{Nt}}{\nu \hat{f}_N} = \frac{K_N}{N_N} \hat{f}_{Nt} - (\frac{K_N}{N_N} - 1) \hat{R}_{t-1} - \omega_n (\frac{K_N}{N_N} - 1)(\hat{K}_{Nt} + \hat{q}_{Nt-1}) + [\omega_n (\frac{K_N}{N_N} - 1) + 1] \hat{N}_{Nt-1} + (\frac{K_N}{N_N} - 1)(\hat{S}_{t-1} - \hat{S}_t)
\]
\[\frac{\dot{N}_{xt}}{\nu_f} = \frac{K_X}{N_X} f_{xt} - (\frac{K_X}{N_X} - 1)\dot{R}_{t-1} - \omega_x (\frac{K_X}{N_X} - 1)(\dot{K}_{xt} + \dot{q}_{xt-1}) + [\omega_x (\frac{K_X}{N_X} - 1)] + (\frac{K_X}{N_X} - 1)(\dot{S}_{t-1} - \dot{S}_t) \Rightarrow \nu_X = \nu_N = \nu\]

Retailers and Local currency pricing:
\[\dot{P}_{nt} = \dot{P}_{nt} + (1 - \beta \varphi)(1 - \varphi) \theta_{mt}\]
\[\dot{P}_{mt} = \dot{P}_{mt} + (1 - \beta \varphi)(1 - \varphi^*) (\dot{P}_{mt} - \dot{P}_{mt} + \dot{S}_t) \Rightarrow \varphi^* = \varphi\]

or \(\varphi^* = 0\)
\[\dot{P}_{mt} = \dot{P}_{mt} + \dot{S}_t + \varepsilon_{PM}\]
\[\dot{P}_{xt} = \dot{P}_{xt} + \dot{S}_t \text{ (Law of one price)}\]

Monetary Policy rule:
\[\ddot{\mu}_t = \ddot{\mu}_t - \dot{\mu}_t + \dot{P}_t - \dot{P}_t\]
\[\ddot{r}_t = (1 - \rho_R)[\mu_x S_t + \mu_x (\dot{P}_t - \dot{P}_t) + \mu_{\pi_N} (\dot{P}_{nt} - \dot{P}_{nt-1}) + \rho_R \ddot{r}_t + \varepsilon_{ru}]\]

or \(\ddot{r}_t = \mu_x S_t + \mu_x (\dot{P}_t - \dot{P}_t) + \mu_{\pi_N} (\dot{P}_{nt} - \dot{P}_{nt-1}) + \rho_R \ddot{r}_t + \varepsilon_{ru}\)

Equilibrium:
\[\ddot{Y}_{nt} = (c + I_N I_N + I_X I_X + c_x c_x + c_x c_x)] + \rho(\dot{P}_t - \dot{P}_nt)\]
\[\ddot{Y}_{mt} = (c + I_N I_N + I_X I_X + c_x c_x + c_x c_x)] + \rho(\dot{P}_t - \dot{P}_mt)\]
\[\ddot{Y}_t = (1 - \frac{P_{xy}}{Y})\ddot{Y}_{nt} + \frac{P_{xy}}{Y}\ddot{Y}_{xt}\]
\[\ddot{D}_{t+1} = R^{**} \ddot{D}_t + DR^{**} \ddot{r}_t + D(R^{**} - 1)\ddot{S}_t - \frac{P_{xy}}{Y} (\dot{P}_x + \ddot{Y}_x) + \frac{P_{ym}}{Y}(\dot{P}_m + \ddot{y}_m)\]
\[\frac{H_x}{H} \ddot{H}_{xt} + \frac{H_N}{H} \ddot{H}_{nt} = \ddot{H}_t\]
C. Appendix on convergence

To be more specific, consider the between and within sequence variance of each parameter, given respectively by

\[ B = \frac{n}{m} \sum_{j=1}^{m} (\hat{\psi}_j - \hat{\psi}_\cdot)(\hat{\psi}_j - \hat{\psi}_\cdot)' \]

where \( \hat{\psi}_j = \frac{1}{n} \sum_{t=1}^{n} \hat{\psi}_{jt} \) and \( \hat{\psi}_\cdot = \frac{1}{m} \sum_{j=1}^{m} \hat{\psi}_j \).

and

\[ W = \frac{1}{m(n-1)} \sum_{j=1}^{n} \sum_{t=1}^{m} (\psi_{jt} - \hat{\psi}_j)(\psi_{jt} - \hat{\psi}_j)' \]

where \( j = 1, ..., m \) is the number of chains and \( t = 1, ..., n \) is the number of draws in each chain. In this model we set \( m = 2 \) and \( n = 30000 \).

The marginal posterior variance \( \hat{V} \) will be a weighted average of \( W \) and \( B \):

\[ \hat{V} = \frac{n-1}{n} W + \frac{1 + \frac{1}{m}}{n} B \]

One way to check convergence is to calculate the univariate potential scale reduction factor, that compares pooled and within-chain inferences for each parameter:

\[ R = \frac{\hat{V}}{\sigma^2} \]

As the denominator of \( R \) is not itself known, it must be estimated from the data; we can gain an over-estimate of \( R \) by under-estimating \( \sigma^2 \) by \( W \). Thus, we over-estimate \( R \) by

\[ \hat{R} = \frac{\hat{V}}{\hat{W}} = \frac{n-1}{n} + \frac{m+1}{m} \frac{B}{nW} \]

which declines to 1 as \( n \to \infty \). If the potential scale reduction is high, we should proceed with further simulations to improve our inference. We compute this ratio for all the parameters.

Brooks and Gelman (1998) also proposed a multivariate version of the potential scale reduction factor that is expressed as

\[ \hat{R} = \frac{\hat{V}}{\hat{W}} = \frac{n-1}{n} + \frac{m+1}{m} \lambda_1 \]

where \( \lambda_1 \) is the largest eigenvalue of the symmetric, positive definite matrix \( W^{-1}B/n \).

Generally, to avoid the effect of the starting points and considering that for large simulations the distribution converges to the posterior, we ignored the first half of each sequence.

Alternatively, convergence can be checked applying the Geweke’s test or the CUMSUM statistic. Geweke’s test statistic compares the estimate \( \hat{g}_A \) of a posterior mean for the first \( n_A \) draws (or for the first chain) with the estimate \( \hat{g}_B \) for the last \( n_B \) draws (or for the other chain). If the two subsamples are well separated they should be independent. The statistic, that is normally distributed if \( n \) is large and if the MCMC converges, is:

\[ Z = \frac{\hat{g}_A - \hat{g}_B}{\sqrt{nse^2_A + nse^2_B}} \]

where \( nse^2_A \) and \( nse^2_B \) are the numerical standard errors for each subsample (or for each chain).
The CUMSUM statistics for a scalar \( \vartheta \) is
\[
\frac{1}{n} \sum_{i=1}^{n} \frac{\vartheta_i - E(\vartheta)}{\sqrt{\text{var}(\vartheta)}}
\]
where \( E(\vartheta) \) and \( \sqrt{\text{var}(\vartheta)} \) are the MC sample mean and standard deviation of \( n \) draws. If the MCMC sampler converges, the graph of the CUMSUM statistic against \( t \) should converge smoothly to zero. On the contrary, long and regular excursions away from zero are an indicator of the absence of convergence.
<table>
<thead>
<tr>
<th>parameter</th>
<th>Prior</th>
<th>Prior mean</th>
<th>Prior s.d.</th>
<th>Bayesian posterior mean</th>
<th>Bayesian mode</th>
<th>Bayesian s.d.</th>
<th>ML estimate</th>
<th>ML s.d.</th>
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### Table 2: Estimation (150 observations)

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<th>Bayesian mode</th>
<th>Bayesian s.d.</th>
<th>ML estimate</th>
<th>ML s.d.</th>
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<td>posterior mean 150 obs</td>
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<td>posterior mean 1000 obs</td>
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Table 4: Comparing model with the FA and model without the FA (Bayesian estimation, 150 observations)

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Figure 1A: Priors and Posteriors - 1000 observations (YN YX RN RNF PM PX)

Figure 1B: Priors and Posteriors - 1000 observations (YN YX RN RNF PM PX)
Figure 1C: Priors and Posteriors - 1000 observations (YN YX RN RNF PM PX)

Figure 2A: Priors and Posteriors - 1000 observations (YN YX RN C PM PX)
Figure 2B: Priors and Posteriors - 1000 observations (YN YX RN C PM PX)

Figure 2C: Priors and Posteriors - 1000 observations (YN YX RN C PM PX)
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Figure 3B: Model without the FA-Priors and Posteriors - 1000 observations
(YN YX RN RNF PM PX)

Figure 3C: Model without the FA-Priors and Posteriors - 1000 observations
(YN YX RN RNF PM PX)
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Figure 4: Model with the FA- Univariate variance reduction factor (Brooks and Gelman, 1998)
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Figure 6B: Model with the FA- CUMSUM statistic

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Figure 9A: Model without the FA- CUMSUM statistic
Figure 9B: Model without the FA-CUMSUM statistic