Real Exchange Rate Dynamics With Endogenous Distribution Services

(Preliminary and Incomplete)

Millan L. B. Mulraine†
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Abstract
This paper demonstrates that a two-country flexible price dynamic general equilibrium (DGE) model driven by exogenous innovations to investment-specific technology and with an endogenous distribution services sector can qualitatively and quantitatively replicate the key dynamic features of the real exchange rate. In doing so, the paper incorporates two established empirical observations: (i) that propagation in the real sector has been an important contributor to movements in the real exchange rate, and (ii) that the large and persistent deviations from the law of one price in tradeable goods across countries can be attributable mostly to the wedge created by distribution costs of these tradeable goods. The evidence presented shows that a model with shocks to investment-specific technology and with an endogenous distribution services sector can account for up to 73% of the relative volatility in the real exchange rate.

Keywords: Distribution costs, Exchange rate dynamics, Investment-specific shocks, Law of one price.

JEL Classification: E31, F31, F41

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†Corresponding address: Department of Economics, University of Toronto, 150 St. George Street, Toronto, ON M5S 3G7, Canada. Tel: 1 (416) 824 8069, Email: millan.mulraine@utoronto.ca
1 Introduction:

In the international macroeconomics literature one discrepancy remains as elusive today as it has been since its emergence thirty years ago. That is, since the beginning of the post-Bretton Wood flexible exchange rate period of 1971, nominal and real exchange rates have shown extreme volatility and persistence, and have been considerably more variable than the underlying economic fundamentals assumed to be associated with their determination. Following the publication of Dornbusch (1976) ‘overshooting’ model\(^1\), the quest of the new open economy macroeconomics literature has been to explain this puzzling behavior of bilateral real exchange rates by means of dynamic general equilibrium monetary models with nominal rigidities in the form of sticky prices.

This literature was further advanced by the monopolistic pricing-to-market model developed by Betts and Devereux (2000), which took account of the well established empirical observations on the pronounced deviations from the law of one price in tradeable goods. This particular approach has been predicated on the assumption that movements in the relative price of non-tradeable goods has no impact on the observed exchange rate volatility, and thus the high and persistent variability in the real exchange rate can be explain solely by movements in the relative price of tradeable goods across countries - which are assumed to be caused by innovations to the money supply. In an attempt to offer quantitative justifications for this approach, Chari et al. (2002) constructs a monetary DGE model to show that a model with price stickiness for at least one period, and high risk aversion can replicate the fluctuations in the real exchange rate between the US and a European aggregate.

The empirical evidence on the dynamic behavior of the real exchange rate over the post-floating period, however, casts doubts on the importance of shocks to the money supply in explaining the deviation in the real exchange rate. Alexius (2005) shows that between 60-90% of the volatility of the real exchange rate can be accounted for by relative productivity shocks, thereby confirming the long held view of a Bassala-Samuelson effect on the real exchange rate. In addition to this study, Carr and Floyd (2002), shows that the dynamic behavior of the US-

\(^1\) In this elegant exposition it was shown that the volatile behavior of the exchange rate was consistent with rational expectation in the presence of price rigidities. The overshooting of the nominal exchange rate beyond its long-run level, therefore, was shown to be the results of the interaction between monetary shocks and sluggish price adjustment.
Canada real exchange rate, for example, can be explained by asymmetric real shocks, and that there appears to be no evidence to support the view that monetary shocks have had any significant impact on the real exchange rate. Despite this evidence, the use of real quantitative general equilibrium models to explain the dynamic behavior of the real exchange rate has been largely absent from the international business cycle literature. Here, we present evidence to support the Bassala-Samuelson proposition by positing a real DGE model with an endogenous distribution sector and in so doing provide an avenue for this strand of the literature to be advanced.

The importance of distribution costs in generating the deviation from the law of one price has been well documented. In a seminal piece, Burstein et al. (2003) shows that distribution costs are very large and account for over 40% of the final price of retail goods in the US and over 60% in Argentina. These costs are related to services such as transportation, insurance, and marketing and distribution services associated with bringing the tradeable consumer good to the consumer, and is akin to the iceberg costs advocated by Obstfeld and Rogoff (2000). Consequently, to generate the deviations from the law of one price, this paper develops an endogenous distribution services sector. This approach departs significantly from the standard literature by treating the distribution services as a productive activity that combines domestic labor and capital input with the imported consumer good to create a new domestic consumer good. Thus, the model endogenously generates the wedge between the price of tradeable goods between the two countries, as observed in the data.

In this paper we develop a stylized two-country DGE model driven solely by exogenous innovations to real factors in the economy to demonstrate that this model can replicate the dynamic behavior observed in the real exchange rate. In particular, we specify a non-monetary DGE model driven by shocks to investment-specific technology and with a localized distribution services sector to show that a stylized model without nominal rigidities can match the stylized facts of the real exchange rate. The inclusion of a distribution service sector enables us to endogenously generate the deviations from the law of one price which has been a key feature of the real exchange rate and thus accounts for the contribution of the volatility in the real exchange rate made by movements in the relative price of tradeable goods.

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2The current approach in the literature has been to assume that each unit of tradeable consumption good requires a fixed unit of distribution services.
The exogenous propagation mechanism considered in this paper is shocks to the efficiency in the production of next period’s capital goods - or investment-specific technology shocks. To take account of this unique measure of productivity innovation investment in machinery and equipment - which is a tradeable good - is directly affected by improved efficiency in their production of next period’s capital stock. This form of innovation was shown by Greenwood et al. (1988, 2000) to account for over 30% of output fluctuation in the postwar US data. Further evidence on the impact of innovations to relative prices on the business cycle behavior of the US economy has been provided by Fisher (2003), who - using more recent data - finds that investment-specific technology shock can accounts for over 50% of business cycle variation in hours worked, compared to only 6% accounted for by shock to total factor productivity. Boileau (2002) also uses this propagation mechanism in a two-country framework to explore its ability to explain the cross-country correlation of output and the terms of trade.\(^3\)

In recent work, Jin and Zeng (2005) develops a flexible price model with distribution costs to show that with reasonably large distribution costs their model can generate highly volatile and persistent real exchange rate. This paper, however, differs from their work in a number of key dimensions. In this paper the innovation considered in this stylized economy comes from shocks to the efficiency of investment goods and not the Solow-neutral total factor productivity shock considered in their work. Secondly, and more importantly, in this paper we consider an endogenous distribution services sector instead of assuming that distribution costs are a fixed proportion of the price of tradeable consumer goods. And finally, in the model presented here, countries trade in not only consumption goods - as considered in their work, but also in the intermediate capital good.

The remainder of the paper proceeds as follows. In Section 2 of this paper the model is presented with its dynamic features discussed in Section 3. In Section 4 the model solution is discussed and the simulation results from the calibration of the model to match the stylized facts of the real exchange rate presented. The paper then concludes in Section 5.

\(^3\)Letendre and Luo (2005) and Mulraine (2005) also use this propagation mechanism in an open economy framework to study the dynamic behavior of the Canadian economy.
2 The Model:

Consider a two-country, multi-sector general equilibrium open economy model in which each economy is comprised of three types of agents; a representative consumer, a representative consumer goods producer, and a representative distribution services firm. The two economies considered are structurally symmetric, as such it will suffice to describe the agents and their activities in the home country, with an asterisk (*) denoting variables associated with the foreign country. These two economies are connected to each other through their trade in intermediate capital goods, consumer goods and financial asset.

2.1 The representative consumer:

Each economy is populated by a large number of infinitely-lived identical agents of mass one who has taste for both domestic $c_{1t}$ and foreign produced consumer goods $c_{1t}^m$. Each period the agent is endowed with a unit supply of labor which is divided among the two competing productive activities - the production of consumer goods $l_{ct}$, and the production of distribution services $l_{dt}$. Labor is considered to be perfectly mobile across sectors but will be completely immobile across the two economies. The representative consumer’s objective will be to maximize their discounted utility function which is expressed by:

$$\max_{c_{1t},c_{1t}^m,k_{t+1},k_{2t},\alpha_{t+1}} \sum_{t=0}^{\infty} \beta^t \log(c_t), \quad 0 < \beta < 1$$

where $c_t$ is the consumption level of the composite consumer good which comprises of home and imported final consumption goods. The composite consumption good $c_t$ is such that:

$$c_t = G(c_{1t}, c_{1t}^m)$$

where we aggregate home produced $c_{1t}$ and imported $c_{1t}^m$ final consumption goods using a constant elasticity of substitution (CES) aggregator function $G(c_{1t}, c_{1t}^m)$ given by:

$$G(c_{1t}, c_{1t}^m) = \left[ \omega_c(c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c} \right]^{\frac{1}{\eta_c}}$$

This functional form of the CES aggregator enables us to characterize the respective preferences for home goods (or home bias) given by $\omega_c$ relative to imported consumption goods,
and the constant elasticity of substitution between home and foreign produced final goods,

$$\sigma_c = \frac{1}{1-\eta_c}.$$

The representative household has access to a perfectly competitive international capital market where they can trade in international financial assets $a_t$ at the endogenously determined risk-free world interest rate $r_t$. The accumulation of this type of financial asset will evolve according to the following process:

$$a_{t+1} = tb_t + (1 + r_t)a_t$$

where $tb_t$ is the trade balance for the given period.

Each period the representative consumer devotes $i_t$ of the total income for that period in the form of investment which is used towards the creation of intermediate capital goods. During this period the economy faces an exogenous innovation $q_t$ to the efficiency of this investment good\footnote{Following the work of Greenwood et al. (2000), this paper espouses the use of capital-embodied technological changes as the main contributor to economic fluctuations in the economy considered. For an elaborate and exhaustive discussion on the issues related to the use of shocks to the efficiency of investment goods as a measure of technological innovation, see Fisher (1999).} in the creation of the total domestic stock of intermediate capital goods given by $m_t = q_t i_t$. This stock to domestic intermediate goods is then divided between the portion used domestically $k_{1t}$ for the production of next period capital stock, and the portion that is exported $k_{1t}^*$. The total domestic gross investment in the capital stock for the current period $x_t$ involves a combination of domestic $k_{1t}$ and foreign (imported) intermediate capital goods $k_{2t}$. Such that:

$$x_t = H(k_{1t}, k_{2t})$$

where we aggregate home and foreign intermediate goods using a constant elasticity of substitution (CES) aggregator function $H(k_{1t}, k_{2t})$, such that:

$$H(k_{1t}, k_{2t}) = \left[\omega_k (k_{1t})^{\eta_k} + (1 - \omega_k)(k_{2t})^{\eta_k}\right]^{\frac{1}{\eta_k}}$$

Similar to the composite consumption goods outlined above, this functional for the intermediate good aggregator characterizes the respective preference for home goods (or home bias) given by $\omega_k$ relative to foreign goods, and the constant elasticity of substitution between home and foreign produced final goods, $\sigma_k = \frac{1}{1-\eta_k}$. 
The representative consumer purchases the output of the consumer goods firm $y_t$ and the output of the output of the distribution goods firm $c^{m}_{1t}$ from the available income for that period. In addition to trade in the intermediate capital goods, the consumer exports domestically produced consumer goods $c^*_t$ and trade in foreign assets $a_t$. As such, the budget constraint for the representative consumer is given by:

$$c_{1t} + p^m_{1t}c^m_{1t} + \frac{1}{q_t} k_{1t} + \frac{p^2_{2t}}{q_t} k_{2t} + tb_t + \left(\frac{\phi a}{2}\right)(a_{t+1} - \bar{a})^2 \leq w_t + r_t k_t \quad (7)$$

Subject to the following function which captures the evolution of the capital stock - net of the capital adjustment costs:

$$k_{t+1} = (1 - \delta)k_t + x_t - \left(\frac{\phi_k}{2}\right)(k_{t+1} - k_t)^2 \quad (8)$$

Where the variable $x_t$ captures the stock of intermediate goods added to the non-depreciated closing stock of capital in period $t$. Capital adjustment costs are generally introduced in models of this type as a mechanism for smoothing the reallocation of wealth between physical capital and the foreign asset holdings in response to temporary differentials in the net rate of return between foreign and domestic assets. As a result, the adjustment cost can be seen as the cost(s) associated with the installation of new machinery and equipment. This cost could be considered to include training, installation fees and the cost of disposing of the old stock of machinery. The essential thing to note here, however, is that this cost will act as a moderating force on the investment decision of domestic firms, thereby eliminating excessive responsiveness in investment decisions to small rate differentials, and to ensure that any adjustment to the capital stock is gradual.

Following Schmitt-Grohé and Uribe (2003), we introduce a convex portfolio adjustment cost given by $\left(\frac{\phi a}{2}\right)(a_{t+1} - \bar{a})^2$ to prevent agents from playing Ponzi-type games, and consequently ensuring stationarity in the behavior of net foreign asset holdings. In this framework, given the symmetric nature of the two economies considered the steady-state value of net foreign asset is assume to be zero, that is $(\bar{a} = 0)$. 


2.2 The representative final goods firm:

The home country produces an internationally tradeable composite consumption commodity whose Cobb-Douglas production technology is given by:

\[ y_t = k_{ct}^{\alpha} l_{ct}^{1-\alpha} \]  

(9)

Where \( k_{ct} \) and \( l_{ct} \) represent the amount of capital and labor services which are allocated to this sector, respectively. Thus, the static profit maximization problem for this representative consumption good producer in the home country is given by:

\[ \max \pi_t = y_t - w_t l_{ct} - r_t k_{ct} \]  

(10)

Note that the constant returns to scale technology for this consumption goods firm necessitates that it makes zero profit.

2.3 The localized distribution sector firm:

As outlined above, the imported consumption good requires the services of the local distribution sector. This representative distribution sector firm provides all the requisite services that are entangled in the movement of the imported consumption goods \( c_{2t} \) from its ‘point of entry’ in the home economy to the ‘point of consumption’ \( c_{m1t} \). These services include transportation, advertising, insurance, warehousing and all other services associated with bringing the imported goods from the border to the retail outlet. The profit maximization problem for this sector is given by:

\[ \max \pi_t^d = p_{mt}^m c_{m1t}^m - w_t l_{dt} - r_t k_{dt} - p_{2t} c_{2t} \]  

(11)

subject to a Cobb-Douglas production function given by:

\[ c_{m1t}^m = [\omega_d(d_t)^{\eta_d} + (1 - \omega_d)(c_{2t})^{\eta_d}]^{\frac{1}{\eta_d}} \]  

(12)

In this regard, the distribution firm takes the imported consumer goods \( c_{2t} \) and augments it with distribution services \( d_{t} \) before making it available at the consumer outlet as \( c_{m1t}^m \). The production function for the distribution service is given by the following function:

\[ d_t = k_{dt}^{\alpha} l_{dt}^{1-\alpha} \]  

(13)
One key feature of this framework is the fact that the distribution cost associated with the movement of the imported goods from the border to domestic consumers in this model $\phi_t^m = p_{1t}^m - p_{2t}$ arises endogenously, a significant departure from the standard literature where these costs are considered to be fixed and exogenously. Note that the distribution firm pays $p_{2t}$ at the border for the imported consumption good and sell it at $p_{1t}^m$ to home consumers. The constant returns to scale technology for the representative distribution services firm ensures that profits are zero. As before, the functional for the imported good aggregator characterizes the respective share of the imported good accounted for by distribution services given by $\omega_d$ relative to the imported good, and the constant elasticity of substitution between distributions services and foreign produced final goods, $\sigma_d = \frac{1}{1-\eta_d}$.

2.4 Stochastic processes:

To close the model setup the propagation mechanisms for the shocks to investment efficiency is described by the following processes:

$$q_{t+1} = \Gamma_q q_t + \xi_q$$

(14)

Where $q = [\ln(q), \ln(q^*)]'$. Here $\Gamma_q$ is the matrix of coefficients, and $\xi_q$ is the vectors of mean zero normal random variables with contemporaneous variance-covariance matrix given by $\Sigma_q$ corresponding to the innovations to investment efficiency.

2.5 Model behavior:

In order to provide intuition into the model’s behavior we shall briefly analyze the main channels of operation present. Recall that the dynamic behavior of this model hinges on two key features incorporated in this framework, namely: (i) the endogenous distribution services sector, and (ii) trade in both consumer and intermediate capital goods. The first of these features provides the natural wedge between the domestic and foreign price of tradeable consumption goods - or the deviation from the law of one price in tradeable goods. This endogenous wedge arises from the fact that imported consumption goods must first be augmented with domestic distribution services before they are consumed by agents in the model.

The role of distribution services in the model is quite simple, but significant. Its existence means that the price of the same commodity across the two countries will differ by the cost of
providing the requisite distribution services. As a consequence of this difference in the price across the two countries for tradeable consumption goods, movements in the price of a unit of distribution service will result in deviation in the relative price of tradeable consumer goods across the two countries, and hence deviations in the real exchange rate via a difference in the aggregate price index of the respective economies. The endogenous nature of this sector means that this wedge will vary with relative prices, and will not be constant as is the standard in the literature. Moreover, because distribution services require the input of labor and capital stock, there is an inevitable trade-off between the production of domestic consumer goods distribution services.

The trade in both producer and consumer goods - with only consumer goods requiring distribution services, provides another key conduit through which any propagation can have divergent impacts across the two economies. Foremost among these will be its impact on the consumption and investment decision of agents in the first period. Because the final consumption $c_t$ and investment $x_t$ good requires a mixed of domestic and foreign produced goods, and with the elasticity of substitution between these goods being constant, there will be opportunities for technology and wealth sharing across the two economies as a result of asymmetric shocks.

For example, consider a unit shock to investment efficiency in the domestic economy relative to the foreign economy. The first-period wealth effect from this innovation will result in increased consumption and investment for domestic agents, while the substitution effect will favor accumulation of investment goods relative to consumption goods. The decrease in consumption will favor the importation of the relatively cheaper foreign-produced consumer goods while the increase in capital will result in the creation of more domestic intermediate capital goods. To finance these additional expenditure, domestic agents will inevitably borrow against the return from the higher capital in the future. Clearly, the ramifications of these two effects cannot be fully analyzed in this stochastic model as they will affect not only the quantities traded, but also the prices of these commodities traded and consequently, the aggregate price level, as a result we shall now turn to the numerical solution to the model.
3 Recursive problem for the decentralized home economy:

Let $V(k, a, q, \lambda)$ be the optimal value function for the representative agent whose assets at the beginning of the period are given by the holding of foreign assets $a$, and the stock of capital $k$. Let the aggregate state of the world facing this representative consumer be described by $\lambda$, where $A(\lambda)$ and $K(\lambda)$ are the aggregate holding of assets, and $P(\lambda)$, $R(\lambda)$, and $W(\lambda)$ are the aggregate vector of prices, and the rental price for capital and labor, respectively. The dynamic problem facing the representative household who takes the aggregate state of the world can be represented by the following Bellman equation:

$$V(a, k, q, \lambda) = \max_{a', c_1, c_2, k_1, k_2, k', l_c, l_d, d_t} \{\log(c) + \beta E[V(a', k', q', \lambda')]\}$$ (15)

subject to:

$$c_1 + p_2 c_2 + \frac{1}{q} k_1 + \frac{p_2}{q^*} k_2 + tb + \left(\frac{\phi_a}{2}\right)(a' - \bar{a})^2 \leq y$$ (16)

$$tb \leq a' - (1 + r)a$$ (17)

$$l_c + l_d \leq 1$$ (18)

$$k_c + k_d \leq k$$ (19)

$$k' \leq (1 - \delta)k + x_t - \left(\frac{\phi_k}{2}\right)(k' - k)^2$$ (20)

$$x_t \leq [\omega_k(k_1)^{\eta_k} + (1 - \omega_k)(k_2)^{\eta_k}] \frac{1}{\eta_k}$$ (21)

$$c \leq [\omega_c(c_1)^{\eta_c} + (1 - \omega_c)(c_2)^{\eta_c}] \frac{1}{\eta_c}$$ (22)

$$c_1^{m} \leq [\omega_d(d)^{\eta_d} + (1 - \omega_d)(c_2)^{\eta_d}] \frac{1}{\eta_d}$$ (23)

$$y \leq k_c^{\alpha} l_c^{\beta - \alpha}$$ (24)

$$d \leq k_d^{\alpha} l_d^{\beta - \alpha}$$ (25)

and

$$q' \leq \rho_q q + \rho_{q,q^*} q^* + \varepsilon$$ (26)
3.1 Definition of competitive equilibrium:

A competitive equilibrium for the decentralized domestic economy is a vector of prices $P(p_1^n, p_2, r, w)$ and allocations $(a', k', c_1, c_2, k_1, k_2, k_c, k_d, l_c, l_d)$ such that:

1. The representative household solves their problem taking the aggregate state of the world $\lambda(.)$ and the vector of aggregate prices $P(.)$ as given, with the optimal allocations to their problem satisfying $a' = A'(\lambda)$, $c_1 = C_1(\lambda)$, $c_1^n = C_1^n(\lambda)$, $k_1 = K_1(\lambda)$, $k_2 = K_2(\lambda)$, and $k' = K'(\lambda)$;

2. Given prices, the allocations $l_c = L(\lambda)$, and $k_c = E_c(\lambda)$, solve the profit maximization problem for the representative consumer goods producer.

3. Given prices, the quantities $c_2 = C_2(\lambda)$, $l_d = L_d(\lambda)$, and $k_d = K_d(\lambda)$ solves the profit maximization problem for the representative distribution services firm.

4. All domestic factors and goods markets clear.

5. The global goods and asset markets clear. Such that:

$$tb + p_2 tb^* = 0$$  \hspace{1cm} (27)

3.2 The real exchange rate:

To assess the dynamic properties of the real exchange rate of this stylized model we shall use two definitions of the real exchange rates. The first of which will be the real exchange associated with consumer prices. Recall that the representative consumer in both economies consumes two types of consumer goods - domestically produced consumer goods and the imported consumer good - given by $c_1t$ and $c_1^n$, respectively, for the domestic consumer, and $c_2t$ and $c_2^n$ for the foreign consumer. The prices associated with these commodities are $p_1t$ (normalized to 1) and $p_1^n$, for the domestic consumer and $p_2t$ and $p_2^n$ for the foreign consumer. As such, given that the CES functional form of the utility function, the CPI for the home country $p_t^c$ and the foreign economy $p_t^{c*}$ are given by:

$$ p_t^c = \left[ \omega_c^{\frac{1}{\eta_c}} + (1 - \omega_c) \frac{1}{1-\eta_c} \left( p_1^n \right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}} $$ \hspace{1cm} (28)

$$ p_t^{c*} = \left[ \omega_c^{\frac{1}{\eta_c}} \left( p_2t \right)^{\frac{\eta_c}{\eta_c-1}} + (1 - \omega_c) \frac{1}{1-\eta_c} \left( p_2^n \right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}} $$ \hspace{1cm} (29)

\footnote{An equivalent definition of the competitive equilibrium will exist for the foreign country.}
If the law of one price for consumer goods were to hold, then we would expect the consumer price of the imported consumption good (produced in the foreign economy, for example) over the two economies should be such that \( \varepsilon_t p_{2t} = p_{21}^m \), where \( \varepsilon_t \) is the representation of the nominal exchange rate. Using this condition in the definition of the real exchange rate, we obtain an expression for the real exchange rate associated with consumer prices as:

\[
rer^c_t = \frac{p_{11}^m p^c_{2t}}{p_{2t} p^c_t}
\]

Similarly, we can derive the real exchange associated with producer prices. Recall that the representative consumer in both economies uses two types of intermediate capital goods in the production of next period’s capital goods - domestically produced investment good and the imported investment good - given by \( k_{1t} \) and \( k_{2t} \), respectively for the home consumer, and \( k^*_2 \) and \( k^*_1 \) for the foreign consumer. The prices associated with these commodities are \( \frac{1}{q_t} \) and \( \frac{p_{2t}}{q_t} \), for the domestic consumer and \( \frac{p_{2t}}{q_t} \) and \( \frac{1}{q_t} \) for the foreign consumer. As such, given that the CES functional form of investment, the PPI for the home country \( p^k_t \) and the foreign economy \( p^k_{1t} \) are given by:

\[
p^k_t = \left[ \frac{1}{\omega_k} \left( \frac{1}{q_t} \right)^{\eta_k} + (1 - \omega_k) \left( \frac{p_{2t}}{q^*_t} \right)^{\eta_k} \right]^{n_k^{-1}} \frac{n_k^{-1}}{\eta_k}
\]

\[
p^k_{1t} = \left[ \frac{1}{\omega_k} \left( \frac{p_{2t}}{q^*_t} \right)^{\eta_k} + (1 - \omega_k) \left( \frac{1}{q_t} \right)^{\eta_k} \right]^{n_k^{-1}} \frac{n_k^{-1}}{\eta_k}
\]

If the law of one price for investment goods were to hold, then we would expect the price of the imported intermediate capital good (produced in the foreign economy, for example) over the two economies should be such that \( \varepsilon_t \left( \frac{p_{2t}}{q_t} \right) = \left( \frac{1}{q_t} \right) \), where \( \varepsilon_t \) is the representation of the nominal exchange rate. Using this condition in the definition of the real exchange rate, we obtain an expression for the real exchange rate associated with producer prices as:

\[
rer^k_t = \frac{1}{q_t} \frac{p^k_{1t}}{p^k_t}
\]

4 Quantitative Assessment:

Having fully specified the model we shall now turn to the quantitative assessment of the model described earlier. Note that for this model specification, the competitive equilibrium allocations do not have closed-form analytical solutions. Thus the model must be solved
numerically. This will be done by finding an approximate solution to the log-linear approximation of the first order conditions about their steady-state values. The approach used here is the method advocated by King et al. (1988) and is similar to that used by Schmitt-Grohé and Uribe (2003). The moments used in this analysis are the percentage standard deviation, the first-order autocorrelation of each variable of interest, and their respective cross-country correlation.

4.1 Parameter Calibration:

Since the two economies considered in this framework are symmetric in all respects - except their experience of the innovations to productivity, all the parameters used have the same values in both economies. Following Backus et al. (1994), the constant discount factor is set at $\beta = 0.96$, corresponding to an annual interest rate of $r^* = 0.04$, and the constant depreciation rate for capital was set at $\delta = 0.10$. For the benchmark model the value of the elasticity of substitution between home and foreign consumer goods, distribution services and imported consumer goods, and home and foreign machinery and equipment were set to $\sigma_c = \sigma_d = \sigma_k = 2.5$, such that $\eta_c = \eta_d = \eta_k = 3/5$, respectively. To determine the share parameters for the domestic consumption goods, investment goods and distribution services given by, $\omega_c$, $\omega_k$ and $\omega_d$, respectively, we follow the calibration procedure of Boileau (2002) who set the steady-state output share of trade in final goods and trade in equipment to 20% and 10%, respectively. In addition to these output shares, following the findings Burstein et al. (2003), we also set the steady-state distribution margin for imported consumer goods equal to 42% - equivalent to the distribution margin for consumer prices in the US. As a result of this calibration procedure, we obtain the respective shares given by $\omega_c = 0.575$, $\omega_k = 0.635$ and $\omega_d = 0.320$.

The value of the elasticity of substitution chosen accords to the “large elasticity” case considered by Backus et al. (1994). This value was chosen as the benchmark case since the higher elasticity has been shown by them to be associated with a lower correlation between the trade balance and the terms of trade. As such, with a higher elasticity of substitution we would expect the price deviation across countries to be lower, given the inverse relationship between the elasticity of substitution and the terms of trade. Consequently, this high value will
provide us with the lower limit on the volatility of the real exchange rate in this framework.\footnote{As a sensitivity check we provide the moments for two other values $\sigma_i = 1$ \& $\sigma_i = 1.5$.}

Following Boileau (2002), the productivity innovation to investment efficiency has a standard deviation of 0.013 and a cross-country correlation of 0.345. The exogenous investment-specific shock process has a autocorrelation of 0.553 and a cross-country feedback of 0.027. As such the matrices for $\Sigma_q$ and $\Gamma_q$ are given by:

$$
\Sigma_q = 10^{-4} \begin{pmatrix} 
1.687 & 0.582 \\
0.582 & 1.687 
\end{pmatrix}, \quad \Gamma_q = \begin{pmatrix} 
0.553 & 0.027 \\
0.027 & 0.553 
\end{pmatrix}
$$

Given the parameter values for the stochastic processes described above, the only remaining parameter to be determined in this model will be $\phi_i$, $i \in \{a, k\}$ - the adjustment costs parameter. This parameter is particularly important since it determines (to a large extent) the investment and net foreign assets response to innovations to the exogenous processes considered. Despite its significance, however, this parameter cannot be determined ex-ante as it has no empirical counterpart. We set its value in a manner that will limit the volatility in investment, while simultaneously ensuring stationarity in all macroeconomic variables. In particular, we set it such that the volatility of investment is equal to its empirical counterpart and to ensure stationarity in the holding of net foreign assets. To this end, we use a value of $\phi_a = 0.36$ and $\phi_k = 1.60$.

### 4.2 Simulation Results

This section is primarily aimed at comparing the performance of the model presented in matching the dynamic properties of the data. To do this, particular attention will be paid to the ability of the model to match key moments in the data. That is, we shall examine the percentage standard deviation (volatility), first-order autocorrelation and cross-country correlation of total consumption, total investment, net export, the CPI-based real exchange rate, and the PPI-based real exchange rate. Further analysis will be conducted by examining the associated dynamics of these variables of interest by studying the impulse responses generated from the propagation considered.
4.3 Benchmark Model:

The requisite unconditional moments generated from the simulation exercises are presented in Table 1 below. Column 2 of the Table 1 refers to the associated moments obtained from Chari et al. (2002). In Columns 5 we present the unconditional moments for the macroeconomic aggregates for the benchmark model, in which case the elasticity of substitution between traded goods are set at \( \sigma_i = 2.5 \). As seen in this column, with the model simulated to match the relative standard deviation of consumption, the model can explain as much as 60% of the volatility of the PPI-based real exchange rate and 25% of the CPI-based real exchange rate. The model also provides autocorrelation statistics for the real exchange rate that are relatively close to those observed in the data.

The benchmark model matches quite reasonably the persistence of consumption and GDP, though it does a better job at matching the high persistence of consumption than it does in matching that of GDP. Despite the appealing performance in matching the relative volatility and persistence of the real exchange rate, consumption and GDP, the model performs very poorly in matching the empirical behavior of investment. As is evident for the table, the model overestimates the relative volatility of investment by a factor of 3 and underestimates its persistence by a factor of 2. This excessive volatility emerges as a consequence of the high elasticity of substitution considered and is a direct consequence of the nature of the productivity innovation considered. The excessive relative volatility seen in investment was also documented by Boileau (2002).

In terms of the cross-correlation of the macroeconomic aggregates, the benchmark model has been able to replicate the positive cross-correlation of GDP, consumption and investment - consistent with the data. This outcome is important since it highlights the fact that this model with trade in equipment avoids the counterfactual negative cross-country correlation in investment and almost-perfect cross-country correlation in consumption - two outcomes that have been the feature of standard international business cycle models. In essence, by incorporating trade in equipment, the innovation to investment technology in the home country is exported to the foreign country who will invariably benefit from this new technology - consistent with the empirical observations. This progress, however, must be tempered by the fact that these statistics are over-estimated by a factor of two, relative to the moments in the
data in the case of consumption and investment.

The dynamic behavior of the variables of interest in the stylized model following a 1% shock to investment-efficiency - with the elasticity of substitution set to 2.5 - are presented in Figure 1 - 3. Here we can ascertain the response of home (Figure 1) and foreign (Figure 2) macroeconomic aggregates, and prices, foreign assets and the trade balance (Figure 3). From these, it becomes apparent that the shock to productivity in domestic investment goods along with trade in these intermediate goods results in an immediate accumulation of investment in capital goods in both countries. The magnitude of the rise in investment in the two countries, however, differs as the home country experiences a higher increase in investment than its trading partner. Consistent with economic theory, the home country substitutes domestic investment goods for foreign produced investment goods, while in the foreign country local investment goods was substituted by the relatively cheaper imported investment goods.

The behavior of total consumption in these two economies is also instructive. As is evident from the graph total consumption falls significantly in the first period before rising above its steady-state level in both countries. Similar to the evidence on the behavior of investment accumulation, the level of consumption fall by a larger amount in the foreign country than in the home economy. In the home country the substitution effect dominated the wealth effect from the higher productivity in the home country as agents substitutes consumption in the initial period of the shock for investment goods which was used to financed the higher consumption (relative to its steady-state level) in subsequent periods. Note that in the home country the greater increase in investment along with a smaller reduction in consumption relative to the foreign country was financed by the accumulation of foreign debts.

Figure 3 provides dynamics for the various prices and the domestic holdings of net foreign assets. As seen, the home country accumulates foreign debts (negative trade balance) which is used to financed its increased investment and consumption over the horizon of the analysis. The dynamic movement of the price indices considered and their associated real exchange rate is consistent with neoclassical theory, with the improvement in investment technology resulting in a reduction in the the producer price index for the home country and consequently a real appreciation of the PPI-based real exchange rate. The opposite movement in the consumer price index is an artifact of the model. That is, consumer goods becoming more expensive relative to investment goods in the home country.
4.4 Sensitivity Analysis:

Following Backus et al. (1994), we vary the value of the elasticity of substitution as a sensitivity check of the behavior of the model. In Column 3 of the Table 1 we provide unconditional moments for the macroeconomic aggregates for an elasticity of substitution equal given by $\sigma_i = 1$, and in Column 4 we provide moments for $\sigma_i = 1.5$. The value for the adjustment cost parameters are the same as in the benchmark model in which case $\phi_k = 1.6$ and $\phi_a = 0.36$. Note also that to maintain the import shares outlined above, the CES shares given by the $\omega_i$’s must also be recalibrated.

The evidence provided in Table 1 shows that the lower the elasticity of substitution the better the performance of the model in matching the relative volatility of the real exchange rates considered. In the case of the “low elasticity” case the model matches over 70% of the relative volatility of the real exchange rate in both case. The model also improves on matching the empirical properties of investment. This outcome is essentially the result of the fact that the lower the elasticity of substitution between the competing goods the more inertia will be seen in the responses to changes in their relative price, and consequently the less volatile will be investment - for example.

One striking anomaly, however, emerges from this analysis. As seen in the Column 3, the “low elasticity” case results in the counterfactual negative cross-correlation of output. This behavior is the result of the fact that the improved productivity in the home country does not translate into higher capital accumulation in the foreign country - as occurred in the higher elasticity cases.

5 Conclusion:

In this paper we have been able to show that a model propagated by shocks to investment-specific technology, and with an endogenous distribution services sector can account for up to 73% of the relative volatility in the real exchange rate. These findings accord with the Bassala-Samuelson proposition, and provides evidence to show that real innovation in the economy can account for a significant portion of the empirical volatility of the real exchange rate. Moreover, by incorporating an endogenous distribution services sector, the model has

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7The impulse response for values for the elasticity of substitution are not shown.
been able to endogenously generate the deviation in the prices of tradeable goods in the two economies considered.

The model specification considered here has been motivated by the evidence on the importance of the distribution services in generating the deviation in the law of one price and the long held view that productivity innovations play an important role in real exchange rate dynamics. It is aimed at providing an avenue for considering the contribution of innovation to productivity in explaining the real exchange rate - a facet of the literature that has been largely missing.

An interesting extension to the framework considered here will be to augment the investment-specific shock model with innovations to total factor productivity, and to compare the performance of a model with these two productivity innovations with those of a model driven by innovations to the money supply.
References


Figure 1: Impulse Response to a Unit I-E Shock in Home Country, $\sigma_i = 2.5$
Figure 2: Impulse Response to a Unit I-E Shock in Foreign Country, $\sigma_i = 2.5$
Figure 3: Impulse Response to a Unit I-E Shock in Prices, $\sigma_i = 2.5$
Table 1: Observed and Simulated Moments for Benchmark Model

<table>
<thead>
<tr>
<th>Relative Volatilities:</th>
<th>Data: $\sigma_i = 1$:</th>
<th>$\sigma_i = 1.5$:</th>
<th>$\sigma_i = 2.5$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.8300</td>
<td>1.7327</td>
<td>1.3040</td>
</tr>
<tr>
<td>Investment</td>
<td>2.7800</td>
<td>4.2707</td>
<td>6.0709</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.1100</td>
<td>0.0026</td>
<td>0.0026</td>
</tr>
<tr>
<td>RER (CPI)</td>
<td>4.3600</td>
<td>3.2720</td>
<td>2.6511</td>
</tr>
<tr>
<td>RER (PPI)</td>
<td>3.9000</td>
<td>2.7780</td>
<td>3.0902</td>
</tr>
</tbody>
</table>

| Serial correlations:  | Output          | 0.8800 | 0.9924 | 0.9790 | 0.9623 |
|                       | Consumption     | 0.8900 | 0.9932 | 0.8878 | 0.9160 |
|                       | Investment      | 0.9100 | 0.7487 | 0.6040 | 0.4858 |
|                       | Trade Balance   | 0.8200 | 0.2115 | 0.1968 | 0.1463 |
|                       | RER (CPI)       | 0.8300 | 0.9833 | 0.6883 | 0.7460 |
|                       | RER (PPI)       | 0.8200 | 0.9442 | 0.5933 | 0.5994 |

| Cross-Country Correlation: | Output          | 0.6600 | -0.4795 | 0.9193 | 0.8725 |
|                            | Consumption     | 0.3800 | 0.1346 | 0.6631 | 0.7093 |
|                            | Investment      | 0.3300 | 0.1395 | 0.8866 | 0.7093 |

**Notes:** The first column refers to data that have been logged and detrended using the HP filter and obtained from Chari et al. (2002). With the exception of the trade balance, the relative volatility statistics refer to the ratio of the standard deviation of the variables of interest divided by the standard deviation of GDP. The standard deviation of the trade balance is simply the standard deviation of the ratio of the trade balance to GDP.
6 Derivations and Proofs:

6.1 The Problem for the Decentralized Domestic Economy

\[
\max_{\{a_{t+1}, c_{1t}, c_{2t}, k_{1t}, k_{2t}, k_{t+1}, k_c, k_{dt}, l_c, l_{dt}, c_{it}^m, y_t, d_t\}} \quad E_t \sum_{t=0}^{\infty} \beta^t \log(c_t), \quad 0 < \beta < 1
\]  

s.t.

\[
c_{1t} + p_2 c_{2t} + \frac{1}{q_t} k_{1t} + \frac{p_2 t}{q_t} k_{2t} + t b_t \leq y_t - \left(\frac{\phi_a}{2}\right)(a_{t+1} - \bar{a})^2
\]  

\[
b_t \leq a_{t+1} - (1 + r_t) a_t
\]  

\[
l_c + l_{dt} \leq 1
\]  

\[
k_c + k_{dt} \leq k_t
\]  

\[
k_{t+1} \leq x_t + (1 - \delta) k_t - \left(\frac{\phi_k}{2}\right)(k_{t+1} - k_t)^2
\]  

\[
x_t \leq \left[\omega_k(k_{1t})^{\eta_k} + (1 - \omega_k)(k_{2t})^{\eta_k}\right]^{1/\eta_k}
\]  

\[
c_t \leq \left[\omega_c(c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c}\right]^{1/\eta_c}
\]  

\[
c_{it}^{m} \leq \left[\omega_d(d_t)^{\eta_d} + (1 - \omega_d)(c_{2t})^{\eta_d}\right]^{1/\eta_d}
\]  

\[
y_t \leq k_{dt}^{\alpha(1 - \alpha)}
\]  

\[
d_t \leq k_{dt}^{\alpha(1 - \alpha)}
\]  

\[
q_{t+1} \leq \rho q_t + \rho q^* q^*_t + \varepsilon_t
\]  

6.2 The FOCs:

Using \(\lambda^c_t, \lambda^l_t, \lambda^k_t, \lambda^q_t, \lambda^m_t, \text{ and } \lambda^d_t\) as the multipliers associated with Eq. 35, 37, 38, 39, 42 and 44 we have the following first order condition with respect to \(a_{t+1}, c_{1t}, c_{1t}^m, c_{2t}, d_t, k_{t+1}, k_{1t}, k_{2t}, k_c, k_{dt}, l_c, \text{ and } l_{dt}\):

\[
\lambda^c_t \left[1 + \phi_a(a_{t+1} - \bar{a})\right] = \beta E_t \lambda^c_{t+1}(1 + r_{t+1})
\]  

\[
\lambda^c_t = \omega_c(c_{1t})^{\eta_c} \left[\omega_c(c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c}\right]^{-1}
\]  

\[
\lambda^m_t = (1 - \omega_c)(c_{1t}^m)^{\eta_c} \left[\omega_c(c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c}\right]^{-1}
\]  

\[
\lambda^p_{2t} = \lambda^m_t (1 - \omega_d)(c_{2t})^{\eta_d} \left[\omega_d(d_t)^{\eta_d} + (1 - \omega_d)(c_{2t})^{\eta_d}\right]^{1-\frac{1}{\eta_d}}
\]  

\[
\lambda^d_t = \lambda^m_t \omega_d(d_t)^{\eta_d} \left[\omega_d(d_t)^{\eta_d} + (1 - \omega_d)(c_{2t})^{\eta_d}\right]^{1-\frac{1}{\eta_d}}
\]
\lambda_t^x[1 + \phi_k(k_{t+1} - k_t)] = \beta E_t\{\lambda_{t+1}^x[(1 - \delta) + \phi_k(k_{t+2} - k_{t+1})] + \lambda_{t+1}^x\} \tag{51}

\lambda_t^x \frac{1}{q_t} = \lambda_t^x \omega_k(k_{1t})^{\eta_k-1}\left[\omega_k(k_{1t})^{\eta_k} + (1 - \omega_k)(k_{2t})^{\eta_k}\right]^{\frac{1-\eta_k}{\eta_k}} \tag{52}

\lambda_t^x \frac{p_{2t}}{q_t^x} = \lambda_t^x (1 - \omega_k)(k_{2t})^{\eta_k-1}\left[\omega_k(k_{1t})^{\eta_k} + (1 - \omega_k)(k_{2t})^{\eta_k}\right]^{\frac{1-\eta_k}{\eta_k}} \tag{53}

\lambda_t^k = \lambda_t^c \frac{y_t}{k_{ct}} \tag{54}

\lambda_t^k = \lambda_t^d \frac{d_t}{k_{dt}} \tag{55}

\lambda_t^l = \lambda_t^c (1 - \alpha) \frac{y_t}{l_{ct}} \tag{56}

\lambda_t^l = \lambda_t^d (1 - \alpha) \frac{d_t}{l_{dt}} \tag{57}

6.3 Deterministic Steady State Conditions:

Symmetry in the two economies ensures that \( p_{2t} = p_{1t} = 1 \), similarly, given that \( q_t = q_t^x = 1 \) in steady-state, we must also have \( t b_t = 0 \) and \( a_t = a_{t+1} = 0 \). In steady-state we have \( \lambda_t^i = \lambda_{t+1}^i = \lambda^i \forall i \), which implies that \( \beta (1 + r) = 1 \).

From Eq. 46, 51, 52 and 54:

\[ r + \delta = \alpha \frac{y_t}{k_c} \omega_k(k_1)^{\eta_k-1}\left[\omega_k(k_1)^{\eta_k} + (1 - \omega_k)(k_2)^{\eta_k}\right]^{\frac{1-\eta_k}{\eta_k}} \tag{58} \]

From Eq. 46, 49, 50, 51, 52 and 56:

\[ r + \delta = \alpha \frac{d_t}{k_d} \left(\frac{\omega_d}{1 - \omega_d}\right) \left(\frac{c_2}{d}\right)^{\eta_d-1}\omega_k(k_1)^{\eta_k-1}\left[\omega_k(k_1)^{\eta_k} + (1 - \omega_k)(k_2)^{\eta_k}\right]^{\frac{1-\eta_k}{\eta_k}} \tag{59} \]

From Eq. 49, 50, 56 and 57:

\[ \left(\frac{d}{y}\right) \left(\frac{l_c}{l_d}\right) = \left(\frac{1 - \omega_d}{\omega_d}\right) \left(\frac{d}{c_2}\right)^{1-\eta_d} \tag{60} \]

From Eq. 52 and 53:

\[ \frac{k_1}{k_2} = \left[\frac{\omega_k}{1 - \omega_k}\right]^{\frac{1}{1-\eta_k}} \tag{61} \]

From Eq. 47, Eq. 48 and 49:

\[ \frac{c_1}{c_1^m} = \left[\left(\frac{1 - \omega_c}{\omega_c}\right)(1 - \omega_d)\left[\omega_d \left(\frac{d}{c_2}\right)^{\eta_d} + (1 - \omega_d)\right]^{\frac{1-\omega_d}{\omega_d}}\right]^{\frac{1}{\eta_c-1}} \tag{62} \]

To complete the deterministic steady-state conditions we have the following constraints:

\[ l_c + l_d = 1 \tag{63} \]
\[ k_c + k_d = k \]  
(64)

\[ \omega_k \left( \frac{k_1}{k_2} \right)^{\eta_k} + (1 - \omega_k) = \left( \frac{\delta k}{k_2} \right)^{\eta_k} \]  
(65)

\[ \omega_d \left( \frac{d}{c_2} \right)^{\eta_d} + (1 - \omega_d) = \left( \frac{c_1}{c_2} \right)^{\eta_d} \]  
(66)

\[ \frac{d}{k_d} = \left( \frac{k_d}{l_d} \right)^{\alpha-1} \]  
(67)

\[ \frac{y}{k_c} = \left( \frac{k_c}{l_c} \right)^{\alpha-1} \]  
(68)

\[ y = c_1 + c_2 + k_1 + k_2 \]  
(69)

To fully calibrate the steady-state we have a system of 12 equations and 12 unknown. The unknowns we need to solve for are: \( y, k_c, k_d, l_c, l_d, k, k_1, k_2, d, c_1, c_2, \) and \( c_1^m \). We use the factor price equalization condition in both the labor and capital markets to obtain \( \frac{d}{c_2} = \left( \frac{\omega_d}{1 - \omega_d} \right)^{\frac{1}{1 - \eta_d}} \) and \( \frac{k_d}{l_d} = k = \frac{k_c}{l_c} \). These two conditions, along with the value of \( \frac{k_1}{k_2} \) can then be used to obtain the remaining variables of interest.