Abstract

The aim of this paper is to investigate both the efficacy and the stability properties of monetary policy rules in presence of heterogeneous consumers. We aim to underline the link between the well-known Taylor Principle and the demand-policy regimes, defined on the basis of the monetary policy transmission mechanism. By developing an analytical analysis, we show that the different demand-transmission mechanisms play a key role on monetary efficacy and equilibrium determinacy.

Keywords: Heterogeneous consumers, liquidity constraints, determinacy, Euler equation.
JEL codes: E61, E63.

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1. Introduction

Campbell and Mankiw (1989, 1990, 1991) provide compelling evidence for the existence of heterogeneous consumers: households who can smooth consumption (Savers or Ricardian consumers) and agents whose current consumption equals current income (Spenders or Non-Ricardian consumers), which represent a strong violation of the permanent income theory.

Spenders’ behavior can be interpreted in various ways. One can view their behavior as resulting from consumers who face binding borrowing constraints. Alternatively, myopic deviations from the assumption of fully rational expectations should be assumed (rule-of-thumb), i.e. consumers naively extrapolate their current income into the future, or weigh their current income too heavily when looking ahead to their future income because current income is the most salient piece of information available.¹

The behavior of the Spenders is empirically important, with about one-fourth of income accruing to them in the United States (see Fuhrer, 2000).² With specific reference to monetary policy, Rotemberg and Woodford (1999) are among the first to estimate a standard output Euler equation, which is based on the simplest model of optimizing household behavior (there is no weight on past inflation).³ Fuhrer (2000) obtains much better empirical results by enhancing the model of consumer behavior with Spenders and a habit formation process, which adds significant inertial output dynamics.⁴ Muscatelli et al. (2003) stress that the existence of savers increases the variability of output and inflation. In their model automatic stabilizers based on taxation tend to offset the impact of Spenders without resulting counter-productive.

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¹ See Mankiw (2000) and references therein.  
² Muscatelli et al. (2003) find an even larger proportion. They suggest that about 37% of consumers are Spenders, whilst 84% of total consumption in steady state is given by optimizing consumers. Spenders account for about 59% of total employment. Additional evidence is provided by Jappelli (1990), Shea (1995), Parker (1999), Souleles (1999), Fuhrer and Rudebusch (2003), and Ahmad (2004).  
³ Estrella and Fuhrer (1998) and Fuhrer (2000) show that this Euler equation provides a remarkably poor fit to the time series data on aggregate output.  
⁴ With a single equation for overall aggregate demand, it is not easy to infer the exact weight on expected future output and there are essentially no other available estimates with quarterly data. Fuhrer and Rudebusch (2003) investigate this problem.
Recently, spenders have been introduced, to study monetary policy, in a New Keynesian framework (Amato and Laubach, 2003; Gali et al., 2003; Bilbiie, 2004). The presence of Spenders’ behavior may alter dramatically the properties of these models and overturn some of the conventional results found in the literature.

Amato and Laubach (2003) explore the optimal monetary rule with rule-of-thumb households and firms. By modeling consumers’ rule-of-thumb behavior as a consumption habit, households’ decisions today mimic yesterday’s behavior of all agents (including optimizing agents.) The authors discover that, while the monetary policy implications of rule-of-thumb firms are minimal, the interest rate is more sensitive to the presence of rule-of-thumb consumers; in fact, as their fraction increases higher inertial monetary policy is required.

By contrast, Gali et al. (2003) show how the Taylor principle becomes a too weak criterion for stability when the proportion of rule-of-thumb consumers is large. However, the presence of Spenders cannot in itself overturn the conventional result on the sufficiency of the Taylor principle. By contrast, in the case of forward-looking interest rate rules, they show that the conditions for a unique equilibrium are somewhat different from those in a contemporaneous one. In particular, they show that when the share of Spenders is sufficiently large it may not be possible to guarantee a (locally) unique equilibrium or, if it is possible, it may require that interest rates respond less than one-for-one to changes in expected inflation.

By using a simplified version of Gali et al. (2003), Bilbiie (2004) shows how rule-of-thumb consumer can explain the puzzle of monetary policy before and after the Volker. In fact, as we also show, monetary policy can leads to determinacy even if the Taylor rule is not satisfied when enough agents do not participate in asset markets. By assuming zero long-run profit due to transfer between rule-of-thumb

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6 We share some results with Bilbiie (2004, 2005), unlucky both of us recognize our common interests at a late stage of our research agenda.
consumers and savers and an ad hoc lag scheme in the IS curve, Bilbiie (2004) provides some empirical support to his hypothesis.

The aim of this paper is to illustrate in a simple model both the efficacy and the stability properties of monetary policy rules under rule-of-thumb consumers in a framework a la Galì et al. (2003), which is becoming widely used, as we have above documented. Differently from previous works we tackle this issue from an analytical point of view rather than considering simulations/calibrations. By considering the problem analytically, we can discriminate between two different demand regimes (i.e. two IS-curves) characterized by different signs in the correlation between expected consumption growth and real interest rate. The introduction of Spenders into the DSGE New Keynesian model thus may explain the negative correlation between expected consumption growth and real interest rate sometimes found in the empirical literature. In fact, this correlation has been found to be low and sometimes negative across many of the industrialized countries (see Ahmad, 2004, Canzoneri et al. (2002)).

The existence of different regimes plays a crucial role on the discussion about monetary policy efficacy and equilibrium determinacy. In particular, if the correlation between expected consumption growth and real interest rate is positive, monetary policy efficacy increases in the fraction of Spenders (as Amato and Laubach, 2003). A reverse result is obtained if the correlation between expected consumption growth and real interest rate is negative. Regarding determinacy, we find that in the case of a positive correlation, standard results hold, i.e. if monetary policy follows a standard Taylor rule and determinacy is always associated with the satisfaction of the Taylor principle. By contrast, if the correlation is negative, we find different requirements for stability conditional on the magnitude of the effects of interest rate changes on the real output. Hence the non-conventional results stressed by Galì et al. (2003) hold only if the correlation between expected consumption growth and real interest rate is negative.

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7 This assumption is needed to obtain that the steady-state share of spenders’ consumption equal to the one of savers so that it is independent of Spenders’ fraction.

8 An exception is Bilbiie (2005).
The rest of the paper is organized as follows. Next section outlines our basic framework. Section 3 describes the two demand regimes implied by the presence of Spenders. Section 4 investigates the stability of the model under different monetary policy rules. Section 5 summarizes the monetary policy transmission mechanism. Section 6 concludes.

2. The Basic Framework

We consider a simple New Keynesian model augmented by Non-Ricardian consumers (Gali et al., 2003). In order to simplify the analysis and highlight the demand-side effects of Spenders’ behavior we do not consider any capital accumulation process. We assume a continuum of infinitely-lived heterogeneous agents normalized to one. Savers are a fraction $1 - \lambda$, they consume and accumulate wealth as in the standard setup. The remaining fraction agents $\lambda$ is instead composed by Spenders who do not own any asset, cannot smooth consumption and thus consume all their current disposable income.

By solving the inter-temporal optimization problems of Savers and Spenders, aggregating and then log-linearizing, we obtain the following description of the demand side of the economy:

\begin{align}
1. & \quad t & = & \lambda \left( t_{i - 1} - \pi_{t + 1} \right) + E_{t+1} \Delta \omega_{t+1}, \\
2. & \quad \omega_{t} = y_{t} + \psi_{t},
\end{align}

Equation (1) is the aggregate consumption, it represents a modified version of the standard consumption Euler equation, where $c_{t}$ is consumption, $i_{t}$ is the nominal interest rate, $\pi_{t}$ is the inflation rate. $\zeta^{N} = (1 + \nu)(1 + \kappa)^{-1}$ is the steady state share of Spenders’ consumption. Our Euler equation differs from the standard one in which the last term of the right hand side of equation (1) is absent. This is due to the presence of the Savers, which establish a link between

\footnote{A large part of the model is rather standard (see e.g. Rotemberg and Woodford, 1997; or Woodford, 2003). Thus here the model is only described in its main equations. The demand side of the economy is derived in more detail in Appendix A since it plays a crucial role. A technical appendix with a full-model derivation is available upon request.}
the demand for goods and the real wage \( \omega_t \) (see equation (2)). The variables \( y_t \) and \( n_t \) are respectively aggregate output and employment, while the parameter \( \nu \) is the inverse of the Frisch labor supply elasticity.

The supply side of the economy is represented by a standard forward-looking Phillips curve and a labor demand: 10

\[
\pi_t = \beta E_t \pi_{t+1} + kx_t + u_t ,
\]

\[
\omega_t = -n_t + y_t
\]

where \( x_t = y_t - a_t \) is the output gap with respect to the flexible-price output, which coincides with the exogenous technology shock, \( a_t \). The variable \( u_t \) is an AR(1) process representing the standard exogenous cost-push shock. Equation (4) is firms’ aggregate labor demand.

By considering the log-linearized production function \( x_t = n_t \), aggregate consumption can be written as

\[
c_t = -(1 - \lambda \zeta_N)(i_t - E_t \pi_{t+1}) + E_t \Delta x_{t+1} - \lambda \zeta_N (1 + \nu) E_t \Delta a_{t+1}.
\]

Current consumption depends on real interest rate (because of the Euler inter-temporal substitution effect) and on the current output level with a income (output-gap growth) elasticity of consumption equal to \( \lambda \zeta_N (1 + \nu) \). If \( \lambda \) is equal to zero, i.e., all consumers may save, the income elasticity of consumption becomes nil and the standard Euler equation holds. Indeed, it is worth noticing that the income elasticity describes the effects of consumers’ expenditure changes due to changes in real wage.

After some more algebra, equation (4) can finally be re-written as:

\[
x_t = E_t x_{t+1} - \Omega (i_t - E_t \pi_{t+1}) + \Omega \Delta a_{t+1},
\]

where \( \Omega = \frac{1 - \lambda \zeta_N}{1 + (1 + \nu) \lambda \zeta_N} \) is the income monetary multiplier, i.e. the semi-elasticity of the real output to the real interest rate. 11

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10 The production function is \( Y_t = A_t N_t \), the labor demand simply equates the real wage to the marginal productivity of labor.

11 It should be noticed that neither the share of Spenders’ consumption nor the Frisch elasticity depends on the fraction of Spenders (see Appendix B).
3. Demand Regimes and Monetary Policy Efficacy

Equation (5) is similar to the standard one proposed by the New-Keynesian literature, the existence of Spenders however affects the impact of interest rate policy on aggregate demand from both a quantitative and a qualitative point of view. According to the sign of the income multiplier, equation (5) individuates two different regimes:

1. A standard regime holds if the income monetary multiplier is positive. Such a regime is dominated by the hypothesis of life-cycle permanent income and thus by the consumption smoothing theory (see e.g. Clarida, et al. 1999).

2. An inverse regime holds if the income monetary multiplier is negative and it is dominated by the liquidity-constraint effect, where an increase in real interest rates is expansionary and interest rate cuts imply contractions since a large part of the consumers cannot access to financial markets and saving.

Aggregate consumption (5) negatively depends on real interest rates but positively on the current output by the aggregate income elasticity of consumption. Thus if the income elasticity of consumption is low, the money multiplier will be negative. By contrast, if the elasticity is high, the multiplier is positive.

The intuition of the result can be found in the labor-market dynamics. Consider an increase of the interest rate that reduces the savers’ demand, it also shifts left the labor demand curve pressing the wage downward (equation (4)). A reduction of the savers labor supply then follows, because of the substitution effect (see equation (2)). The net effect of labor and demand supply on the real wage depends on the extent of the inverse Frish elasticity and the spenders’ fraction. If

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12 Note that employment of Spenders does not rise in a demand-driven boom because we have assumed a logarithmic functional form for the consumer’s instantaneous utility (see also Gali et al., 2003; or Muscatelli et al., 2003). A different form (e.g. constant relative risk aversion) eliminates the inelasticity of Spenders’ labor supply, but does not affect our main conclusions. Although this inelasticity is a drawback of the model, the logarithmic functional form greatly helps to simplify the exposition.
the elasticity and the Savers’ fraction are large, the net effect is negative: a wage reduction follows. The wage reduction further stimulates demand falls because also Non-Ricardian decreases their consumption and further falls in the real wage, so that the standard regime holds. A reverse result holds when the proportion of savers is low: a real wage increase follows (inverse regime).

Formally, the two regimes depend on a threshold value of \( \lambda \), the traditional regime holds for:

\[
\lambda < \lambda^* = \frac{1}{\xi_N(1+\nu)} = \frac{\kappa(1+\kappa)}{(\kappa + \theta)^2},
\]

otherwise we are in the liquidity-constrained regime. The parameter \( \theta = (\eta - 1)\eta^{-1} \in (0,1) \) indicates firms markup, where \( \eta \) is the elasticity of substitution across differentiated products.

For relatively low values of \( \theta \) and high values of \( \kappa \), the threshold value is greater than one (\( \lambda^* > 1 \)). In such a case, only the standard regime occurs since \( \lambda \in [0,1] \).

In other terms the inverse Frish elasticity is smaller then one. For relatively high values of \( \theta \) and low values of \( \kappa \), the liquidity-constrained regime can emerge. In addition, if \( \theta \) is greater than 0.5, \( \lambda^* \) is always smaller than one. Thus, in such a case, the liquidity-constraint regime always holds for a value of \( \lambda \) sufficiently great.

We can now discuss on the efficacy of monetary policy. In the standard regime the efficacy, which is measured by the size of the income monetary multiplier, is increasing in the fraction of spenders. By contrast, in the liquidity-constrained regime, its efficacy is decreasing in \( \lambda \). The efficacy of monetary policy is represented in the figure 1, where the absolute value of income monetary multiplier, \( |\Omega| \), vis-à-vis the fraction of Spenders, \( \lambda \), is plotted.

**Figure 1**

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13 The inverse Frish elasticity and the steady state fraction of spenders depend on the deep parameters: the intermediate sector elasticity of substitution and the labor disutility coefficient.

14 The following condition implies that income elasticity of consumption is smaller than one. It is obtained by considering that the steady state value of \( N \) is \( \theta(\kappa + \theta)^{-1} \) (see the Appendix B).
Notice that without Non-Ricardian consumers, as $\lambda \to 0$, $\Omega \to -1$, thus elasticity of the real output to the real interest rate is minus one, i.e. as in the standard case with logarithmic utility. In such a case, a positive correlation between expected consumption growth and real interest rate is found. As long as $\lambda$ increases the efficacy of monetary policy raises till $\Omega = +\infty$ (the income elasticity of consumption is equal to one). For $\lambda > \lambda^*$ the regime shifts to the liquidity-constrained one (where there is a negative correlation between expected consumption growth and real interest rate), an interest cut affects positively real output and the efficacy of monetary policy is decreasing in the fraction of Spenders, i.e. $\lambda$.\(^{15}\)

It is finally worth noticing that the optimal response of monetary policy to a variation of the natural rate of output, which is equal to a technology shock (see equation (6)), depends on the demand regime. As usual, in the standard regime the central bank should respond by increasing the nominal interest rate of $\Omega$ to offset a unitary shock. By contrast in the liquidity constrained regime it has to reduce the nominal interest rate of an equal amount.

### 4. Taylor Principle and Determinacy\(^{16}\)

#### 4.1. Exogenous Taylor rule

A description of the monetary authority behavior completes the model above-presented. Monetary policy can be described by an exogenous rule, which relates the interest rate to the other variables, or by an endogenous one, directly derived by the solution of an optimization problem, e.g. welfare maximization. One fundamental property which is requested for the monetary authority behavior is to support rational expectation equilibrium determinacy.

Let us start by considering an exogenous Taylor rule as the following:\(^{17}\)

\(^{15}\) Of course, for values of $\theta$ and $\kappa$ implying $\lambda^*>1$, only the first (decreasing) part of the figure is economically relevant.

\(^{16}\) The proofs of determinacy conditions are provided in Appendix C.

\(^{17}\) John Taylor has proposed that U.S. monetary policy in recent years can be described by an
(8) \[ i_t = \alpha_1 \pi_t + \alpha_2 x_t, \]
where \( \alpha_1 \) and \( \alpha_2 \) are both positive.

In the standard regime determinacy requires an *active* policy rule:

(9) \[ a_1 > 1 - \frac{1 - \beta}{k} a_2. \]

The above determinacy condition has a simple usual interpretation. A feedback rule satisfies the Taylor principle if in the event of a sustained increase in the inflation rate by one percentage point, the nominal interest rate will eventually be raised by more than one percentage point. Each percentage point of permanent increase in the inflation rate implies an increase in the long-run average output gap of \( (1 - \beta) k^{-1} \) percent. An exogenous Taylor rule thus conforms to the Taylor principle if and only if its coefficients satisfy \( a_t + (1 - \beta) k^{-1} a_2 > 1 \) (see, among others, Woodford, 2004).

In the liquidity-constrained regime, \( \Omega \) is negative. To simplify the exposition, we redefine it as \( \Omega = -\Omega \), which is a positive measure of monetary policy efficacy. Determinacy thus requires

(10) \[ a_i > \max \left\{ 1 - \frac{1 - \beta}{k} a_2, \frac{2}{\Omega} \right\} \]

(11) \[ a_i < \min \left\{ 1 - \frac{1 - \beta}{k} a_2, \frac{2}{\Omega} \right\} \]

If \( \Omega > \frac{1 - \beta}{k} \), the Taylor principle (9) holds, but the equilibrium is stable also if \( a_i < \min \left\{ \frac{1 + \beta}{k} \left( \frac{2}{\Omega} - a_2 \right), \frac{2}{\Omega} \right\} \). By contrast, if \( \Omega < \frac{1 - \beta}{k} \), \( a_i > \left( \frac{2}{\Omega} - a_2 \right) \frac{1 + \beta}{k} - 1 \) or \( a_i < \min \left\{ \frac{1 + \beta}{k} - \frac{a_2}{1 - \frac{1}{k} a_2}, 1 - \frac{1 - \beta}{k} a_2 \right\} \) is requested.

Summarizing, in the standard regime, the Taylor principle is the necessary and sufficient condition for determinacy. In the liquidity-constrained regime, we have to consider two cases.
a) If monetary policy has a relative high efficacy ($\Omega > (1 + \beta)k^{-1}$), the Taylor principle is only a sufficient condition for determinacy since also a (relatively) loose policy leads to the same result.

b) By contrast, if monetary policy has a relatively low efficacy ($\Omega < (1 + \beta)k^{-1}$), the Taylor principle does not lead to determinacy, a sufficient condition for determinacy requires a stronger reaction to inflation or, also in this case, a (relatively) loose policy.

The economic intuition of our results will be clearer after describing the case on an endogenous-Taylor rule and the monetary policy transmission mechanism in the liquidity-constrained regime.

4.2. Endogenous-Taylor rule

The monetary policy procedures consistent with loss minimization may be often represented as forward-looking relations between interest rate and expected inflation. Formally, in such a case, the central bank should follow an optimal path for the nominal interest rate satisfying:

$$i_t = \alpha_s E\pi_{t+1},$$

where the coefficient $\alpha_s$ is determined by the monetary policy regime where the central bank acts and the parameters of the central bank loss. Equation (11) is usually derived from the solution of an optimization problem and thus represents an endogenous (forward-looking) Taylor rule since a Taylor rule of the standard form can be easily derived from it. By using equation (3), the forward-looking Taylor rule can be re-written in the form of equation (7), where $\alpha_s = \frac{1}{h} \alpha_3$ and

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18 More in detail, equation (11) is derived from the so-called flexible inflation targeting approach (Svensson, 1999, 2003) under different monetary policy regimes (i.e. discretion, commitment or timeless perspective). It can be also seen as the results of the utility-based welfare maximization (Woodford, 2003: Ch. 6). However, to generalize our results to such a case one should show that the central bank’s loss parameters (and thus $\alpha_s$) are independent of the Spenders fraction. An analysis of the utility based welfare criterion is beyond the scope of the present paper thus we stick us to the interpretation of equation (11) as an optimal policy derived from an exogenous loss as e.g. Evans and Honkapohja (2005), i.e. flexible inflation targeting approach. We derive the optimal policies in the next section.
\( \alpha_2 = -\frac{1}{k} \alpha_3 \). Determinacy can be easily studied according to the lines of the previous sub-section. However, note that now either \( \alpha_1 \) or \( \alpha_2 \) is negative according to the sign of \( \alpha_3 \).

In the standard regime, determinacy requires:

\[
\alpha_3 \in \left( 1, 1 + 2 \frac{1-\beta}{k\Omega} \right).
\]

Equation (12) is standard and nests the Taylor principle: monetary policy should respond more than one-to-one to increases in inflation, and should also not be too aggressive as noticed by Bernake and Woodford (1997).

In the liquidity-constrained regime, stability requires:

\[
\alpha_3 \in \left( 1 - \frac{2(\beta+1)}{\Omega k}, 1 \right).
\]

Monetary policy has now to be conducted by a sort of inverted Taylor Principle. The central bank should respond less than one-to-one to increases in inflation. However, too loose monetary policies may also lead to indeterminacy. In particular, if monetary policy has relatively high effectiveness, \( \Omega > 2 \frac{\beta+1}{\Omega k} \), indeterminacy may also derives from a loose positive reaction to expected inflation, i.e. \( \alpha_3 < 1 - \frac{2(\beta+1)}{\Omega k} \). The rational of the inverse Taylor principle is straightforward. A positive non-fundamental shock in the expectations reduces the real interest rate; in the liquidity-constrained regime, if monetary policy is passive, it does not lead to the self-fulfillment of expectation since output falls. By contrast if monetary policy is set according to the Taylor principle, the real interest rate will increase as well as output and expectations will be self fulfilled.

The determinacy requirements in the liquidity-constrained regime can be also interpreted in the light of the recent debate on the central bank’s conservativeness and market imperfections begun by Coricelli (2005). He shows that, in the New Keynesian DSGE models, determinacy requires a more conservative central banker as the degree of good market competition increases\footnote{This result contrasts with the finding in static context (see e.g. Coricelli, \textit{et al.} 2000).}. As long as, the existence of rule-of-thumb consumers is interpreted as the result of non-
competitive financial markets, we find the same result of Coricelli (2005), but in a different context. In other words, market imperfections (financial market in our case) call for a less conservative central banker than under competitive ones.

Figure 3 synthesizes the above results in the parameter space, panel (a) ((b)) refers to a relatively low (high) fraction of Non-Ricardian consumers. In the standard regime, (white area) the Taylor Principle always holds. In the liquidity constraint regime we must distinguish between two type of monetary policy effectiveness: a relatively low effectiveness (dark area) and a relatively high one (light area). In the dark area, although an inverted Taylor principle holds, monetary policy leads to determinacy. By contrast, in the light area, even if an inverted Taylor principle still holds a too loose monetary policy leads to indeterminacy.

Figure 2

In the standard regime, if the policy rule is not active, a non-fundamental increase in expected inflation generates an increase in the current output gap and, by the current Phillips curve, inflation increases, validating the initial non-fundamental expectation. The Taylor principle is needed to guarantee determinacy since an active rule generates a fall in output gap and thus in actual inflation, contradicting initial expectations. By contrast, in a liquidity-constrained regime, if the policy rule is active, a non-fundamental increase in expected inflation generates an increase in the current output gap and an increase in inflation (by the Phillips curve), validating the initial non-fundamental expectation. Thus, in such a regime, the Taylor principle leads to indeterminacy, instead a passive policy rule is requested. In fact, if the central bank follows a passive policy rule, a non-fundamental increase in expected inflation is associated with a fall in the real interest rate, a fall in the output gap, and deflation, contradicting the initial expectation that are hence not self-fulfilling.

4.3. Optimal monetary policy (flexible-inflation targeting)

In this section we discuss the determinacy of optimal monetary policies in presence of Non-Ricardian consumers more in detail by using previous results.

\(^{20}\) See Mankiw (2000) for a brief discussion on the different interpretations about the assumption of rule-of-thumb consumers.
We assume that the monetary authority controls the nominal interest rate to minimize an exogenous loss defined over inflation and output gap, i.e. flexible inflation targeting approach (Evans and Honkapohja, 2005):

\[ W = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( \pi_{t+1}^2 + \alpha x_{t+1}^2 \right), \]

where \( \beta \in (0,1) \) is the discount factor and \( \alpha > 0 \) measures the relative weight the central bank gives to output gap stabilization with respect to inflation.

We consider three different common regimes of monetary policy conduct: discretion, restricted commitment, full commitment, which are well-known (see e.g. Clarida et al., 1999) for more details on their interpretations and the rules derivations). According to the monetary regime, the following policy rule results to be optimal:

\[ i^* = \alpha E_i \pi_{t+1} + r^e \]

where \( r^e = \Delta a_{t+1} \) is the natural real interest rate (i.e. the rate corresponding to the flexible-price equilibrium) and \( D, C, T \) respectively stand for discretion, full commitment, and constrained commitment.

The coefficients of monetary policy reaction to expected inflation are:

\[ \alpha_D = 1 + \frac{k(1 - \rho^u)}{\alpha \rho^u \Omega}, \]

\[ \alpha_C = 1 + \frac{k}{\alpha \rho^u \Omega}, \]

\[ \alpha_T = 1 - \frac{k}{\alpha \Omega} \]

By using the results exposed in the previous section, determinacy of the rules, expressed by equation (16), can be easily investigated.

In the standard regime usual results hold, i.e. the discretion and restricted commitment always lead to determinacy, by contrast the full commitment leads to indeterminacy. We do not discuss them further.

In the liquidity-constrained regime, although full commitment always satisfies the Taylor principle (\( \alpha_T > 1 \)), it is always associated with indeterminacy. By contrast,
in the case of discretion and restricted commitment, despite the Taylor principle is never satisfied, the equilibrium can be stable. Conditions for determinacy are easily found by using equations (14) with (17) and (19):

\[
(20) \quad \alpha \geq z_1 = \frac{k^2 (1 - \rho \alpha)}{2(\beta + 1) \rho \alpha},
\]

\[
(21) \quad \alpha \geq z_2 = \frac{k^2}{2(\beta + 1) \rho}. \]

Equation (20) refers to the discretion whereas equation (21) refers to restricted commitment. Since \( z_1 < z_2 \), the upper limit to conservatism of the central bank is larger in the discretionary regime.

Our results are summarized in the following table.

\textit{Table 1}

The table illustrates the determinacy requirements and the Taylor principle satisfaction for each combination of monetary and demand regimes. The first two columns describe the standard case where the Taylor principle is necessary for determinacy and the full commitment is indeterminate since it does not satisfy the principle. The last two columns describe the liquidity-constrained regime. Now results are reversed since the Taylor is sufficient condition for indeterminacy. The full commitment is indeterminate since now it always satisfies the Taylor principle, it leads to self-fulfilling fluctuations in output and inflation which involves indeterminacy. The discretionary and restricted commitment regimes can be determinate, if monetary policy is not too conservative.

\section{5. The Taylor Principle and the Monetary Policy Transmission}

The implications of liquidity constraints on the Taylor Principle can be better understood by considering the monetary policy transmission mechanism. A key role in monetary transmission is in fact played by consumers’ heterogeneity, which affects monetary policy via consumers’ different choices.
For instance, consider a cost-push shock that increases inflation. Real interest rate decreases on impact. Without liquidity constraints, decreased interest rate will increase output-gap, and thus inflation. If monetary policy does not intervene we can expect self-full filling inflation. Given that prices increase real wage decreases, therefore Spenders’ consumption falls and output declines. If the proportion of Spenders is high the output decline (via Spenders) more than compensates the output increase (via Savers), so that output gap will finally decline and a unique equilibrium is compatible in spite of a lower real rate. This means that the Taylor principle is not always a necessary condition for the determinacy. More importantly this monetary policy transmission mechanism justifies the positive sloped IS-curve which may hold for high proportion of Spenders. If instead the proportion of Savers is low, a lower real interest rate is not sufficient for equilibrium determinacy and the standard Taylor principle must be respected.

6. Conclusions

This paper introduces consumers’ heterogeneity into a DSGE New Keynesian model. We find that the existence of consumers who cannot access to financial markets (Spenders) can explain the negative correlation between expected consumption growth and real interest rate often found in the empirical literature. More in detail, by an analytical investigation, we individuate two different demand-policy regimes characterized by different signs in the slope of the IS curve. In fact, a high proportion Spenders can be compatible with a unconventional positive-sloped IS-curve (liquidity-constrained regime). By considering the liquidity-constrained agents, we find against conventional wisdom that if the slope of the IS is negative, monetary policy effectiveness increases in the fraction of Spenders. In fact, although a smaller fraction of Savers reduces the effects of interest rate policy on the inter-temporal allocation of consumption, the greater fraction of Spenders increases the effects of monetary
policy by the variations in Spenders’ consumption induced by real wage changes. By contrast, in the liquidity-constrained regime, the reverse effect holds. The IS slope also affects the determinacy property of the rational expectation equilibrium. As long as a positive correlation between expected consumption growth and real interest rate is not observed standard results hold. Otherwise determinacy may be guaranteed by a passive monetary policy and the standard Taylor principle can be denied.

More in details, in the liquidity-constrained regime, results on determinacy can be summarized as follows.

1. If monetary policy is set according to a standard Taylor rule, the Taylor principle is only a sufficient condition for determinacy when monetary policy is relatively effective whereas a more aggressive central bank is needed if the monetary policy has a (relative) low efficacy. However, irrespectively of the policy efficacy, determinacy can also be achieved by a relative (loose) policy, which clearly does not satisfy the Taylor principle.

2. If the central bank supports an (optimal) dynamic relationship between output and expected inflation, determinacy requires that central bank should react less aggressively (not satisfying the Taylor principle), but not too loosely, since, in such a case, a non-fundamental increase in expected inflation needs an higher interest rate to be not self-fulfilling.

Finally, we want to stress that our results are closely related to the empirical verification of the relevance of the liquidity-constrained regime, i.e. negative correlation between expected consumption growth and real interest rate, which is however outside the scope of this paper. If the liquidity-constrained regime matters, determinacy needs to be studied with more attention and, in setting their policies, monetary authorities must take into account of the regime where they are since a good policy for a regime can be explosive in the another one. A possible additional factor explaining the explosion of bubbles in emerging markets could be related to attempt of managing the monetary policy according to rules designed for developed financial markets in economy where the financial market were not
fully developed. This provocative reflection however is rather preliminary and need of more empirical verifications.

Appendix A – The demand side

Representative consumers are indexed by $R$ (Ricardian) and $N$ (Non-Ricardian), they maximize the following utility functions:

\[
E_i \sum_{t=0}^{\infty} \beta^t u \left( C_{i+1}, \frac{M_{i+1}}{P_{i+1}}, N_{i+1}, \phi \right) \quad j \in \{R, N\}
\]

where $\beta \in (0,1)$ is the discount factor, $C_i$ represents household consumption at time $t$, while $\frac{M_{i+1}}{P_{i+1}}, N_i$ are respectively, real money balances, and labor. $\phi$ is a binary variable such that when $j = R$, $\phi = 1$ and when $j = N$, $\phi = 0$. We assume the following logarithmic instantaneous utilities,

\[
u(\cdot) = \ln C_{i+1} \kappa \ln (1 - N_{i+1}) + \phi \ln \left( \frac{M_{i+1}}{P_{i+1}} \right) \text{ with } \chi > 0 \text{ and } \kappa > 0.
\]

By solving their optimization problems, consumers face the budget constraints:

\[
C_i = \frac{W_i}{P_i} N_i + \phi \left[ \Pi_i + TR_i - \frac{M_i - M_{i-1}}{P_i} - \frac{B_i - (1 + i_{i-1})B_{i-1}}{P_i} \right], \quad \text{where } W_i \text{ is the nominal wage at time } t, \Pi_i \text{ is profit sharing, } TR_i \text{ are Government lump-sum transfer Note that real wages are the only source of fluctuations of Non-Ricardian disposable income and therefore they are subject to a static budget constraint, while savers (Ricardian consumers) are the only ones facing a dynamic constraint. In fact, since spenders do not save they consume all their current income and the amount of money they hold at the end of period } t \text{ is equal to zero.}

By solving the Ricardian and Non-Ricardian representative consumers’ maximization problems, we obtain the following first-order conditions:

\[
C^R_i = \left[ \beta (1 + i) P_i \right]^{-1} E_i \left[ P_{i+1} C^R_{i+1} \right]
\]

\[
C^N_i = \frac{W_i}{P_i} N^N_i
\]
Equations (a.2) and (a.3) are the optimal consumption for Ricardian (i.e. inter-temporal stochastic consumption Euler equation) and Non-Ricardian consumers (who consume the whole labor income). Equation (a.3) is the optimal demand for real money balances for Ricardian consumers. Equation (a.4) is; the optimal condition for the labor supply. From equations (a.4) and (a.5), it is easy to find that Non-Ricardian consumers supply a fixed quantity of labor, i.e. $N_i^N = \frac{1}{1 - \kappa}$.

The aggregate consumption and employment are

\begin{equation}
(a.6) \quad C_i = (1 - \lambda) C_i^R + \lambda C_i^N
\end{equation}

\begin{equation}
(a.7) \quad N_i = (1 - \lambda) N_i^R + \lambda N_i^N
\end{equation}

From equations (a.5) and (a.7), we obtain the wage aggregate supply:

\begin{equation}
(a.8) \quad C_i = \frac{1}{\kappa} \frac{W_i}{P_i} \{1 - N_i\}
\end{equation}

By log-linearizing equation (a.8) we obtain equation (2), recall that $Y_i = C_i$ in equilibrium. By log-linearizing equations (a.2) and (a.3) we find:

\begin{equation}
(a.10) \quad c_i = (1 - \lambda) \zeta c_i^R + \lambda \zeta c_i^N
\end{equation}

\begin{equation}
(a.11) \quad c_i^R = (i_t - E_i \pi_{t+1}) + E_i c_i^R_{t+1}
\end{equation}

\begin{equation}
(a.12) \quad c_i^N = w_i - p_t
\end{equation}

Solving equation (a.11) for $c_i^R$ and using equations (a.10) and (a.12) we obtain equation (1).
Appendix B – Demand Regimes

This appendix shows the independence between the income monetary multiplier and the fraction of rule-of-thumb consumers. We need to relate the fraction of steady state fraction of Non-Ricardian consumption and the inverse Frisch elasticity only to deep parameters.

Regarding the former, from the demand side of the economy, i.e. equations (a.3) and (a.8), we obtain $\zeta^N = C^N C^{-1} = (1 + \nu) \kappa (1 + \kappa)^{-1}$, recall that Ricardian consumers supply a fixed amount of labor.

To find the steady state value of the employment, we introduce the supply side of the economy, but since it is rather standard we will briefly discuss it (a technical appendix is available upon request). As usual, we consider an economy composed by a continuum of firms (indexed by $z \in [0,1]$ ) producing differentiated intermediate goods with a constant return to scale technology $Y_i(z) = A_i N_i(z)$.

Intermediate goods are used as inputs by a perfectly competitive final goods firm. In such a context, under flexible prices, all firms set their price equal to a constant markup over marginal cost, which, under the hypothesis of symmetric firms, is constant and given by

\[(b.1) \quad \theta = (\eta - 1) \eta^{-1}.\]

Moreover, given the constant return to scale technology and the aggregate nature of shocks, real marginal costs are the same across the symmetric intermediate good producing firms. Accordingly, from the cost minimization, real marginal cost is:

\[(b.2) \quad \theta_i = A_i W_i P^{-1}.\]

By equating equations (a.8) and (b.2), we obtain that in the steady state:

\[(b.3) \quad N = \theta (\kappa + \theta)^{-1},\]

which is independent of the fraction of Spenders.

Figure 1 describes the relationship between the two regimes and the parameters $\theta$ and $\kappa$. 

20
Panel (a) illustrates the two regimes. The white area is the standard one, whereas the dark area is the liquidity-constrained regime. As claimed, for combinations of relatively low values of \( \theta \) and high values of \( \kappa \), i.e. points on the left of the black curve in panel (b) only the standard regime holds. The left curve represents the combinations of \( \theta \) and \( \kappa \) corresponding to \( \lambda^* = 1 \) (the upper-bound). Flatter curves correspond to decreasing values of \( \lambda^* \). The liquidity-constraint regime is then more likely to be observed for relative high values of \( \theta \) and \( \lambda \), and relative low values of \( \kappa \).

**Appendix C – Determinacy**

Determinacy is studied by augmenting the log-linearized dynamic system (3)-(5) with a simple feedback rule (8), which also nests the simple case of the forward-looking Taylor rule of the form (12). From equations (8), (3), and (5), we obtain:

\[
\begin{bmatrix}
1 & \Omega \\
0 & \beta
\end{bmatrix}
\begin{bmatrix}
y_{t+1} \\
\pi_{t+1}
\end{bmatrix}
=
\begin{bmatrix}
1 + \Omega a_2 & \Omega a_t \\
-k & 1
\end{bmatrix}
\begin{bmatrix}
y_t \\
\pi_t
\end{bmatrix}
\]

Stability depends on the eigen-structure of the following matrix:

\[
M = \begin{bmatrix}
1 & \Omega \\
0 & \beta
\end{bmatrix}^{-1}
\begin{bmatrix}
1 + \Omega a_2 & \Omega a_t \\
-k & 1
\end{bmatrix}
= \begin{bmatrix}
1 + \Omega \left( a_2 + \frac{k}{\beta} \right) & \Omega \left( a_t - \frac{1}{\beta} \right) \\
-k & 1
\end{bmatrix}
\]

By indicating with \( D(.) \) and \( T(.) \) the determinant and trace operators, we have:

\[
\begin{aligned}
D(M) &= \beta^{-1} + \Omega (a_2 + ka_t) \beta^{-1} \\
T(M) &= 1 + a_2 \Omega + (1 + k \Omega) \beta^{-1}
\end{aligned}
\]

The eigen-structure of matrix \( M \) is studied by following Woodford (2003: Appendices to Chapter 4). Since the analysis of the standard one does not differs from Woodford (2003), we only consider the liquidity-constrained regime. In this case:

\[\text{In order to investigate the stability properties we do not need to look at the stochastic part that thus is omitted for the sake of brevity. We assume stationary disturbance processes.}\]
regime, determinacy requires either: i) $D(M) > 1$, i.e. $a_1 < \left[(1 - \beta)\Omega^{-1} - a_2\right]k^{-1}$, $D(M) \pm T(M) + 1 > 0$ or ii) $D(M_i) \pm T(M_i) + 1 < 0$. 

Being:

(c.4) \[ D(M) + T(M) + 1 = \left\{2(1 + \beta) - \bar{\Omega}\left[(1 + \beta)a_2 + (1 + a_1)k\right]\right\} \beta^{-1} \]

(c.5) \[ D(M) - T(M) + 1 = -\bar{\Omega}\left[(1 - \beta)a_2 + k(a_i - 1)\right] \beta^{-1} \]

from equations (c.4) and (c.5) we derive conditions (10) and (11), respectively. Moreover, by considering a rule (12), it is easy to verify that $D(M) = \beta^{-1} > 1$, thus stability requires $D(M_i) \pm T(M_i) > -1$ and $D(M_i) \pm T(M_i) < -1$. By considering $\alpha_1 = \beta^{-1} \alpha_3$ and $\alpha_2 = -\beta^{-1} k \alpha_3$, it is easy to verify that $D(M_i) \pm T(M_i) < -1$ is never satisfied. By contrast, $D(M_i) \pm T(M_i) > -1$ requires condition (14).

References


Figures

(a)

(b)

Figure 1a

Figure 1
Table 1 – Demand and monetary regimes, Determinacy & Taylor principle

<table>
<thead>
<tr>
<th>Monetary regimes</th>
<th>Demand regimes</th>
<th>Standard Determinacy</th>
<th>Taylor</th>
<th>Liquidity-constrained Determinacy</th>
<th>Taylor</th>
</tr>
</thead>
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<td>Discretion</td>
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<td>always</td>
<td>(\alpha_{i} &gt; \frac{k^2(1-\rho')}{2(\beta+1)^2\rho^2})</td>
<td>never</td>
</tr>
<tr>
<td></td>
<td>Constrained Commitment</td>
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<td>always</td>
<td>(\alpha_{i} &gt; \frac{k^2}{2(\beta+1)^2\rho^2})</td>
<td>never</td>
</tr>
<tr>
<td></td>
<td>Full Commitment</td>
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<td>never</td>
<td>always</td>
</tr>
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</table>

![Figure 2](image-url)
Technical Appendix (Not to be published)

1. The demand side

We consider a standard New Keynesian dynamic stochastic general equilibrium model augmented by rule-of-thumb consumers a là Galí et al. (2004). We assume a continuum of infinitely-lived heterogeneous agents normalized to one. A fraction $1 - \lambda$ of them consumes and accumulates wealth as in the standard setup (savers). The remaining fraction $\lambda$ is composed by agents who do not own any asset, cannot smooth consumption, and therefore, consume all their current disposable income (spenders).

Formally, representative consumers are indexed by $R$ (savers) and $N$ (spenders), they maximize the following utility functions:

$$E_t \sum_{i=0}^{\infty} \beta^t u\left(C_i^j, \frac{M_i^j}{P_i^j}, N_i^j, \phi_i^j\right) \quad j \in \{R, N\}$$

where $\beta \in (0,1)$ is the discount factor, $C_i$ represents household consumption at time $t$, while $\frac{M_i^j}{P_i^j}, N_i^j$ are respectively, real money balances, and labor. $\phi_i^j$ is a binary variable such that when $j = R$, $\phi^R = 1$ and when $j = N$, $\phi^N = 0$. We assume the following logarithmic instantaneous utilities,

$$u(\cdot) = \ln C_i^j + \kappa \ln \left(1 - N_i^j\right) + \phi_i^j \chi \ln \left(M_i^j P_i^{j-1}\right)$$

with $\chi > 0$ and $\kappa > 0$. By solving their optimization problems, consumers face the budget constraints:

$$C_i^j = \frac{W_i}{P_i^j} N_i^j + \phi_i^j \left[\Pi_i^j - \frac{M_i^j}{P_i} - B_i^j - (1 + \delta) B_i^{j-1}\right],$$

where $W_i$ is the nominal wage at time $t$ and $\Pi_i$ is profit sharing. Note that real wages are the only source of fluctuations of spenders’ disposable income and therefore they are subject to a static budget constraint, while savers are the only ones facing a dynamic constraint. In fact, since spenders do not save they
consume all their current income and the amount of money they hold at the end of period is equal to zero.\textsuperscript{22}

By solving the representative saver’s and spender’s maximization problem, we obtain the following first-order conditions:

\begin{align*}
\text{(24)} & \quad C_i^R = \left[ \beta (1 + i) P_t \right]^{-1} E_t \left[ P_{t+1} C_{i+1}^R \right] \\
\text{(25)} & \quad C_i^N = \frac{W_t}{P_t} N_i^N \\
\text{(26)} & \quad \left( P_t C_i^R \right)^{-1} = \beta E_t \left[ P_{t+1} C_{i+1}^R \right]^{-1} + \chi P_t \left( M_i^R \right)^{-1} \\
\text{(27)} & \quad W_t P_t^{-1} = \kappa C_i^j \left( 1 - N_i^j \right)^{-1} \quad j \in \{ R, N \}
\end{align*}

Equations (24) and (25) are the optimal consumption for savers (i.e. inter-temporal stochastic consumption Euler equation) and spenders (who consume the whole labor income). Equation (26) is the optimal demand for real money balances for savers. Equation (27) is the optimal condition for the labor supply. From equations (26) and (27), it is easy to find that spenders supply a fixed quantity of labor, i.e. $N_i^N = \frac{1}{\kappa \chi} \cdot \text{23}$

By combining equations (25) and (27)

\begin{align*}
\text{(28)} & \quad C_i^N = \frac{1}{1 + \kappa} \frac{W_t}{P_t} = N_i^N \frac{W_t}{P_t}
\end{align*}

Equation (28) implies that spenders consume the whole labor income.

The aggregate consumption and employment are

\begin{align*}
\text{(29)} & \quad C_i = (1 - \lambda) C_i^R + \lambda C_i^N \\
\text{(30)} & \quad N_i = (1 - \lambda) N_i^R + \lambda N_i^N
\end{align*}

\textsuperscript{22} As usual the rule-of-thumb hypothesis is introduced as an \textit{ad hoc} assumption. Here, for the sake of simplicity, we also assume that the savers are those that own the firms and demand money. Different assumptions may have different implication on short and long run income redistribution but do not affect our results.

\textsuperscript{23} Note that employment of spenders does not rise in a demand-driven boom because we have assumed a logarithmic functional form for the consumer’s instantaneous utility (see also Gali et al., 2003; or Muscatelli et al., 2003). A different form (e.g. constant relative risk aversion) eliminates the inelasticity of spenders’ labor supply, but does not affect our main conclusions. Although this inelasticity is a drawback of the model, the logarithmic functional form greatly helps to simplify the exposition.
From aggregate equations (29)-(30) and (27), we obtain the aggregate labor supply:

\[
\frac{W_t}{P_t} = \frac{\kappa}{1 - N_t} C_t
\]

which states that the real wage is equal to the marginal rate of substitution between leisure and consumption.

2. The supply side

2.1 The final sector

The supply side of the economy is composed by a continuum of firms producing differentiated intermediate goods, which are used as inputs by a perfectly competitive final goods firm.

As usual, final good is produced by a perfectly competitive firm with the following technology:

\[
Y_t = \int_0^1 Y_t(z)^{\frac{\varphi}{\theta}} dz
\]

where \( Y_t(z) \) is the quantity of intermediate good \( z \) used as input by the representative firm.

The profit maximization yields the following set of demands for intermediate goods:

\[
Y_i(z) = \left( \frac{P(z)}{P_i} \right)^{\theta} Y_t
\]

while from the zero profit condition we have:

\[
P_t = \left[ \int_0^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.
\]

2.2 The intermediate sector

Each intermediate goods firm is indifferent between the two types of households and produces output with a constant return to scale technology in a homogeneous labor input, \( N_i(z) \), as follows:

\[
Y_i(z) = A_i N_i(z)
\]

where \( A_i \) is an exogenous technology shock.
Given the constant return to scale technology and the aggregate nature of shocks, real marginal costs are the same across the symmetric intermediate good producing firms. Accordingly, from the cost minimization, we obtain the following labor demand:

\[
\frac{W_t}{P_t} = A_t \Phi_t
\]

where \( W_t \) is the same across households.

2.3 Staggered prices adjustment

Following Calvo’s setup firms adjust their price following a Bernoulli distribution. In each period firms have a constant probability, \((1 - \phi)\), to adjust their price, and a probability equal to \(\phi\) to keep their price fixed. The time elapsing between every adjustment follows a geometric distribution, so that the expected waiting time for the next price adjustment is equal to \( \frac{1}{1 - \phi} \).

Accordingly the problem of the firm changing price at time \( t \) consists of choosing \( P_i(z) \) to maximize the following profits function:

\[
E_t \sum_{i=0}^{\infty} (\phi \beta)^i \Lambda_{t,i+1} \left\{ \left[ \frac{P(z)}{P_t} \right] Y_{t,i+1}(z) - \Phi_t Y_{t,i+1}(z) \right\}
\]

subject to

\[
Y_{t,i+1}(z) = \left[ \frac{P(z)}{P_t} \right]^g Y_{t,i+1}
\]

\[
\Lambda_{t,i+1} = \left( \frac{C_{t+1}}{C_t} \right)
\]

where \( Y_{t,i+1}(z) \) is the firm demand function for its output at time \( t + i \), conditional to its price setting \( i \) periods before, \( P_i(z) \). \( \beta \Lambda_{t,i+1} \) is the effective discount factor between time \( t \) and \( t + i \).

From the first order conditions:

\[
P_i(z) = \theta E_t \sum_{i=0}^{\infty} \omega_{t,i+1} \frac{W_{t+1}}{A_{t+1}}
\]
The optimal price $P_t(z)$ is a markup, $\theta = \frac{\eta - 1}{\eta}$, over a weighted average of expected future nominal marginal costs, where the weights $\omega_{t,i}$ are given the following expression $\omega_{t,i} = \frac{(\phi \beta)^{\lambda_{t,i}} D_{t,i}}{E_t \sum_{i=0}^{\infty} (\phi \beta)^{\lambda_{t,i}} D_{t,i}}$ with $D_{t,i} = \left[ \frac{P_t(z)}{P_t} \right]^{\gamma - \eta} Y_{t,i}$ that indicates the firm’s revenue at time $t + i$ conditional on $P_t(z)$.

In the symmetric equilibrium all adjusting firms finally choose the same price $P_t(z)$ and the same level of output $Y_t(z)$, so that the dynamics of the consumption-based index will be:

$$ (41) \quad P_t = \left[ \varphi P_t^{e^{-\eta}} + (1 - \varphi) P_t(z)^{1-\eta} \right]^{\frac{1}{\eta}}. $$

2.4 The natural rate of output

Under flexible prices all the firms set their prices equal to a constant markup over marginal cost. Given the hypothesis of symmetric equilibrium, real marginal cost is constant and given by:

$$ (42) \quad \Phi_t = \frac{\eta - 1}{\eta} = \theta $$

Combining real marginal cost (36) with the aggregate labor supply $\frac{w_t}{P_t} = \frac{\kappa C_t}{1 - N_t}$ yields:

$$ (43) \quad \Phi_t = \frac{\kappa C_t}{1 - N_t} = \frac{\kappa Y_t}{A_t - Y_t} $$

where we use the definition of the aggregate production function, given by $Y_t = A_t N_t$, and the market clearing condition $Y_t = C_t$.

Combining (42) with (43) and solving for $Y_t$ we obtain the following value for the “natural level of output,” i.e. the flexible-price output:

$$ (44) \quad Y_t^n = \left( 1 + \frac{n \kappa}{\eta - 1} \right)^{-1} A_t. $$

As expected, the “natural level of output” does not depend on monetary policy.

4. The log-linearized economy
The log-linearized model is obtained by considering the aggregate resource constraint \( Y_t = C_t \) and the demand and supply optimality conditions above derived.

Considering the demand side, by log-linearizing equations (24) and (29) we find:

(45) \[ c_t^r = -(i_t - E_r \pi_{t+1}) + E_{r+1}^r \]

(46) \[ c_t = (1 - \lambda) \zeta_R c_t^r + \lambda \zeta_N c_t^N \]

Solving equation (46) for \( c_t^r \) yields:

(47) \[ c_t^r = \frac{c_t - \lambda \zeta_N c_t^N}{(1 - \lambda) \zeta_R} \]

By using (47) and shifting it one period ahead into equation (45), we obtain:

(48) \[ c_t = E_t c_{t+1} - (1 - \lambda) \zeta_R (i_t - E_r \pi_{t+1}) - \lambda \zeta_N E_t \Delta c_{t+1}^N \]

From the spenders’ consumption function (28) we have:

(49) \[ \Delta c_{t+1} = \Delta \left( w_{t+1} - p_{t+1} \right) \]

and thus the aggregate log-linearized Euler equation is

(50) \[ c_t = E_t c_{t+1} - (1 - \lambda) \zeta_R (i_t - E_r \pi_{t+1}) - \lambda \zeta_N E_t \Delta \left( w_{t+1} - p_{t+1} \right) \]

Equation (50) can be rewritten as:

(51) \[ c_t = E_t c_{t+1} - (1 - \lambda \zeta_N) (i_t - E_r \pi_{t+1}) - \lambda \zeta_N E_t \Delta \left( w_{t+1} - p_{t+1} \right) \]

since, from the aggregate equation (29), \( (1 - \lambda) \zeta_R = (1 - \lambda \zeta_N) \) holds.

Equation (51) represents a modified version of the standard Euler equation, where \( i_t \) is the nominal interest rate. Consumption today depends on tomorrow expected consumption and on the real interest rate, but differently from the standard Euler equation, the presence of non-optimizing consumers establishes a link between the demand for goods and the real wage (see Gali et al. 2004; or Muscatelli et al. 2005; for further details).

Notice that \( \zeta_N = (1 + \nu)(1 + \kappa)^{-1} \), where \( \nu = N(1 - N)^{-1} \) is the inverse of the Frisch aggregate labor supply elasticity, which is independent of the spenders’ share; in fact, by equating the supply and demand of labor (eqs. (31) (36)) and
considering the aggregate resource constraint, we obtain \( N_i \left(1 - N_i \right)^{-1} = \Phi_i \kappa^{-1} \) that in the steady state is \( \nu = \theta \kappa^{-1} \) (see equation (42)).

By log-linearizing equation (31) and using the aggregate resource constraint, we obtain the aggregate labor supply:

\[
(52) \quad w_t - p_t = y_t + \nu n_t
\]

Regarding the supply side of the economy, by log-linearizing (40) and using the definition (41), we obtain the familiar New-Keynesian Phillips-curve:

\[
(53) \quad \pi_t = \beta E \pi_{t+1} + \tau \phi
\]

with \( \tau = \frac{1 - \rho \eta (1 - \rho)}{\rho} \). Equation (53) is a forward looking equation for inflation, which links movements of current inflation to contemporaneous movements in real marginal cost and expected inflation.

The log-linearization of the labor market implies:

\[
(54) \quad w_t - p_t = \phi_t + a_t
\]

The equilibrium of the labor market implies:

\[
(55) \quad \phi_t = (1 + \nu) x_t, \quad \phi_t = y_t - a_t + \nu n_t
\]

By considering equations (53) and (55). We obtain the New Keynesian forward relation between inflation and output:

\[
(56) \quad \pi_t = \beta E \pi_{t+1} + \tau \left( y_t - a_t + \nu n_t \right)
\]

The log-linearization of the aggregate production (35) is:

\[
(57) \quad y_t = a_t + n_t
\]

where \( a_t \) is assumed to follow a stationary first-order process \( a_t = \rho^n a_{\cdot t-1} + \hat{a}_t \) with \( \rho^n \in (0,1) \) and \( \hat{a}_t \sim N(0, \sigma_a) \). Log-linearization of the flexible price output (44) leads to \( y_t^n = a_t \), an increase in the technology shock, increases the natural rate of output. Thus:

\[
(58) \quad x_t = y_t - y_t^n = y_t - a_t
\]

Notice that, from (57) and (58), \( x_t = n_t \).

Now we can express the log-linearized form of the model in terms of output gap.

The model can be reduced to two familiar expressions. The IS relationship is
derived from equations (51) and (52), by considering the aggregate resource constraint and the output gap definition (58) and rearranging:

\[(59) \quad x_i = E_t x_{t+1} - \Omega (i_t - E_t \pi_{t+1}) + \Omega \Delta a_{t+1} \]

where \( \Omega = \frac{1 - \lambda \zeta}{1 - (\lambda + \nu) \zeta} \) is the income monetary multiplier.

The AS (Phillips curve) relationship is obtained from equations (56) and (58):

\[(60) \quad \pi_i = \beta E_t \pi_{t+1} + k x_i \]

where \( k = \tau (1 + \nu) \).

**References**
